SPUR AND BEVEL GEARING

SPUR AND BEVEL GEARING

A TREATISE ON THE PRINCIPLES, DIMENSIONS, CALCULATION, DESIGN AND STRENGTH OF SPUR AND BEVEL GEARING, TOGETHER WITH CHAPTERS ON SPECIAL TOOTH FORMS AND METHODS OF CUTTING GEAR TEETH

COMPILED AND EDITED
BY
ERIK OBERG, A. S. M. E.
ASSOCIATE EDITOR OF MACHINERY
EDITOR OF MACHINERY'S HANDBOOK
AUTHOR OF "HANDBOOK OF SMALL TOOLS," "SHOP ARITHMETIC FOR THE MACHINIST," "SOLUTION OF TRIANGLES," "STRENGTH OF MATERIALS," "ELEMENTARY ALGEBRA," ETC.

FIRST EDITION
SECOND PRINTING

NEW YORK
THE INDUSTRIAL PRESS
LONDON: THE MACHINERY PUBLISHING CO., LTD.
1915

Composition and Electrotyping by
F. H. Gilson Company, Boston, U.S.A.

PREFACE

EARLY in 1908, MACHINERY began the publication of its well-known 25-cent Reference Books. These include the best of the material that has appeared in MACHINERY during the past twenty years, adequately revised, amplified and brought up-to-date. Of these books, one hundred and twenty-five different titles have been published in the past six years.

Many subjects, however, cannot be covered in all their phases to an adequate extent in books of this size, and in answer to a demand for more comprehensive and detailed treatments on the more important mechanical subjects, it has been deemed advisable to bring out a number of larger volumes, each covering one subject completely. This book is one of these volumes. In bringing out these books, the first consideration on the part of the editors has been to make them meet the practical needs of the machine building trade. Mere theory and academic discussions have been avoided. The rules, formulas and instructions are illustrated with engravings whenever practicable, and examples are given to show their application to every-day problems. Theoretical considerations, however, have not been neglected, when necessary to fully explain a practical process, and the present book on spur and bevel gearing is, therefore, a treatise on both the practice and the theory of gearing, along such lines as will make it especially useful to practical men.

The information contained is mainly compiled from articles published in MACHINERY, and the best on the subject that has appeared in the Reference Books is also included. Amplifications and additions have been made wherever necessary. For the material contained, MACHINERY is indebted to a large number of men who have furnished practical information to its columns in the past. In many instances, it has not been

possible to give credit to each individual contributor, but it should be mentioned that the framework upon which the whole book has been built up, consists of the Reference Books and articles which Mr. Ralph E. Flanders, the well-known gear expert and formerly Associate Editor of MACHINERY, has written and compiled. To all other writers whose material has appeared in MACHINERY and is now used in this book, MACHINERY hereby expresses its appreciation.

MACHINERY

NEW YORK, May, 1914

CONTENTS

Chapter I

PRINCIPLES AND DIMENSIONS OF SPUR GEARING

Pages

Friction Wheels — Toothed Gearing — Involute and Cycloidal Systems — Pressure Angles — Definitions of Gear-tooth Parts — Circular and Diametral Pitch — Pitch Diameter — Center Distance — Clearance — Depth and Thickness of Tooth — Outside Diameter — Number of Teeth — Table of Rules and Formulas — Internal Spur Gears — Grant's Odontograph — Cutters for Milling Gear Teeth — Chordal Thickness and Corrected Addendum — Testing Tooth Thickness — Limits for Gearing — Metric System of Gear Teeth... 1–37

Chapter II

MATERIALS USED FOR GEARS

Cast Iron — Cast Steel — Materials for Medium-sized and Small Gears — Brass and Bronze Gears — Rawhide Gears — Fiber Gears — Cloth Gears — Materials for Racks — Gears for Machine Tool Drives — Safe Tooth Pressures for Cast-iron Gears — Grades of Steel giving Best Results — Heat-treatment of Gears.................................. 38–50

Chapter III

STRENGTH AND DURABILITY OF SPUR GEARING

Strength of Gear Teeth — The Lewis Formula — Relation between Width of Face and Diametral Pitch — Derivation of the Lewis Formula — Tests for Strength of Gear Teeth — Rawhide Gearing — Strength of Rawhide Gears with Flanges — Durability of Gearing — Efficiency of Standard Spur Gears — Variation of the Strength of Gear Teeth with the Velocity — Variation in Strength due to Impact — The Effect of Shocks — Practical Considerations affecting Design....... 51–74

CHAPTER IV

SIMPLIFIED FORMULAS FOR STRENGTH OF GEARS

PAGES

Need for Simplified Formulas — Assumptions on which Formulas are Based — Values of Safe Working Stress — Derivation of Simplified Formulas — Correction for Numbers of Teeth — Correction for Increased Velocity — Pinion Diameters — Summary of Formulas — Chart for Rapid Solution of Gear Problems — Weight of Gears — Price of Gears...... 75–83

CHAPTER V

THE STUB-TOOTH GEAR

Standards for Gear Teeth — Cycloidal and Involute Tooth Forms — Length of Tooth — Angle of Obliquity, Efficiency and Wearing Qualities — Investigation of an Actual Design — Comparison between Ordinary and Stub Gear-teeth — Fellows System of Stub Gear-teeth — Nuttall System of Stub Gear-teeth — Tooth Shape for Rolling Mill Gears — Strength of Stub Gear-teeth — Advantages of Stub-tooth Systems..... 84–107

CHAPTER VI

NOISY GEARING

Causes of Noisy Gearing — Noise due to Interference — Analysis of the Conditions in Gears with Small Number of Teeth — Charts for finding Number of Teeth in Large Gear — Methods for avoiding Interference — The Shortened Addendum — Application to Rack and Pinion — Application to Worm-gearing — Application to Internal Gears — Application to Bevel Gears — Makeshift Methods for avoiding Noise in Gearing.. 108–128

CHAPTER VII

DESIGN OF SPUR GEARING

Small and Medium Sized Gears — Large Gears — Dimensions of Spur Gears — Conditions governing the Design of Gearing — Model Spur Gear Drawing — Dimensions of Gear Drawings — Rack Drawing.......................... 129–135

Chapter VIII

METHODS FOR CUTTING SPUR GEAR TEETH

PAGES

Classification of Gear-cutting Machinery — Formed Tool Principle — Templet Principle — Odontographic Principle — Describing-generating Principle — Molding-generating Principle — Methods of Operation: By Impression; by Shaping or Planing; by Milling; by Grinding or Abrasion — Machines for forming the Teeth of Spur Gears — Gear Hobbing Machines — Designing a Hob for Hobbing Spur Gears — Form and Dimensions of Tooth — Relief — Number of Flutes — Straight or Spiral Flutes — Threading the Hob — Thread Relieving Tool — Heat-treatment of Hob — Interchangeability of Hobbed and Milled Gears — Special Hobs for Gear Teeth — Generating Hob Tooth Shapes — Making a Master Planing Tool for a Hob.................................... 136–166

Chapter IX

PRODUCTION AND HEAT-TREATMENT OF GEARS

Processes in Production of Automobile Transmission Gears — Steels used for Automobile Gears — Gear Blanks — Rough-turning — Annealing — Finish-boring — Broaching — Finish-turning — Cutting the Teeth — Rounding the Ends of the Teeth — The Carbonizing Process — Casehardening and Oil-tempering — Sand-blasting — Hardness Testing — Grinding — Running-in — Heat-treated Gears in Machine Tools — Steel to be used — Advantages of Heat-treated Gears — Water or Oil-hardened Vs. Casehardened Gears — Points in Gear Design — Equipment for Heat-treating Gears — Oil used for Quenching — Cost of Heat-treated Gears — Kinds of Heat-treatments used — Tests on Heat-treated Gears — Device for Testing and Measuring Gears........................ 167–199

Chapter X

BEVEL GEAR RULES AND FORMULAS

Bevel Gear Definitions — Different Kinds of Bevel Gears — Dimensions — Rules and Formulas — Derivation of Formulas — Shafts at Right Angles — Miter Bevel Gearing — Acute-

PAGES

angle Bevel Gearing — Obtuse-angle Bevel Gearing — Crown Gears — Internal Bevel Gears — Examples of Bevel Gear Calculations — How to avoid Internal Bevel Gears — Systems of Tooth Outlines — Involute Teeth — Octoid Teeth — Milling Cutters for Bevel Gears — Special Forms of Bevel Gear Teeth.. 200–223

CHAPTER XI

STRENGTH AND DESIGN OF BEVEL GEARS

Materials used for making Bevel Gears — Strength of Bevel Gear Teeth — Rules and Formulas — Bearing Pressures due to the Action of Bevel Gears under Load — Example of Calculation of Thrust — Design of Bevel Gearing — Bevel Gear Blanks — Model Bevel Gear Drawing — Dimensioning Drawings for Gears with Planed Teeth.......................... 224–242

CHAPTER XII

METHODS FOR FORMING THE TEETH OF BEVEL GEARS

Comparison between Spur and Bevel Gear Cutting — Tredgold's Approximation — Formed Tool Principle — Templet Principle — Odontographic Principle — Molding-generating Principle — Methods of Operation: By Impression; by Shaping or Planing; by Milling; by Grinding or Abrasion — Machines for Cutting the Teeth of Bevel Gears by the Formed Tool Process — Templet Planing Machines — Adjustable Former for Bevel Gear Planing — Bevel Gear Generating Machines.. 243–260

CHAPTER XIII

MILLING THE TEETH IN BEVEL GEARS

Cutting Bevel Gears in a Milling Machine — Setting the Cutter — Upsetting and Rolling the Blank — Trial Cuts — Positive Determination of the Set-over — Filing the Teeth — Testing Bevel Gear Teeth — Cutting Bevel Gears on the Automatic Gear-cutting Machine — Derivation of Positive Set-over Formula — Practicability of Milling Process — Another Positive Method of setting Bevel Gear Cutter........... 261–277

Chapter XIV

LONG AND SHORT ADDENDUM GEARS

Pages

Object of Gears with Lengthened Addendum — Pinions with Small Number of Teeth — Field of Application — Efficiency — Finding Circular Thickness of Tooth — Duplication of Gears having Long and Short Addendum — Correction for Shrinkage in Hardening 278–290

Chapter XV

SKEW BEVEL GEARS

Characteristics of Skew Bevel Gears — Skew Bevel Gear Tooth which can be Produced by the Molding-generating Process — Method of Transmitting Power between Non-parallel, Non-intersecting Shafts — Types of Skew Bevel Gears — Theoretical Analysis of Proposed Tooth Form — Practical Application of Theory 291–300

SPUR AND BEVEL GEARING

CHAPTER I

PRINCIPLES AND DIMENSIONS OF SPUR GEARING

GEAR wheels are such common objects in the machine shop, and are manufactured with such rapidity and ease by the aid of the modern automatic gear cutter, that many seldom stop to think what they really are, why the teeth must be constructed with certain curves, and what it is desired that they shall accomplish. In following chapters we shall take up some of the practical questions, touching upon the calculations that come up in the design, but will here deal chiefly with a few of the theoretical points of the subject that are seldom explained in a simple manner, for the benefit of those who have had neither the time nor the opportunity to look into matters of this kind.

Friction Wheels. — Suppose there are two wheels arranged as in Fig. 1 with their faces in close, frictional contact, and that both are of exactly the same size, so that when the crank is turned around once, wheel *B* will turn exactly once also, provided, of course, there is no slipping between the two wheels. It must be noticed, moreover, that if the crank be turned uniformly, wheel *B* will not only make the correct number of revolutions relative to *A*, but it will revolve uniformly, as well; that is, both its total motion and the motion from point to point will be correct.

Toothed Gearing. — Now, there are many places in machine construction where the slipping inseparable from friction wheels cannot be tolerated, and this difficulty might be overcome by fastening small projections to one of the wheels, as on *A* in Fig. 2, and cutting grooves in the other wheel *B*. Then, if the crank were turned, wheel *B* would always make just the right

number of turns, even if considerable power were transmitted. It is probable, however, that these projections and grooves would not fulfill the purpose of gear teeth. What is wanted of gear teeth is that they shall give exactly the same kind of motion

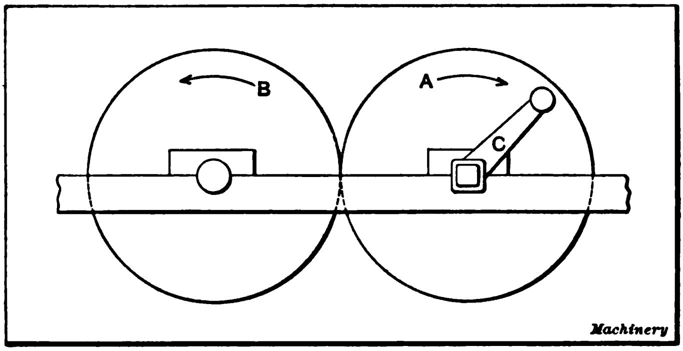


Fig. 1. Two Wheels with Their Faces in Frictional Contact

as corresponding friction wheels running without slipping. They must not only keep the number of revolutions right, but they must give a perfectly even and smooth motion from point to point or from tooth to tooth.

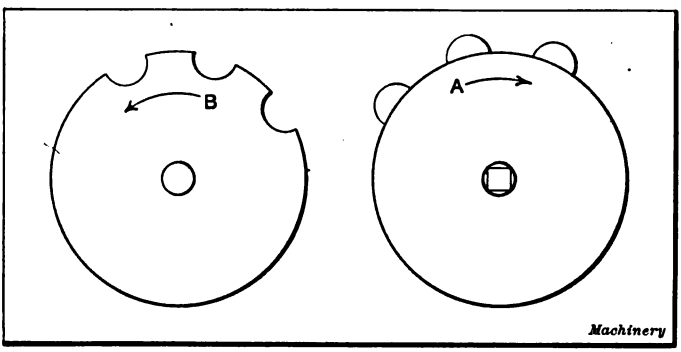


Fig. 2. Example of Simplest Type of Toothed Gearing

Fig. 3 shows clearly how such a result is obtained. It represents the friction wheels with teeth fastened to them, the teeth, of course, extending all the way around instead of part way, as shown. These teeth are set so as to be partly without and

partly within the edges of the two wheels, as, obviously, they will give better results when thus arranged, than if all the projections were on one wheel, and all the grooves or depressions on the other, as in Fig. 2.

With the wheels fitted in this way it can be proved that the only conditions which must be fulfilled, in order that the teeth shall give to wheel *B* the same motion that it would have if it were driven by frictional contact with wheel *A*, is that a line drawn from the point *O*, where the two wheels meet or touch each other, through the point where the tooth curves touch, shall be at right angles to both tooth curves at this point, whatever the position of the gears. For example, in Fig. 3, two of

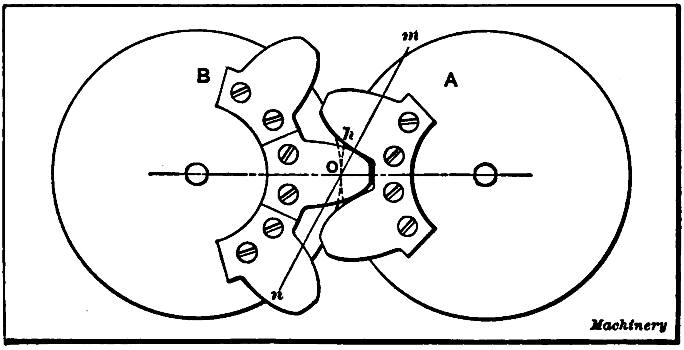


Fig. 3. Friction Wheels with Properly Shaped Gear Teeth attached to Them

the teeth touch at *h*. If the curves are of the right shape, a line *mn*, drawn through *h* and *O*, will be at right angles to both curves at point *h*. This is the law of tooth curves, and it is of no consequence what the shape of the teeth is, so far as their correct action is concerned, if this law holds true for every successive point where the teeth come in contact.

In technical language the "friction wheels" mentioned are known as "pitch cylinders," and they are always represented on a gear drawing by a line — usually a dash and dot line — called the "pitch line." As teeth are generally proportioned, this line falls nearly, but not quite, midway between the tops and bottoms of the teeth, the inequality being due to the space left at the

bottom of the teeth for clearance. The diameter of the pitch cylinder is called the "pitch diameter."

Involute System. — We are now ready to consider the particular forms of teeth most commonly used. The one that is at present most in favor is the involute tooth, the term "involute"

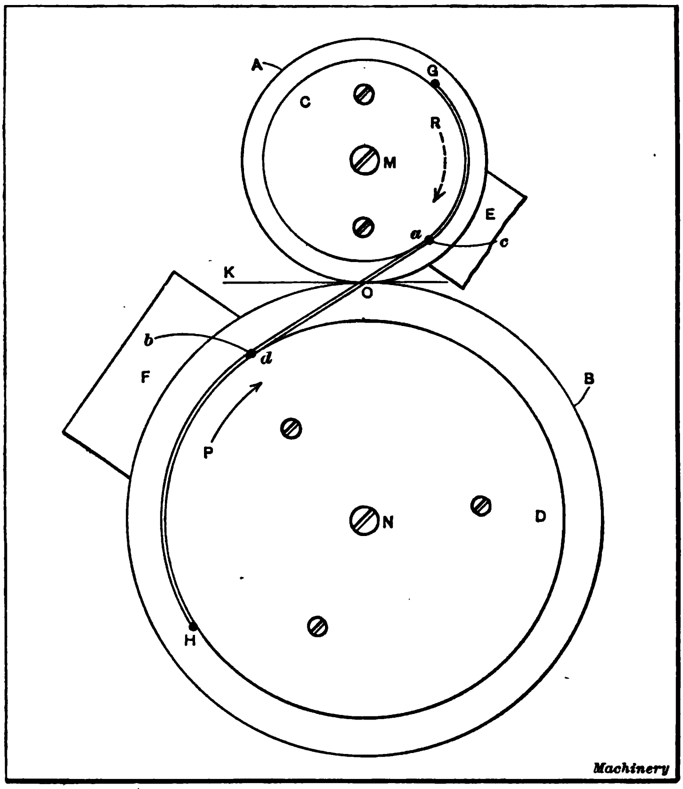


Fig. 4. Method of obtaining Involute Tooth-curves

being the name of a curve described by the end of a cord as it is unwound from another curve. For example, to draw an involute, wind a cord around a circular disk of any convenient material, and make a loop in the outer end of the cord. Lay the disk flat on a piece of paper, and with a pencil-point passed through the loop, unwind the string, keeping it drawn tight,

and let the point of the pencil trace a curve, which will then be an involute.

In Fig. 4 is shown how this principle is applied to forming tooth curves. *A* and *B*, with centers at *M* and *N*, are two disks which serve the purpose of pitch cylinders; *C* and *D* are two smaller disks fastened to the larger ones and around which a cord is stretched and fastened at points *G* and *H*. When either disk is turned, the cord is supposed to pull the other one around at the same speed that it would rotate if moved solely by frictional contact between disks *A* and *B*. To do this, it is simply necessary to have the diameters of disks *C* and *D* in the same ratio as the diameters of *A* and *B*. If *A*, for example, is half as large as *B*, then *C* must be half as large as *D*.

To make room for drawing the curves, let pieces *F* and *E* be fastened to the large and small wheels, respectively. With a pencil fixed at point *d* on the cord, turn wheel *A* in the direction of the arrow *R*, meanwhile moving the pencil outward, and the curve *db* will be described, which will be a suitable tooth curve for the larger wheel, and which it can be proved will answer the requirements of the general law. Starting again with the pencil at *a*, and turning wheel *B* in the direction of the arrow *P*, and moving the pencil outward, a similar curve *ac*, for the smaller wheel, will be traced.

The circles representing the disks *C* and *D* are called "base circles," and in practice are drawn at a distance from the pitch circle of about one-sixtieth of the pitch diameter. This makes the angle *KOd* in Fig. 4, called the angle of obliquity, about $14\frac{1}{2}$ degrees; and although it is not by any means certain that this is the best angle, it is the one most commonly used.

Cycloidal System. — Take a circular disk and roll it along the edge of a ruler or straightedge, holding the point of a pencil at the rim of the disk, so that, as the latter rolls, the pencil will trace a curve. This curve is a cycloid. Should the disk be rolled on the edge of another circular disk, however, the curve traced would be an epicycloid, and should it be rolled on the inside of a hoop, it would be called a hypocycloid. These curves are employed for the teeth of the cycloidal system of gears.

In Fig. 5 is shown how the face or the outer portion of the tooth is rolled up by the point *A* on the outer rolling circle, and how the flank or inner portion is generated by point *B* on the inner rolling circle. In this case the hypocycloid and the flank of the tooth are straight lines, the reason for this being that, as drawn, the diameter of the rolling circle is one-half the diameter of the pitch circle of the gear, and the hypocycloid generated under these conditions becomes a straight line.

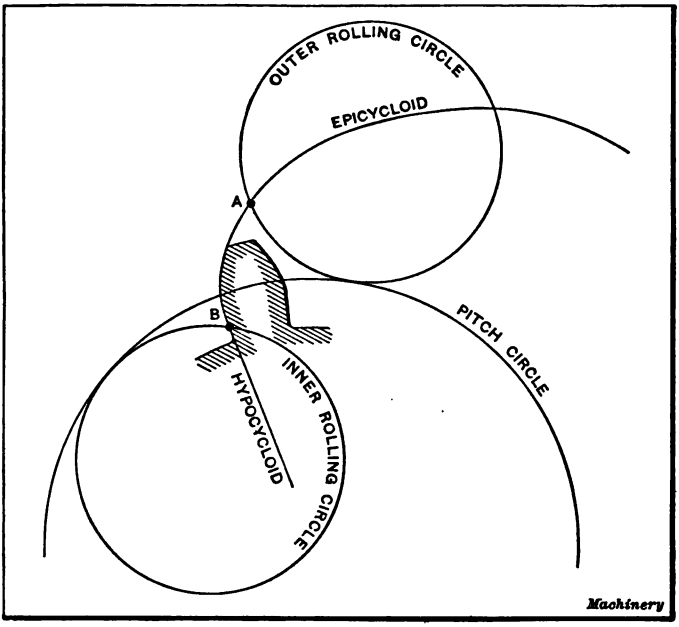


Fig. 5. Method of obtaining Cycloidal Tooth-curves

Comparison of the Involute and Cycloidal Systems. — The involute and cycloidal systems are the only two that are used to any extent, and in Fig. 6 a gear tooth and rack tooth of each are shown for comparison. The involute gear tooth has the involute curve from point *a* to point *b* on the base circle; from *b* to *c*, at the bottom of the tooth, the flank is a straight, radial line. One difficulty with the involute system is that, with the standard length of tooth, point *a* will interfere when running with gears or pinions having a small number of teeth. To

avoid this, the point is rounded off a little below the involute curve. In general appearance the tooth seems to have a broad, strong base, and a continuous curve from *a* to *c*. A strong feature of the involute gearing is that it will run correctly even if the distance between the centers of the gears is not exactly right. This will be evident by referring to Fig. 4, where it will appear that the relative velocities of the two wheels will be the same however far apart they may be, and if involute teeth are used in place of the string connection there shown, the action will be just the same. The involute rack tooth has straight sides at an angle of 14½ degrees, with the points slightly rounded off.

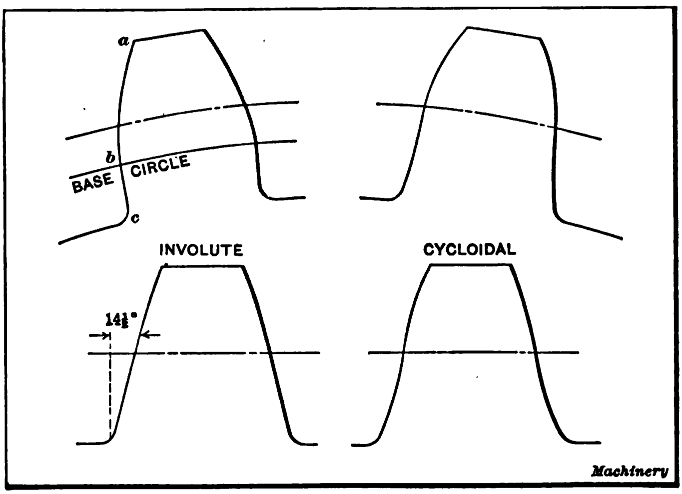


Fig. 6. Comparison between Involute and Cycloidal Teeth

Of the cycloidal teeth but little need be said except that they have two distinct curves above and below the pitch line, as previously explained, and that in the rack tooth the two curves are just alike, but reversed.

Whatever system is used, it is essential that all the wheels of a given pitch should be capable of running together. To make this possible with the involute, all the gears must have the same angle of obliquity; and with the cycloidal system, the same size rolling or describing circle must be employed for all

sizes. The circle generally chosen is one having half the diameter of a 12-tooth pinion, which makes the flanks of this pinion radial. In Fig. 5, if the diameter of the rolling circle had been either greater or less than half the diameter of the pitch circle, the flank of the tooth would have been curved, and in the case of the greater circle, the curve would have fallen inside of the radial flank drawn in the figure, causing a weak, under-cut tooth. With the smaller circle, the curve would fall outside, making a strong tooth.

The most important point in favor of the epicycloidal system of gearing is the freedom from interference of the teeth; but this advantage is considerably modified by the fact that it is necessary, in order that epicycloidal gears shall run properly together, that the pitch circles of the two gears of a pair touch or tangent each other, or, in other words, that the center distance between the two gears be exactly correct. As already mentioned, with involute gears the distance between the centers may be varied somewhat without affecting the smoothness with which motion and power may be transferred from one gear to another. The variation, however, must not be great, on account of the fact that the points of the teeth are rounded off to avoid interference. The variation in the center distance would, of course, increase the amount of backlash, that is, the space or clearance between the faces of the teeth, but the theoretically correct action would not be interfered with. This property of involute gears is one of the reasons why this system has been so extensively adopted.

Of the two systems, the epicycloidal is the older. Cast gears were, in the past, always made with this system of teeth and many are still so made, on account of the number of patterns with this system of teeth that are still on hand. For cut gearing, however, and for a large proportion of modern cast gearing, the involute system has replaced the epicycloidal. One objection sometimes brought forward against the involute system is that the thrust on the shaft bearings is greater than when epicycloidal teeth are used, on account of the obliquity of the line of action; but although it is true that the line of action is at an angle to the

direction of the motion of the gear teeth, this angle is a constant angle. In the epicycloidal system, on the other hand, the line of action is at right angles to a line connecting the centers of the two gears, when two teeth are in contact on the line of centers; but the direction of this pushing action is variable, so that when the teeth are coming in contact with one another the pressure has an obliquity fully as great as, and sometimes greater than, that present in standard involute gears. Authorities on gearing, therefore, do not consider that the objection mentioned to the involute system of gear teeth has any practical weight.

Pressure Angles. — While $14\frac{1}{2}$-degree angle of obliquity or pressure angle has been adopted as the standard for involute gear teeth, it does by no means follow that all involute gear teeth

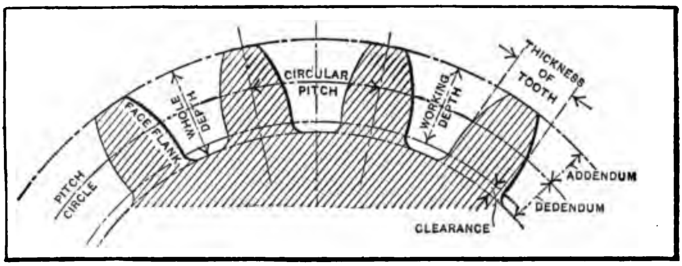


Fig. 7. Gear-tooth Parts

are made with this angle. Many gears are made with a 20-degree pressure angle. This angle makes the tooth considerably broader at the base and correspondingly narrower at the point. The strength of the tooth is thus increased, and, at the present time, the 20-degree pressure angle is used to a considerable extent.

Definitions of Gear-tooth Parts. — When one of two gears that are in mesh with each other is revolved, it will drive the other gear at a certain rate. Imagine, as has already been explained, that instead of gears two circular disks are in contact, so that when one disk is revolved it will drive the other disk by frictional force. The diameters of the disks may be so selected that when one revolves at the same rate as the gear to which it corresponds, it will drive the other disk at the same rate as the

second gear would be driven. The diameters of the disks are then the same as the *pitch diameters* of the gears, and the circumferences of these disks, which are tangent or touch each other, represent the *pitch circles* of the gears.

The *outside diameter* of a gear is the diameter measured over the tops of the teeth.

The *root diameter* of a gear is the diameter measured at the bottom or roots of the teeth.

The *center distance* is the distance between the centers of two meshing gears, the pitch circles of which are tangent to each other.

The *diametral pitch* of a gear is the number of teeth for each inch of pitch diameter, and is found by dividing the number of teeth by the pitch diameter.

The *circular pitch* is the distance from the center of one tooth to the center of the next along the pitch circle.

The *chordal pitch* is the distance from the center (on the pitch circle) of one tooth to the center of the next, measured along a straight line.

The *thickness* of the tooth is generally understood to be the thickness at the pitch circle, measured along the circular arc.

The *chordal thickness* of the tooth is the thickness at the pitch circle measured along a straight line or chord.

The *addendum* of a gear tooth is the distance from the pitch circle to the top of the tooth.

The *dedendum* of a gear tooth is the distance from the pitch circle to the root of the tooth.

The *working depth* is the depth to which the teeth in a meshing gear enter into the spaces between the teeth.

The *clearance* is the amount by which the tooth space is cut deeper than the working depth.

The *face* of the tooth is that part of the tooth curve that is between the outside circumference and the pitch circle.

The *flank* of the tooth is that part of the working depth of the tooth which comes inside of the pitch circle.

In the following will be given a number of rules and examples showing the relation between these various dimensions.

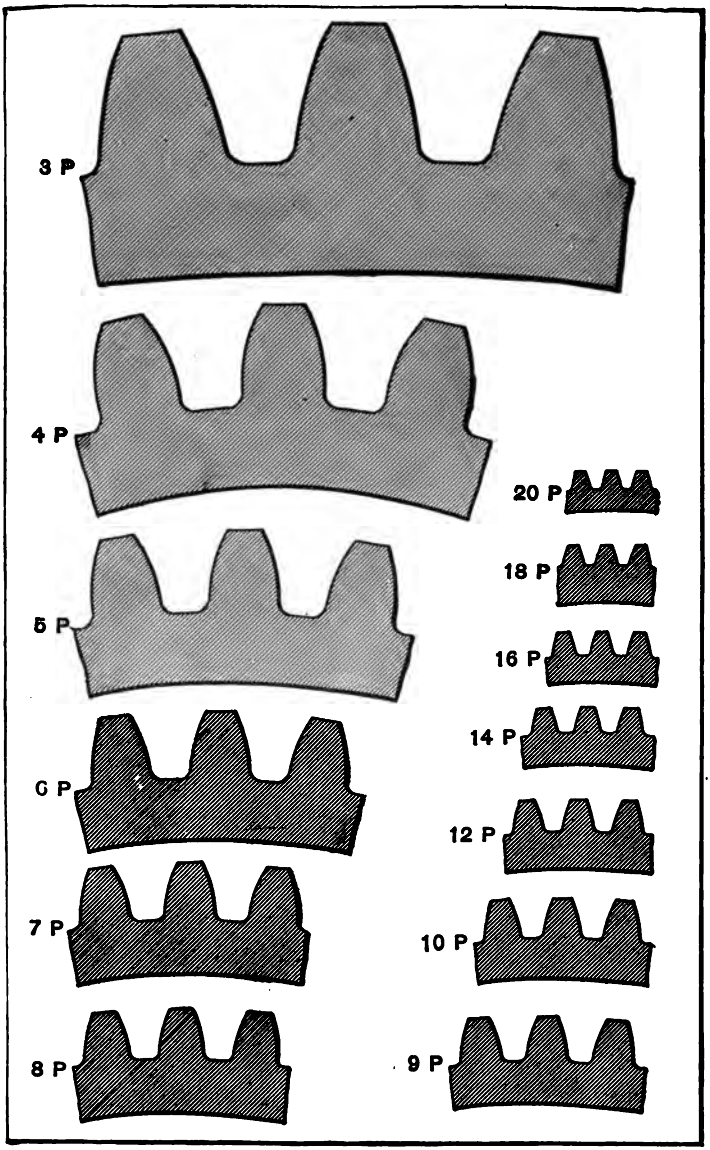


Fig. 8. Gear Teeth of Different Diametral Pitch shown in Full Natural Size in order to enable the Designer to Determine at a Glance the Actual Proportions of Gear Teeth of Various Pitches

Circular and Diametral Pitch. — The circular pitch, at the present time, is, as a rule, used only in relation to gears with cast teeth, which are not afterwards finished or cut. Diametral pitch is used almost exclusively for all cut gearing, and of late, to some extent, for cast gearing as well.

As the circular pitch is equal to the distance from the center of one tooth to the center of the next, measured along the pitch circle, it can readily be seen that the circular pitch will be equal to the circumference of the pitch circle divided by the number of teeth in the gear. The circumference, however, is equal to the pitch diameter multiplied by 3.1416; hence, we can write the rule for finding the circular pitch, when the pitch diameter of the gear and the number of teeth in the gear are known, as follows:

$$\text{Circular pitch} = \frac{\text{pitch diameter} \times 3.1416}{\text{number of teeth}}$$

Example: — The pitch diameter of a gear is $47\frac{3}{4}$ inches and the gear has 75 teeth. Find the circular pitch.

The circumference of the pitch circle is $47\frac{3}{4} \times 3.1416 = 150.01$ inches; then the circular pitch equals $150.01 \div 75 = 2$ inches, almost exactly.

The relation between the diametral pitch, the pitch diameter and the number of teeth is much simpler than that between the circular pitch and these quantities. As the diametral pitch is the number of teeth for each inch of pitch diameter, it can readily be seen that the diametral pitch can be found by dividing the number of teeth by the pitch diameter.

If a gear has 20 teeth and the pitch diameter is 2 inches, the diametral pitch would be equal to $20 \div 2$, or 10. In the same way, if the gear has 22 teeth and the pitch diameter is $5\frac{1}{2}$ inches, then the diametral pitch equals $22 \div 5\frac{1}{2} = 4$. The relationship between diametral pitch, pitch diameter and the number of teeth may be written as below:

$$\text{Diametral pitch} = \frac{\text{number of teeth}}{\text{pitch diameter}}$$

Changing Circular Pitch into Diametral Pitch. — If the circular pitch is given and the diametral pitch is to be found,

this can be done by dividing 3.1416 by the circular pitch, or:

$$\text{Diametral pitch} = \frac{3.1416}{\text{circular pitch}}$$

Example: — If the circular pitch of a gear is $\frac{5}{8}$ inch, what is the nearest whole number diametral pitch?

By following the rule given above, we have $3.1416 \div \frac{5}{8} = 5.026$, which shows that 5 is the nearest whole number diametral pitch corresponding to a $\frac{5}{8}$-inch circular pitch.

Changing Diametral Pitch into Circular Pitch. — If the diametral pitch is known and the circular pitch is to be found, divide 3.1416 by the diametral pitch. The quotient is the circular pitch, or:

$$\text{Circular pitch} = \frac{3.1416}{\text{diametral pitch}}$$

Example: — If the diametral pitch of a gear is 4, what is the corresponding circular pitch?

According to the rule given, $3.1416 \div 4 = 0.7854$ inch is the circular pitch of the gear.

Explanation of Rules Given. — Having given the rules, we will now proceed to explain how they are obtained. We know that the distance around the circumference of a circle is equal to 3.1416 multiplied by the diameter of the circle; hence, for every inch of diameter, we have 3.1416 inches of circumference. As the diametral pitch of a gear is equal to the number of teeth for each inch of pitch diameter, and each inch of diameter is represented by 3.1416 inches of circumference, then the diametral pitch equals the number of teeth for each 3.1416 inches of circumference. As the circular pitch is the distance from the center of one tooth to the center of the next, then the circular pitch must be equal to 3.1416 divided by the number of teeth in that 3.1416 inches of circumference, and, as we have shown that the diametral pitch is equal to the number of teeth in each 3.1416 inches of circumference, then the circular pitch must equal 3.1416 divided by the diametral pitch as given by the rule and formula.

It may not be actually necessary to show how we obtain the diametral pitch from the circular pitch, because the formula may be simply transposed to give the required result. For the sake of completeness, however, the explanation will be given. As in the preceding case, we begin with the ratio of the circumference of the circle to its diameter, which is 3.1416. In each 3.1416 inches of circumference we have a certain number of teeth, which is the diametral pitch of the gear. Now, having given the circular pitch, if we divide 3.1416 by that, we obtain the number of teeth for 3.1416 inches of the circumference, which is the diametral pitch of the gear, as shown by the rule and formula given.

Tables of gear tooth parts are given in the following pages which will facilitate the finding of corresponding diametral and circular pitches. These tables give, in addition, other dimensions relating to gear teeth, such as the thickness of the tooth at the pitch line, the addendum, the working depth of the tooth, the depth of space below the pitch line (or dedendum), and the whole depth of the tooth.

Pitch Diameter. — When the diametral pitch and the number of teeth in a gear are known, the pitch diameter is found by dividing the number of teeth by the diametral pitch, or:

$$\text{Pitch diameter} = \frac{\text{number of teeth}}{\text{diametral pitch}}$$

Example: — A 10-pitch gear has 35 teeth. Find the pitch diameter.

Divide 35 (the number of teeth) by 10 (the diametral pitch). The quotient is $3\frac{1}{2}$, which is the pitch diameter of the gear in inches.

That the rule and formula given above is correct, is proved by the definition of diametral pitch. As the diametral pitch equals the number of teeth for each inch of pitch diameter, then, if we divide the number of teeth in the gear by the diametral pitch, we must obviously obtain the number of inches of the pitch diameter.

When the circular pitch and the number of teeth in a gear

are given, the pitch diameter is found by dividing the product of the number of teeth and the circular pitch by 3.1416, or:

$$\text{Pitch diameter} = \frac{\text{number of teeth} \times \text{circular pitch}}{3.1416}$$

Example: — A gear of 2-inch circular pitch has 75 teeth. Find the pitch diameter.

According to the rule given, the pitch diameter equals:

$$\frac{75 \times 2}{3.1416} = 47.75 \text{ inches, very nearly.}$$

The accuracy of the rule and formula given for finding the pitch diameter when the circular pitch is given will readily be understood. As the circular pitch is the distance from the center of one tooth to the next, this distance multiplied by the total number of teeth will give the total pitch circumference of the gear. This divided by 3.1416, of course, gives the pitch diameter.

Center Distance. — To find the center distance when the numbers of teeth in the two gears and the diametral pitch are given, add together the number of teeth in both gears and divide the sum by two times the diametral pitch, or:

$$\text{Center distance} = \frac{\text{no. of teeth, 1st gear} + \text{no. of teeth, 2nd gear}}{2 \times \text{diametral pitch}}$$

Example: — Find the center distance between two gears of 8 diametral pitch, the number of teeth in the one gear being 22 and in the other, 36.

$$\frac{22 + 36}{2 \times 8} = \frac{58}{16} = 3\tfrac{5}{8} \text{ inches.}$$

This formula is based upon the fact that the pitch diameter of each of the gears equals the number of teeth in that gear, divided by the diametral pitch. The sum of the teeth in the two gears, divided by the diametral pitch, will then equal the sum of the two pitch diameters, but as the center distance is equal to the sum of the pitch radii, and is thus equal to only one-half of the total sum of the two pitch diameters, this sum is divided by 2,

as indicated in the rule and formula, to obtain the distance between the centers.

To find the center distance when the circular pitch and the numbers of teeth in the two gears are given, multiply the sum of the number of teeth in both gears by the circular pitch and divide the product by 6.2832, or:

$$\text{Center distance} = \frac{(\text{no. of teeth, 1st gear} + \text{no. of teeth, 2nd gear}) \times \text{circ. pitch}}{6.2832}$$

Example: — The circular pitch of two equal gears having 75 teeth each is 2 inches. Find the center distance.

$$\frac{(75 + 75) \times 2}{6.2832} = 47.75 \text{ inches, very nearly.}$$

This rule is derived directly from the one just given in which the diametral pitch is used, by merely substituting the circular pitch for the diametral pitch, as explained on a preceding page.

Addendum. — The addendum of a gear tooth is always, in standard diametral pitch gearing, made equal to 1 divided by the diametral pitch, or:

$$\text{Addendum} = \frac{1}{\text{diametral pitch}}$$

Example: — Find the addendum for a gear of 6 diametral pitch. The addendum equals 1 ÷ 6, or 0.1667 inch.

If the circular pitch is given, the addendum equals the circular pitch divided by 3.1416, or:

$$\text{Addendum} = \frac{\text{circular pitch}}{3.1416}$$

Example: — The circular pitch is 2 inches. Find the addendum.

According to the rule given, the addendum equals 2 ÷ 3.1416 = 0.6366.

This rule is derived directly from that for diametral pitch, by substituting the value of the circular pitch for the diametral pitch in that rule.

Clearance. — The clearance below the working depth of the tooth is made equal to 0.157 divided by the diametral pitch, or;

$$\text{Clearance} = \frac{0.157}{\text{diametral pitch}}$$

Example: — Find the clearance for 12 diametral pitch.

$$\text{Clearance} = \frac{0.157}{12} = 0.013 \text{ inch.}$$

If the circular pitch is given, the clearance may be found by dividing the circular pitch by 20, or:

$$\text{Clearance} = \frac{\text{circular pitch}}{20}$$

Example: — Find the clearance for a gear of 2-inch circular pitch.

$$\text{Clearance} = \tfrac{2}{20} = 0.1 \text{ inch.}$$

Clearance of Gears cut on the Gear Shaper. — When gears are cut on the Fellows gear shaper, the clearance is made equal to 0.25 divided by the diametral pitch; hence, the root diameter of these gears is smaller than the root diameter of ordinary milled gears. The pitch and outside diameters are, of course, the same as for gears milled with ordinary rotary milling cutters.

Whole Depth of Tooth. — The whole depth of tooth is composed of two times the addendum plus the clearance, and, therefore, is found by dividing 2.157 by the diametral pitch, or:

$$\text{Whole depth} = \frac{2.157}{\text{diametral pitch}}$$

Example:—Find the whole depth of tooth for 5 diametral pitch.

$$\text{Whole depth} = \frac{2.157}{5} = 0.431 \text{ inch.}$$

If the circular pitch is given, the whole depth of the tooth is found by multiplying the circular pitch by 0.6866, or:

$$\text{Whole depth} = \text{circular pitch} \times 0.6866.$$

Example: — Find the whole depth of tooth for 2-inch circular pitch.

$$\text{Whole depth} = 2 \times 0.6866 = 1.3732 \text{ inch.}$$

The rule for finding the whole depth of tooth when the circular pitch is given may be found from the rule relating to diametral pitch, by simply substituting the value of the circular pitch for that of the diametral pitch.

Thickness of Tooth. — The thickness of the tooth at the pitch line, measured along the circular arc, is found by dividing 1.5708 by the diametral pitch, or:

$$\text{Thickness of tooth} = \frac{1.5708}{\text{diametral pitch}}$$

Example: — Find the thickness of the tooth at the pitch line for a gear of 4 diametral pitch.

$$\text{Thickness of tooth} = \frac{1.5708}{4} = 0.3927 \text{ inch.}$$

This rule makes the tooth space and the thickness of the tooth at the pitch line exactly equal.

If the circular pitch is known, the thickness of the tooth equals the circular pitch divided by 2, or:

$$\text{Thickness of tooth} = \frac{\text{circular pitch}}{2}$$

Example: — Find the thickness of tooth for a $1\frac{1}{2}$-inch circular pitch gear.

$$\text{Thickness of tooth} = \frac{1\frac{1}{2}}{2} = \frac{3}{4} \text{ inch.}$$

It is apparent at a glance that this rule is based upon the rule of making the thickness of the tooth equal to the tooth space, because the thickness of one tooth plus one tooth space equals the circular pitch, and the thickness of the tooth is then made one-half of this distance.

Outside Diameter. — When the diametral pitch is known, the outside diameter of a gear is found by adding 2 to the number of teeth and dividing the sum thus obtained by the diametral pitch, or:

$$\text{Outside diam.} = \frac{\text{number of teeth} + 2}{\text{diametral pitch}}$$

Example: — Find the outside diameter of a 5 diametral pitch gear having 28 teeth.

$$\text{Outside diam.} = \frac{28 + 2}{5} = \frac{30}{5} = 6 \text{ inches.}$$

If the circular pitch is given, the outside diameter is found by multiplying the sum of the number of teeth plus 2 by the circular pitch and dividing the product by 3.1416, or:

$$\text{Outside diam.} = \frac{(\text{no. of teeth} + 2) \times \text{circ. pitch}}{3.1416}$$

Example: — Find the outside diameter of a 2-inch circular pitch gear having 75 teeth.

$$\frac{(75 + 2) \times 2}{3.1416} = \frac{77 \times 2}{3.1416} = \frac{154}{3.1416} = 49.02 \text{ inches.}$$

Number of Teeth. — If the diametral pitch and pitch diameter are known, multiply the pitch diameter by the diametral pitch. The product gives the number of teeth in the gear, or:

$$\text{No. of teeth} = \text{pitch diameter} \times \text{diametral pitch.}$$

Example: — Find the number of teeth of an 8 diametral pitch gear having a pitch diameter of $3\frac{3}{8}$ inches.

$$\text{Number of teeth} = 3\tfrac{3}{8} \times 8 = 27.$$

When the circular pitch and the pitch diameter are known, the number of teeth are found by multiplying the pitch diameter by 3.1416 and dividing the product by the circular pitch, or:

$$\text{No. of teeth} = \frac{3.1416 \times \text{pitch diameter}}{\text{circular pitch}}$$

Example: — Find the number of teeth in a 2-inch circular pitch gear having a pitch diameter of $47\frac{3}{4}$ inches.

$$\text{No. of teeth} = \frac{3.1416 \times 47\frac{3}{4}}{2} = 75.$$

Miscellaneous Rules. — If the outside diameter is known and the pitch diameter is to be found, subtract 2 times the addendum from the outside diameter, or:

$$\text{Pitch diam.} = \text{outside diam.} - 2 \times \text{addendum.}$$

Example: — Find the pitch diameter of an 8 diametral pitch gear having an outside diameter of 4 inches.

As the addendum equals 1 divided by the diametral pitch, we have, in this case:

$$\text{Addendum} = \tfrac{1}{8}$$

$$\text{Pitch diameter} = 4 - 2 \times \tfrac{1}{8} = 3\tfrac{3}{4} \text{ inches.}$$

If the outside diameter and the number of teeth are known, the pitch diameter may be found by multiplying the outside diameter by the number of teeth and dividing this product by the number of teeth plus 2, or:

$$\text{Pitch diam.} = \frac{\text{outside diam.} \times \text{no. of teeth}}{\text{no. of teeth} + 2}$$

Example: — A gear of $3\frac{1}{6}$ inches outside diameter has 36 teeth. Find the pitch diameter.

According to the rule given:

$$\text{Pitch diameter} = \frac{3\frac{1}{6} \times 36}{36 + 2} = \frac{114}{38} = 3 \text{ inches.}$$

If the outside diameter and the diametral pitch are known, the number of teeth may be found by multiplying the outside diameter by the diametral pitch and subtracting 2 from the product, or:

$$\text{No. of teeth} = (\text{outside diam.} \times \text{diametral pitch}) - 2.$$

Example: — Find the number of teeth in a gear of $3\frac{1}{6}$ inches outside diameter and 12 diametral pitch.

According to the rule given:

$$\text{Number of teeth} = (3\tfrac{1}{6} \times 12) - 2 = 36.$$

Table of Rules and Formulas. — For those who prefer rules and formulas in a condensed form, the accompanying table entitled "Rules and Formulas for Dimensions of Spur Gears" has been prepared. By grouping the formulas and dimensions together in this manner, they may be more easily found when wanted. The formulas are numbered, making reference to any of them more convenient. The first sixteen formulas are placed in the order in which they would naturally present themselves to the designer when determining the dimensions of a pair of

$P = Pd$

$P' = Pc$

Rules and Formulas for Dimensions of Spur Gears *

No. of Rule	To Find	Rule	Formula
1	Diametral Pitch	Divide 3.1416 by circular pitch.	$P = \frac{3.1416}{P'}$
2	Circular Pitch	Divide 3.1416 by diametral pitch.	$P' = \frac{3.1416}{P}$
3	Pitch Diameter	Divide number of teeth by diametral pitch.	$D = \frac{N}{P}$
4	Pitch Diameter	Multiply number of teeth by circular pitch and divide the product by 3.1416.	$D = \frac{NP'}{3.1416}$
5	Center Distance	Add the number of teeth in both gears and divide the sum by two times the diametral pitch.	$C = \frac{N_g + N_p}{2P}$
6	Center Distance	Multiply the sum of the number of teeth in both gears by circular pitch and divide the product by 6.2832.	$C = \frac{(N_g + N_p)P'}{6.2832}$
7	Addendum	Divide 1 by diametral pitch.	$S = \frac{1}{P}$
8	Addendum	Divide circular pitch by 3.1416.	$S = \frac{P'}{3.1416}$
9	Clearance	Divide 0.157 by diametral pitch.	$F = \frac{0.157}{P}$
10	Clearance	Divide circular pitch by 20.	$F = \frac{P'}{20}$
11	Whole Depth of Tooth	Divide 2.157 by diametral pitch.	$W = \frac{2.157}{P}$
12	Whole Depth of Tooth	Multiply 0.6866 by circular pitch.	$W = 0.6866\,P'$
13	Thickness of Tooth	Divide 1.5708 by diametral pitch.	$T = \frac{1.5708}{P}$
14	Thickness of Tooth	Divide circular pitch by 2.	$T = \frac{P'}{2}$
15	Outside Diameter	Add 2 to the number of teeth and divide the sum by diametral pitch.	$O = \frac{N+2}{P}$
16	Outside Diameter	Multiply the sum of the number of teeth plus 2 by circular pitch and divide the product by 3.1416.	$O = \frac{(N+2)P'}{3.1416}$
17	Diametral Pitch	Divide number of teeth by pitch diameter.	$P = \frac{N}{D}$
18	Circular Pitch	Multiply pitch diameter by 3.1416 and divide by number of teeth.	$P = \frac{3.1416\,D}{N}$
19	Pitch Diameter	Subtract two times the addendum from outside diameter.	$D = O - 2S$
20	Number of Teeth	Multiply pitch diameter by diametral pitch.	$N = P \times D$
21	Number of Teeth	Multiply pitch diameter by 3.1416 and divide the product by circular pitch.	$N = \frac{3.1416\,D}{P'}$
22	Outside Diameter	Add two times the addendum to the pitch diameter.	$O = D + 2S$
23	Length of Rack	Multiply number of teeth in rack by 3.1416 and divide by diametral pitch.	$L = \frac{3.1416\,N}{P}$
24	Length of Rack	Multiply the number of teeth in the rack by circular pitch.	$L = NP'$

* From MACHINERY'S HANDBOOK, page 550.

Gear Tooth Parts *

(Diametral Pitch Gears)

Diametral Pitch	Circular Pitch	Thickness of Tooth on Pitch Line	Addendum	Working Depth of Tooth	Depth of Space below Pitch Line	Whole Depth of Tooth
P	P'	T	S	W'	$S+F$	W
½	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
¾	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
1¼	2.5133	1.2566	0.8000	1.6000	0.9257	1.7257
1½	2.0944	1.0472	0.6666	1.3333	0.7714	1.4381
1¾	1.7952	0.8976	0.5714	1.1429	0.6612	1.2326
2	1.5708	0.7854	0.5000	1.0000	0.5785	1.0785
2¼	1.3963	0.6981	0.4444	0.8888	0.5143	0.9587
2½	1.2566	0.6283	0.4000	0.8000	0.4628	0.8628
2¾	1.1424	0.5712	0.3636	0.7273	0.4208	0.7844
3	1.0472	0.5236	0.3333	0.6666	0.3857	0.7190
3½	0.8976	0.4488	0.2857	0.5714	0.3306	0.6163
4	0.7854	0.3927	0.2500	0.5000	0.2893	0.5393
5	0.6283	0.3142	0.2000	0.4000	0.2314	0.4314
6	0.5236	0.2618	0.1666	0.3333	0.1928	0.3595
7	0.4488	0.2244	0.1429	0.2857	0.1653	0.3081
8	0.3927	0.1963	0.1250	0.2500	0.1446	0.2696
9	0.3491	0.1745	0.1111	0.2222	0.1286	0.2397
10	0.3142	0.1571	0.1000	0.2000	0.1157	0.2157
11	0.2856	0.1428	0.0909	0.1818	0.1052	0.1961
12	0.2618	0.1309	0.0833	0.1666	0.0964	0.1798
13	0.2417	0.1208	0.0769	0.1538	0.0890	0.1659
14	0.2244	0.1122	0.0714	0.1429	0.0826	0.1541
15	0.2094	0.1047	0.0666	0.1333	0.0771	0.1438
16	0.1963	0.0982	0.0625	0.1250	0.0723	0.1348
17	0.1848	0.0924	0.0588	0.1176	0.0681	0.1269
18	0.1745	0.0873	0.0555	0.1111	0.0643	0.1198
19	0.1653	0.0827	0.0526	0.1053	0.0609	0.1135
20	0.1571	0.0785	0.0500	0.1000	0.0579	0.1079
22	0.1428	0.0714	0.0455	0.0909	0.0526	0.0980
24	0.1309	0.0654	0.0417	0.0833	0.0482	0.0898
26	0.1208	0.0604	0.0385	0.0769	0.0445	0.0829
28	0.1122	0.0561	0.0357	0.0714	0.0413	0.0770
30	0.1047	0.0524	0.0333	0.0666	0.0386	0.0719
32	0.0982	0.0491	0.0312	0.0625	0.0362	0.0674
34	0.0924	0.0462	0.0294	0.0588	0.0340	0.0634
36	0.0873	0.0436	0.0278	0.0555	0.0321	0.0599
38	0.0827	0.0413	0.0263	0.0526	0.0304	0.0568
40	0.0785	0.0393	0.0250	0.0500	0.0289	0.0539
42	0.0748	0.0374	0.0238	0.0476	0.0275	0.0514
44	0.0714	0.0357	0.0227	0.0455	0.0263	0.0490
46	0.0683	0.0341	0.0217	0.0435	0.0252	0.0469
48	0.0654	0.0327	0.0208	0.0417	0.0241	0.0449
50	0.0628	0.0314	0.0200	0.0400	0.0231	0.0431

* From Machinery's Handbook, page 552.

Gear Tooth Parts *

(Circular Pitch Gears)

Circular Pitch	Diametral Pitch	Thickness of Tooth on Pitch Line	Addendum	Working Depth of Tooth	Depth of Space below Pitch Line	Whole Depth of Tooth
P'	P	T	S	W'	$S+F$	W
4	0.7854	2.0000	1.2732	2.5464	1.4732	2.7464
3½	0.8976	1.7500	1.1140	2.2281	1.2890	2.4031
3	1.0472	1.5000	0.9549	1.9098	1.1049	2.0598
2¾	1.1424	1.3750	0.8753	1.7506	1.0128	1.8881
2½	1.2566	1.2500	0.7957	1.5915	0.9207	1.7165
2¼	1.3963	1.1250	0.7162	1.4323	0.8287	1.5448
2	1.5708	1.0000	0.6366	1.2732	0.7366	1.3732
1⅞	1.6755	0.9375	0.5968	1.1937	0.6906	1.2874
1¾	1.7952	0.8750	0.5570	1.1141	0.6445	1.2016
1⅝	1.9333	0.8125	0.5173	1.0345	0.5985	1.1158
1½	2.0944	0.7500	0.4775	0.9549	0.5525	1.0299
1 7/16	2.1855	0.7187	0.4576	0.9151	0.5294	0.9870
1⅜	2.2848	0.6875	0.4377	0.8754	0.5064	0.9441
1 5/16	2.3936	0.6562	0.4178	0.8356	0.4834	0.9012
1¼	2.5133	0.6250	0.3979	0.7958	0.4604	0.8583
1 3/16	2.6456	0.5937	0.3780	0.7560	0.4374	0.8154
1⅛	2.7925	0.5625	0.3581	0.7162	0.4143	0.7724
1 1/16	2.9568	0.5312	0.3382	0.6764	0.3913	0.7295
1	3.1416	0.5000	0.3183	0.6366	0.3683	0.6866
15/16	3.3510	0.4687	0.2984	0.5968	0.3453	0.6437
⅞	3.5904	0.4375	0.2785	0.5570	0.3223	0.6007
13/16	3.8666	0.4062	0.2586	0.5173	0.2993	0.5579
¾	4.1888	0.3750	0.2387	0.4775	0.2762	0.5150
11/16	4.5696	0.3437	0.2189	0.4377	0.2532	0.4720
⅔	4.7124	0.3333	0.2122	0.4244	0.2455	0.4577
⅝	5.0265	0.3125	0.1989	0.3979	0.2301	0.4291
9/16	5.5851	0.2812	0.1790	0.3581	0.2071	0.3862
½	6.2832	0.2500	0.1592	0.3183	0.1842	0.3433
7/16	7.1808	0.2187	0.1393	0.2785	0.1611	0.3003
⅖	7.8540	0.2000	0.1273	0.2546	0.1473	0.2746
⅜	8.3776	0.1875	0.1194	0.2387	0.1381	0.2575
⅓	9.4248	0.1666	0.1061	0.2122	0.1228	0.2289
5/16	10.0531	0.1562	0.0995	0.1989	0.1151	0.2146
2/7	10.9956	0.1429	0.0909	0.1819	0.1052	0.1962
¼	12.5664	0.1250	0.0796	0.1591	0.0921	0.1716
2/9	14.1372	0.1111	0.0707	0.1415	0.0818	0.1526
⅕	15.7080	0.1000	0.0637	0.1273	0.0737	0.1373
3/16	16.7552	0.0937	0.0597	0.1194	0.0690	0.1287
⅙	18.8496	0.0833	0.0531	0.1061	0.0614	0.1144
1/7	21.9911	0.0714	0.0455	0.0910	0.0526	0.0981
⅛	25.1327	0.0625	0.0398	0.0796	0.0460	0.0858
1/9	28.2743	0.0555	0.0354	0.0707	0.0409	0.0763
1/10	31.4159	0.0500	0.0318	0.0637	0.0368	0.0687
1/16	50.2655	0.0312	0.0199	0.0398	0.0230	0.0429

* From MACHINERY'S HANDBOOK, page 553.

spur gears with either diametral or circular pitch. Nos. 17 to 22 give additional formulas for various conditions of known and unknown factors. Formulas 23 and 24 give the length of a rack when the number of teeth in it and either the diametral or the circular pitch are known. In the formulas in this chart, the following notation has been used:

P = diametral pitch;
N = number of teeth; (if the number of teeth in both gear and pinion are referred to, N_g = number of teeth in gear, and N_p = number of teeth in pinion);
P' = circular pitch;
D = pitch diameter;
C = center distance;
S = addendum;
F = clearance;
W = whole depth of tooth;
T = thickness of tooth;
O = outside diameter of gear.

Internal Spur Gears. — As indicated by its name, the internal gear is one having teeth formed on an interior pitch surface instead of on an exterior one. Briefly, and perhaps somewhat unconventionally defined, it is an ordinary spur gear turned inside out. At the right of Fig. 9 is shown a sketch of such a gear, meshing with a spur pinion; at the left is shown a pair of spur gears having the same number of teeth and the same pitch as the pinion and internal gear on the right. By tracing the motion in each figure, it will be seen that internal action causes the two members to turn in the same direction, while external action produces opposite rotation.

The Uses of Internal Gearing. — There are some advantages incident to the use of internal gears for particular applications, as compared with external gears of the same pitch and number of teeth. For one thing, an internal gear has its teeth and that of its pinion protected to a very marked degree from inflicting or receiving injury, often making the use of a gear guard unnecessary, if the parts are properly designed for that purpose. Owing to the fact that the cylindrical pitch surfaces in internal gearing have their curvature in the same direction, the teeth of the pinion approach and mesh with those of its mate somewhat

more gradually and easily than when they are meshing with an external gear. This tends toward smoothness and quietness in running, as well as giving a slightly longer contact for each tooth. Another characteristic which is often an advantage will be seen from a study of Fig. 9. In each case shown we have gears of the same pitch and number of teeth. The internal gears evidently have a much smaller center distance than the external gears. This matter is of importance when it is necessary to transmit considerable power between shafts placed quite close together.

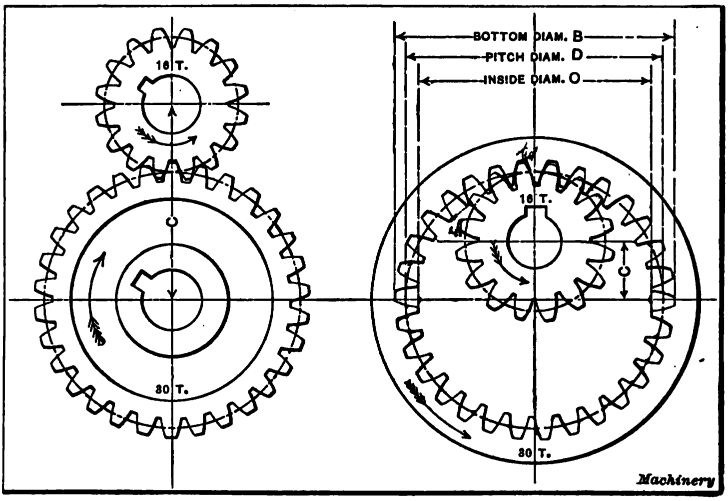


Fig. 9. Comparison between External and Internal Spur Gears

In contrast with the advantages just mentioned, the chief factor which has limited the use of the internal gear has been the difficulty and expense of making it. This difficulty has not been insuperable for cast gearing, but, until the introduction of recent processes, the cutting of internal teeth has been a tedious and unsatisfactory process.

Rules for Designing Internal Gearing. — Neglecting for the time being the modifications which have to be made in the standard proportions to avoid interference, it may be said that the usual thing to do in designing internal gearing is to follow

exactly the dimensions of the standard system as used for external spur gearing. Practically the only modifications required in the rules given in the table "Rules and Formulas for Dimensions of Spur Gears," are those made necessary by the fact that the center distance, in internal gearing, is equal to the difference between the two pitch radii, instead of to their sum. Besides this, we have of course to reckon with the fact that the teeth are "turned inside out," so that the bottom or root diameter is larger than the pitch diameter.

The only new term is "inside diameter," which takes the place of the outside diameter of external spur gearing. It is, of course, the inside diameter of the blank before the teeth are cut, and it is marked O in Fig. 9. The following are the rules which must be changed:

No. 5 will read: *To find the center distance, subtract the number of teeth in the pinion from the number of teeth in the gear and divide the remainder by 2 times the diametral pitch.*

No. 6 will read: *To find the center distance, multiply the difference of the numbers of teeth in the gear and pinion by the circular pitch and divide the product by* 6.2832.

No. 15 will read: *To find the inside diameter, subtract 2 from the number of teeth and divide the remainder by the diametral pitch.*

No. 16 will read: *To find the inside diameter, subtract 2 from the number of teeth, multiply the remainder by the circular pitch, and divide the product by* 3.1416.

No. 19 will read: *To find the pitch diameter, add twice the addendum to the inside diameter.*

No. 22 will read: *To find the inside diameter, subtract twice the addendum from the pitch diameter.*

Interference in Internal Gears. — In laying out the shape of teeth for internal gearing we have to look out for two kinds of interference which are almost sure to be met with. The points of the rack teeth in the 14½-degree involute system are relieved to avoid the interference with the flanks of small pinions, and the points of internal gear teeth have to be relieved for the same reason, but to an even greater degree. A second form of interference occurs when the pinion has too nearly the same number

of teeth as the gear. In this case there is a tendency for the points of the pinion and the gear teeth to strike as they roll into and out of engagement.

For the first form correction may be made either by correcting the points of the internal gear tooth by shortening them (at the same time preferably lengthening the addendum of the pinion), or by changing the pressure angle. The mechanic who desires

Grant's Odontograph Table for Cycloidal Teeth *

Number of Teeth in the Gear		*R, r, D* and *d* for One Diametral Pitch; for any other Pitch divide Values given by that Pitch				*R, r, D* and *d* for One Inch Circular Pitch; for any other Pitch multiply Values given by that Pitch			
		Faces		Flanks		Faces		Flanks	
Exact	Also Used for	*R*	*D*	*r*	*d*	*R*	*D*	*r*	*d*
10	10	1.99	0.02	−8.00	4.00	0.62	0.01	−2.55	1.27
11	11	2.00	0.04	−11.05	6.50	0.63	0.01	−3.34	2.07
12	12	2.01	0.06	∞	∞	0.64	0.02	∞	∞
13½	13–14	2.04	0.07	15.10	9.43	0.65	0.02	4.80	3.00
15½	15–16	2.10	0.09	7.86	3.46	0.67	0.03	2.50	1.10
17½	17–18	2.14	0.11	6.13	2.20	0.68	0.04	1.95	0.70
20	19–21	2.20	0.13	5.12	1.57	0.70	0.04	1.63	0.50
23	22–24	2.26	0.15	4.50	1.13	0.72	0.05	1.43	0.36
27	25–29	2 33	0.16	4.10	0.96	0.74	0.05	1.30	0.29
33	30–36	2.40	0.19	3.80	0.72	0.76	0.06	1.20	0.23
42	37–48	2.48	0.22	3.52	0.63	0.79	0.07	1.12	0.20
58	49–72	2.60	0.25	3.33	0.54	0.83	0.08	1.06	0.17
97	73–144	2.83	0.28	3.14	0.44	0.90	0.09	1.00	0.14
290	145–300	2.92	0.31	3.00	0.38	0.93	0.10	0.95	0.12
∞	Rack	2.96	0.34	2.96	0.34	0.94	0.11	0.94	0.11

* From "A Treatise on Gear Wheels," by George B. Grant.

to use internal cut gearing without making a study of the theoretical conditions has two courses open to him. He may purchase a formed cutter for the gear from the regular makers of formed cutters, telling them the number of teeth and pitch he proposes to use for the gear and pinion. In that case, the maker of the cutter will make such corrections in its form as may be necessary to avoid interference. Another way is to cut the internal gear on the Fellows gear shaper. With this machine the cutter forms its own correction, so that no calculation on the part of the user is ordinarily required.

Grant's Odontograph. — The tables of Grant's odontographs for cycloidal and involute teeth provide a simple means for laying out gear teeth or templets for gear teeth accurately by means of circular arcs which very closely approximate the exact tooth curves. This method was devised by Mr. George B. Grant and constitutes one of the best methods known for approximating the exact shapes of gear teeth by means of circular arcs.

Odontograph Table for the Cycloidal System. — To apply the odontograph table for the cycloidal system of gear teeth, first

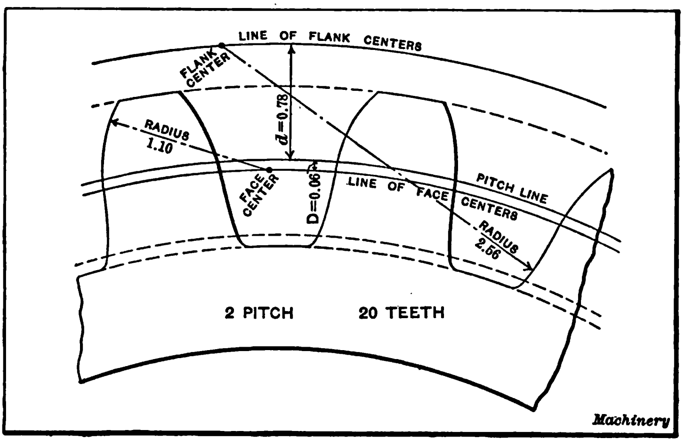


Fig. 10. Cycloidal System Gear Teeth laid out by the Aid of Grant's Odontograph

draw the pitch, addendum, root and clearance circles, as indicated in the engraving, Fig. 10, and space off the pitch of the teeth on the pitch circle in the usual way. Then draw the circle marked "line of flank centers" at the distance *d*, as given in the table, outside of the pitch line, and draw the "line of face centers" at the distance *D* inside of it. With the face radius *R* in the dividers, draw in all the face curves from centers located on the "line of face centers." Then with a flank radius *r*, draw all the flank curves from centers located on the "line of flank centers."

The table gives the distances *D* and *d* and radii *R* and *r* for

pitches either exactly one diametral or one-inch circular pitch. For any other pitch, divide or multiply as directed in the table. The illustration, Fig. 10, shows the method applied to laying out a 2 diametral pitch gear. The odontograph for the cycloidal system may also be applied to laying out teeth for internal gears.

Odontograph Table for The Involute System. — To draw the tooth curves, first lay off the pitch, addendum, root and clearance circles, and space off the teeth on the pitch line as indicated in Fig. 11. Draw the "base line" one-sixtieth of the pitch diameter

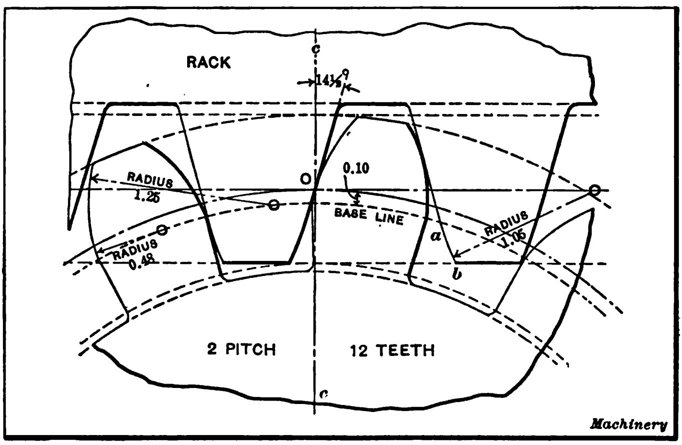


Fig. 11. Involute System Gear Teeth laid out by the Aid of Grant's Odontograph

inside the pitch line; then draw the faces of the teeth from the pitch line to the addendum line by using the face radius as given in the table for involute teeth, and with centers located on the "base line." If the pitch is any other than one diametral or one-inch circular pitch, divide or multiply the values given in the table as directed. To draw the flanks of the teeth from the pitch line to the base line, use the flank radius given with the centers on the base line; then draw straight radial flanks from the base line to the root line and round them off into the clearance line. The illustration, Fig. 11, shows the method applied to laying out a 2 diametral pitch gear.

The odontograph table for involute teeth can be used for internal gears in the same way as for external gears, but care must be taken that the tooth of the gear is cut off to avoid interference. In fact, the point of the tooth may be left off altogether, or rounded off. The pinion tooth need not be carried in to the usual root line, but may just clear the truncated tooth of the gear. No correction for interference is needed on the points of the pinion teeth or on the flanks of the gear teeth.

Grant's Odontograph Tables for Involute Teeth *

No. of Teeth in the Gear	Radii for One Diametral Pitch; for any other Pitch divide Values given by that Pitch		Radii for One Inch Circular Pitch; for any other Pitch multiply Values given by that Pitch		No. of Teeth in the Gear	Radii for One Diametral Pitch; for any other Pitch divide Values given by that Pitch		Radii for One Inch Circular Pitch; for any other Pitch multiply Values given by that Pitch	
	Face Radius	Flank Radius	Face Radius	Flank Radius		Face Radius	Flank Radius	Face Radius	Flank Radius
10	2.28	0.69	0.73	0.22	28	3.92	2.59	1.25	0.82
11	2.40	0.83	0.76	0.27	29	3.99	2.67	1.27	0.85
12	2.51	0.96	0.80	0.31	30	4.06	2.76	1.29	0.88
13	2.62	1.09	0.83	0.34	31	4.13	2.85	1.31	0.91
14	2.72	1.22	0.87	0.39	32	4.20	2.93	1.34	0.93
15	2.82	1.34	0.90	0.43	33	4.27	3.01	1.36	0.96
16	2.92	1.46	0.93	0.47	34	4.33	3.09	1.38	0.99
17	3.02	1.58	0.96	0.50	35	4.39	3.16	1.39	1.01
18	3.12	1.69	0.99	0.54	36	4.45	3.23	1.41	1.03
19	3.22	1.79	1.03	0.57	37–40	4.20		1.34	
20	3.32	1.89	1.06	0.60	41–45	4.63		1.48	
21	3.41	1.98	1.09	0.63	46–51	5.06		1.61	
22	3.49	2.06	1.11	0.66	52–60	5.74		1.83	
23	3.57	2.15	1.13	0.69	61–70	6.52		2.07	
24	3.64	2.24	1.16	0.71	71–90	7.72		2.46	
25	3.71	2.33	1.18	0.74	91–120	9.78		3.11	
26	3.78	2.42	1.20	0.77	121–180	13.38		4.26	
27	3.85	2.50	1.23	0.80	181–360	21.62		6.88	

* From "A Treatise on Gear Wheels," by George B. Grant.

Special Rule for Involute Rack. — Draw the sides of the rack teeth as straight lines inclined $14\frac{1}{2}$ degrees to the center line *cOc*, Fig. 11. Draw the outer half *ab* of the face by means of a circular arc having a radius of 2.10 inches divided by the diametral pitch, or 0.67 inch multiplied by the circular pitch, the center for this arc being on the pitch line of the rack.

Cutters for Involute and Cycloidal Teeth. — According to the system for cutting gear teeth adopted by the Brown & Sharpe Mfg. Co., Providence, R. I., any gear of one pitch will mesh with any other gear or with a rack of the same pitch. Eight cutters are required for each pitch. These eight cutters are adapted to cut from a pinion of twelve teeth to a rack, and are numbered, respectively, 1, 2, 3, etc. The number of teeth and the pitch for which a cutter is adapted is always marked on each. A list of these cutters is given in Table I.

Cutters for the cycloidal form of teeth are also made so that any gear of one pitch will mesh into any other gear or into a rack of the same pitch, but twenty-four cutters are required for each pitch. In order that gears with this form of teeth shall run well together, they must be cut accurately to the required depth; otherwise the pitch circles will not be tangent to each other. To secure a proper depth of tooth, the cutters are made with a shoulder which determines the exact depth that the tooth should be cut. Thus, if care is taken when turning the blanks, to obtain the correct outside diameter of the gear, no measurements need be taken when cutting the teeth. The twenty-four cutters are adapted to cut from a pinion of twelve teeth to a rack, and are designated by letters *A*, *B*, *C*, etc. The number of teeth and the pitch for which the cutter is adapted is always marked on each, the same as in the case of cutters for involute teeth. A list of these cutters is given in Table II.

Table I. Cutters for Involute Gear Teeth

No. of Cutter	Number of Teeth	No. of Cutter	Number of Teeth	No. of Cutter	Number of Teeth
1	135 to rack	4	26 to 34	7	14 to 16
2	55 to 134	5	21 to 25	8	12 to 13
3	35 to 54	6	17 to 20	....	....

Importance of Grinding Gear-cutter Teeth Radially. — Fig. 12 shows, diagrammatically, how the teeth of a milling cutter for gear teeth should be ground to secure the best results; it also illustrates improper grinding. The teeth *A* and *B* are ground

correctly. The lines AC and BC, lying in the plane of the cutting face, are radial; that is, the faces of the teeth would pass directly through the center of the cutter, if projected to the center. Tooth D, however, shows an entirely different condition, and one which unfortunately is not uncommon in gear-cutting practice. The top of the tooth was ground back faster than the base, thus throwing the face of the cutter into the plane indicated by the line DE; consequently the shape of the tooth space cut is distorted, and a gear with badly-shaped teeth must necessarily be produced by it.

The expression, "may be ground without changing the form," is often taken too literally and without the required qualification that it is necessary to grind in a plane radial with the center of the cutter in order that the form shall not be changed. It is evident to anyone who will give the matter a little thought that if a gear is cut with a gear-cutter having teeth ground like D the resulting tooth space will be too wide at the top, if the cutter is carried to the correct depth. Moreover, such a gear-cutter works badly, as the cutting faces of the teeth have a negative rake. The importance of correct grinding of all formed cutters can, therefore, not be too strongly emphasized. Unfortunately, formed cutters that can be ground without changing the form do not always have sufficient clearance to work well with all classes of work, and if such cutters are carelessly used there will be heating and rapid wearing away of the tops of the teeth. If hard pressed and ignorant, the tendency of the grinding operator,

Table II. Cutters for Cycloidal Gear Teeth

Letter of Cutter	Number of Teeth	Letter of Cutter	Number of Teeth	Letter of Cutter	Number of Teeth
A	12	*I*	20	*Q*	43 to 49
B	13	*J*	21 to 22	*R*	50 to 59
C	14	*K*	23 to 24	*S*	60 to 74
D	15	*L*	25 to 26	*T*	75 to 99
E	16	*M*	27 to 29	*U*	100 to 149
F	17	*N*	30 to 33	*V*	150 to 249
G	18	*O*	34 to 37	*W*	250 or more
H	19	*P*	38 to 42	*X*	Rack

in order to hurry the sharpening of such cutters, is to incline the wheel away from the radial plane.

Chordal Thicknesses and Addenda for Gear Teeth and Gear Cutters. — In measuring the thickness of gear teeth and gear cutters, it is necessary to make allowance for the curve of the pitch circle of the gear. In the following will be given formulas for finding the chordal thicknesses and what is called the "corrected" addenda, that is, the perpendicular distance from the

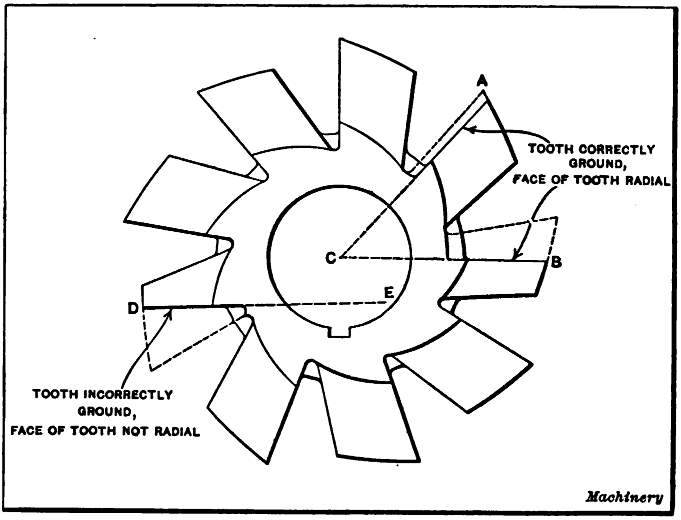


Fig. 12. Correct and Incorrect Methods of Grinding Gear-cutter Teeth

chord at the pitch circle to the outside diameter of the gear as indicated at H, in Fig. 13. Let,

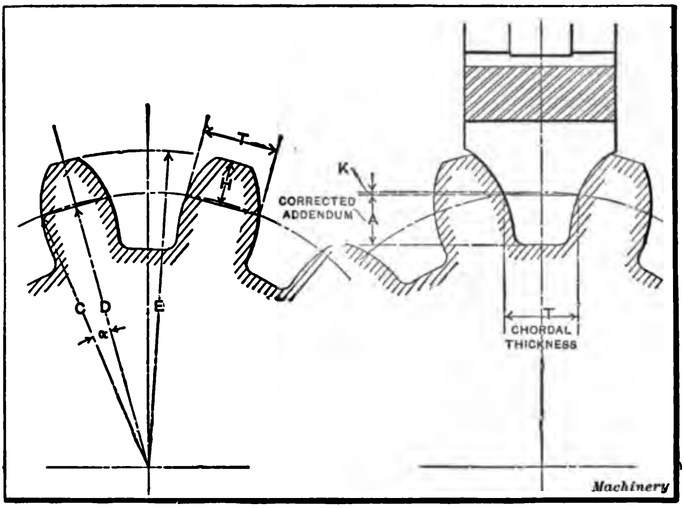
α = half the angle subtended from the center of the gear by one gear tooth (see Fig. 13);
N = number of teeth in gear;
T = chordal thickness of tooth at pitch line;
D = perpendicular distance from chord T to center of gear;
H = perpendicular distance from chord to outside circumference of gear;
C = radius of gear at pitch line;
E = outside radius of gear.

The formulas are as follows:

$$\alpha = \frac{90^\circ}{N}; \quad T = 2\,C \times \sin\alpha;$$

$$D = \sqrt{C^2 - (\tfrac{1}{2}\,T)^2}; \quad H = E - D.$$

In the case of the gear cutter (see Fig. 14), the chordal thickness is the same as that for the gear, but the corrected addendum of the gear cutter is different from the corrected addendum of the gear. The two dimensions, however, added together must equal the total depth of the gear tooth. To obtain the corrected



Figs. 13 and 14. Notation used in Formulas for Chordal Thicknesses and Addenda of Gear Teeth and Cutters

addendum A of the gear cutter, we can, therefore, either subtract the dimension H, as found by the previous formulas, from the dimension for the total depth of the tooth, or we can take the dedendum for the particular pitch required from any standard table of gear tooth parts and subtract the dimension K, Fig. 14, which is found by the formula:

$$K = C\,(1 - \cos\alpha).$$

Testing the Tooth Thickness when Milling Gear Teeth. — The special vernier gear-tooth caliper illustrated in Fig. 15 is sometimes used for testing the thickness of the first tooth milled.

This test is especially desirable if there is any doubt about the accuracy of the blank diameter. To test the tooth thickness, a trial cut is taken for a very short distance at one side of the blank; then the work is indexed for the next space, after which another trial cut is taken part way across the gear. The vertical scale of the caliper is set so that when it rests on top of the tooth, as shown, the lower ends of the caliper jaws will be at the height of the pitch circle. The horizontal scale then shows the chordal

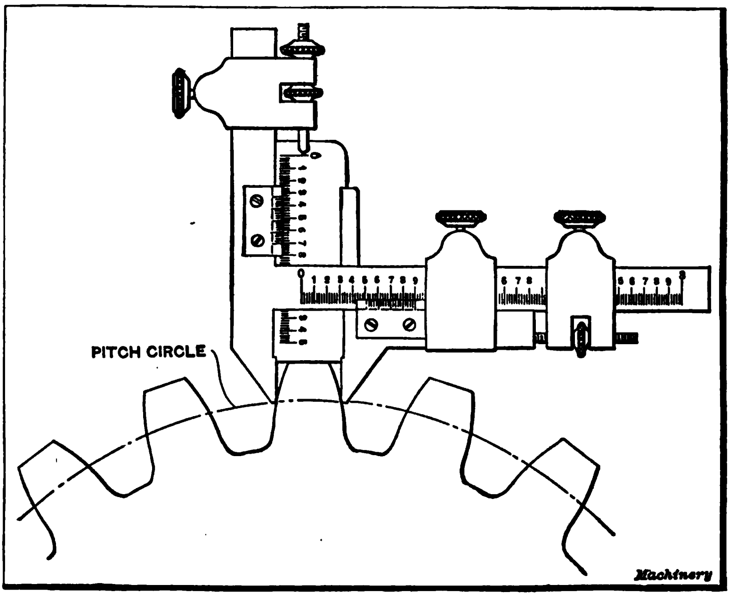


Fig. 15. Vernier Caliper for Measuring Thickness of Gear Teeth at the Pitch Circle

thickness of the tooth at this point. When a gear tooth is measured in this way, it is the chordal thickness T (see Fig. 13) that is obtained, instead of the thickness along the pitch circle, as explained in the previous paragraph. Hence, when measuring teeth of coarse pitch, especially if the diameter of the gear is quite small, dimension T should be obtained. It is also necessary to obtain the corrected addendum H, in order to measure the chordal thickness T at the proper point on the sides of the tooth.

Limits for Gearing. — The limits for center distance, pitch diameter and outside diameter of blanks, given in the table below, are applicable to spur gearing used under ordinary conditions. The + sign indicates dimensions over, and the − sign, dimensions under, the actual theoretical dimension.

Manufacturing Limits for Gearing

Diametral Pitch	Center Distance	Pitch Diameter	Blanks, Outside Diameter
16	±0.002	−0.003 to −0.005	0.000 to −0.005
14	±0.003	−0.004 to −0.006	0.000 to −0.005
12	±0.0035	−0.0045 to −0.007	0.000 to −0.006
10	±0.004	−0.005 to −0.008	0.000 to −0.006
8	±0.005	−0.006 to −0.009	0.000 to −0.007
6	±0.006	−0.007 to −0.010	0.000 to −0.008
5	±0.007	−0.008 to −0.011	0.000 to −0.010
4	±0.008	−0.009 to −0.012	0.000 to −0.015

Metric or Module System of Gear Teeth. — In the metric system, the diametral pitch is not used, but instead, the dimensions of gear teeth are expressed by reference to the *module* of the gear. The module is equal to the pitch diameter in millimeters divided by the number of teeth in the gear. For example,

Table of Tooth Parts for Metric or Module System Gear Teeth

Module	Circular Pitch, Inches	Thickness of Tooth at Pitch Line, Inches	Addendum, Inches	Whole Depth of Tooth, Inches
1¼	0.1546	0.0773	0.0492	0.1061
1½	0.1854	0.0927	0.0591	0.1273
1¾	0.2164	0.1082	0.0686	0.1486
2	0.2472	0.1236	0.0787	0.1698
2¼	0.2784	0.1392	0.0886	0.1911
2½	0.3092	0.1546	0.0984	0.2123
2¾	0.3402	0.1701	0.1083	0.2336
3	0.3710	0.1855	0.1181	0.2548
3½	0.4330	0.2165	0.1378	0.2973
4	0.4948	0.2474	0.1575	0.3397
4½	0.5568	0.2784	0.1772	0.3822
5	0.6186	0.3093	0.1969	0.4247
5½	0.6802	0.3401	0.2165	0.4670
6	0.7420	0.3710	0.2362	0.5095

if the pitch diameter of a gear is 50 millimeters and the number of teeth, 25, then the module equals 50 ÷ 25 = 2. The accompanying table gives a comparison between the module and the corresponding diametral pitch of gears.

Module and Corresponding Diametral Pitch

Module	Corresponding English Diametral Pitch	Module	Corresponding English Diametral Pitch	Module	Corresponding English Diametral Pitch	Module	Corresponding English Diametral Pitch
0.5	50.800	2.25	11.288	5.0	5.080	11	2.309
0.75	33.867	2.5	10.160	5.5	4.618	12	2.117
1.0	25.400	2.75	9.236	6.0	4.233	14	1.814
1.25	20.320	3.0	8.466	7.0	3.628	16	1.587
1.5	16.933	3.5	7.257	8.0	3.175	18	1.411
1.75	14.514	4.0	6.350	9.0	2.822	20	1.270
2.0	12.700	4.5	5.644	10.0	2.540	24	1.058

The module is also equal to the circular pitch in millimeters divided by 3.1416. Either rule gives the same result.

CHAPTER II

MATERIALS USED FOR GEARS

The principal materials used for gears are cast iron, steel (both from bar stock and in the form of forgings or castings), brass and bronze (from stock or castings), rawhide, and fiber.

Cast Iron. — Cast iron is, perhaps, the most used material for gears. It is one of the cheapest, and is the easiest to mold or cut to shape. It wears fairly well also. Its greatest disadvantages are its lack of resisting power to shock or impact, and the uncertainty of the quality of the casting into which it is formed. A casting is liable to various more or less serious defects, some of which may be visible from the exterior, while others are concealed; "blow-holes," "cold shuts," "scabs," etc., are of common occurrence where the foundry work is not skillfully done. Castings from iron are so cheap, however, that those containing such defects may be discarded without hesitation as soon as they are discovered. The prime advantage of the material, aside from its cheapness, is the facility with which it may be molded into any desired form, so that we may have large gears with arms, webs, projecting bosses, counterbalances, etc., to suit the mechanism being designed. These advantages will doubtless continue to keep cast iron the most used of all materials for ordinary work.

Cast Steel. — When greater strength than that obtainable in a cast-iron gear is required, and the gear is too large to be made from blanks cut from round stock or from forgings, steel castings are frequently used for gears. This is considered the best material for large, heavy-duty gears. The art of making castings from steel so that they will be sound throughout has, however, only become understood in the past few years, and blow-holes and rough castings are common enough, so that the use of this material still labors under some disadvantages, except in cases where the foundries are well equipped for doing this

class of work, when it is possible to turn out a first-class product. In the design of gears from cast steel, it seems that true economy is frequently lost sight of, for although cast steel is by far the stronger material, the arms, hubs and rims of cast-steel gears for similar duty are often made of the same proportions as those for cast-iron gears.

Comparison between Cast Iron and Cast Steel for Gears. — By far the greatest number of all gears are made from cast iron or cast steel. Concerning the relative advantages of these two materials there is much difference of opinion. Of late years, users of gears seem to have accepted, without qualifications, the idea that for large gears cast steel is the best material under any and all conditions, but nothing could be more erroneous than this, because under all ordinary conditions cast-steel gears have but a single sharply defined superiority over cast-iron gears, and that is greater strength; and while this is the main advantage, this fact has not been clearly apprehended by designers, because as already mentioned, no appreciable reduction has been made in the size of the hubs, arms or rims of gears, on account of the fact that they are made of steel — the cast-iron sizes still prevailing in most cases.

On the other hand, cast iron has a number of distinct advantages over cast steel, both from the designer's and the user's point of view. Cast iron is a much easier metal to work with than cast steel, having less shrinkage and less warpage than the latter; nor will it so readily transmit vibration. This latter peculiarity can readily be demonstrated by striking with a hammer on a wheel of cast steel and on one of cast iron. The tone of the resulting noise is much higher in pitch in the case of the cast steel, due to the shorter and more rapid vibrations. The noise from the cast iron is lower in tone and of shorter duration, due to the opposite conditions. Thus cast-iron gears immediately take the preference from the standpoint of closest approach to silence in operation. Only in exceptional cases is it necessary to use cast-steel gears for their superior strength. In the vast majority of cases a good cast-iron gear is amply strong enough for the service required of it.

Comparison between Cast-iron and Cast-steel Gears with Relation to the Methods of Production. — There are three conventional methods of making gears. When the teeth are to be cut, the rim of the pattern is made a solid blank. The resulting casting is bored, turned on its periphery and faced on both sides of the rim. The finished blank is then put on the gear-cutting machine and the teeth cut as desired. The two remaining methods relate to gears designed to have cast teeth. In the one case a full pattern is made; that is, all the teeth are cut on the pattern. This is the old-fashioned, expensive way which entails a vast deal of labor both in the laying out and the formation of the teeth. Although tooth-forming machines have done much to relieve this situation in the jobbing shops, the spacing of the formed teeth on the periphery of the wheel is extremely difficult to do accurately — simple as it looks to those who have never had to do it.

The other method referred to is that of machine-molding the gear. This is by far the best method, being the simplest, most accurate and the cheapest. All that is required in the way of a pattern is a sweep, a part of the hub, an arm, and a tooth block. The latter is fixed on the arm of the machine and two or three teeth, as may be desired, are rammed at a time, the spacing being done with absolute precision by the machine. The clearances in a well-made machine-molded gear may be nearly as close as in a cut gear of the same pitch. The comparative ease with which machine-molded iron gears can be made makes them by far the most economical to use for larger sizes, where extreme precision is not required, and if this fact were better known to engineers and designers of machinery, cast-iron gears produced by this method would invariably be specified.

Everyone familiar with foundry practice knows that the outside skin of a casting is its hardest part. This single peculiarity lends to machine-molded cast-iron gears the soundest argument in their favor. It takes long service to wear through this hard shell on the faces of the teeth, whereas in the case of cut gears, all of this valuable wearing surface is entirely cut away before the wheels go into service. Cast-steel gears do

not have these advantages. The high temperature at which steel is poured necessitates the use of silica sand in the molds. This has to be rammed so hard that machines cannot conveniently be used, making a full pattern necessary. Moreover, steel castings are hard to make; it takes a "real molder" to make a good one. Since nearly all the labor in the steel foundries is done by unskilled men, who do not appreciate the niceties of gear molding, it is extremely difficult to get a good cast-steel gear, accurate to pitch, round, and true to theoretical tooth contour. The silica sand burns to the faces of the teeth, which "scabs" and "pits" them, so that almost invariably every tooth face has to be chipped to secure even an indifferent bearing; hence the noise and clatter that is so noticeable in cast-steel gears with cast teeth. If the teeth do not bear across their entire face, the very object that the designer had in mind is altogether lost.

The matter of shrinkage in cast steel is difficult to control, so much so, in fact, that few manufacturers will guarantee large wheels to come near the pattern allowances. No two wheels will shrink just alike, so that the question of whether the wheels will preserve the circularity of the pattern is in grave doubt. Therefore, aside from exceptional cases where great strength is required, designers should be extremely chary of specifying cast-steel gears unless the teeth are to be cut. There is but one principal object in cutting the teeth of gear wheels; there are a number of minor reasons. The first is to insure perfect tooth contact; the others are to lessen the noise of operation, to improve the appearance of the wheels, to discover flaws in the material, etc.; but the expense of cutting is an appreciable consideration.

The argument is often advanced that steel wears longest. The hard skin on the teeth of machine-molded gears of cast iron so lengthens their time of service that even that argument is in grave doubt. We are brought face to face with the fact that, after all, for a large proportion of conditions met with, nothing can quite take the place of a well-made machine-molded cast-iron gear.

Materials for Medium-sized and Small Gears. — In a pair of gears the pinion is often made of steel, from bar stock or a drop forging, even when the larger gear is made from cast iron or some other material. Steel has the advantage over cast iron of being more resilient — that is, it offers a greater resistance to shock or impact. Since the pinion of a pair of gears naturally has teeth of a weaker form than those of its mate, it should be made of stronger material. Furthermore, there is usually less friction between two different metals in contact than between two parts made of the same material. Still further, the smaller of a pair of gears will wear out much faster than its mate, as each of its teeth is in action a greater number of times in a given period; so on this account as well, it should be made of the more resisting material. As the softer grades of steel, however, are not very durable, steel pinions are sometimes casehardened, or they may be made of high-carbon steel that can be hardened without requiring the action of carbonizing materials. By such processes the pinions gain in durability, but suffer somewhat in accuracy of outline, since the natural result of heat-treatment of any kind is to warp and distort the part treated. It is possible to grind hardened gears to the correct outline, leaving a little stock on the sides of the teeth, after cutting, for this purpose. There would, however, seem to be some doubt of the commercial success of the process, owing to the difficulty of keeping the grinding wheel to its proper shape, and keeping the mechanism of the machine itself in proper condition in the presence of the emery dust with which it is surrounded.

Steel blanks for medium-sized gears are sometimes drop-forged to bring the wheel to the desired shape. Fairly long hubs, a thickened rim, and a thin web can be formed in this way without requiring the form to be turned out of solid metal, with the attending waste of time and material. The steel drop forging, thus, has some of the advantages of the casting. It is more costly than the casting, but may be made of better material.

When gears are made of steel from the bar or from forgings, a wide range of physical qualities is offered. The steel may have almost any desired strength, hardness and resilience, or capacity

to resist shock. The matter of hardness is important in the case of high-speed gearing. Two soft materials working on each other at a high velocity tend to abrade each other, so one at least of a pair of gears so used should be of a fairly hard grade of steel.

Brass or Bronze Gears. — It is common and good practice to use a composition metal like brass or bronze for the smaller of a pair of lightly loaded gears which have to run at high speed. When such gears are run with a large gear of cast iron, the difference in texture between the two materials used lessens the friction, and there is a gain on the score of noiselessness as well. Brass may be used where the duty is very light; higher grades of material, like phosphor-bronze, are used for heavier service at high speed. When the service becomes quite severe, the materials in the gears should be reversed, so that the larger one is of phosphor-bronze, and the smaller one of steel. The pinion has thus its maximum of strength and durability, at the same time that the advantages resulting from the use of dissimilar materials are retained.

Rawhide Gears. — Where noiselessness is a prime consideration, rawhide is extensively used. This non-metallic substance possesses the required structure to deaden the sound vibrations, together with a considerable degree of toughness, when properly cured. Manufacturers of gear blanks from this material cure the hide by processes which they claim give far better results for this service than can be obtained by ordinary means. The material is not injured by oil, though it does not require lubrication in service; but there has been some complaint of its swelling and losing its shape when exposed to moisture. Trouble from this source may, however, have been due to the use of an inferior grade of material, because thousands of rawhide pinions are in satisfactory daily use, under all sorts of conditions, at the present time. It is a somewhat more costly material than the others commonly used, but its compensating freedom from noise is often worth more than the added expense. But one of a pair of gears — generally the pinion — is made from this substance, the gear being of steel or iron. Gears as large as 40 inches in diameter have been made from this material.

Fiber Gears. — Fiber is another material used under about the same conditions as rawhide. It is not as strong, and it suffers under the disadvantage of being difficult to machine, owing to its peculiar gritty structure. It is also liable to swell in the presence of moisture. It has an advantage over rawhide in that it is comparatively inexpensive, and may be purchased in a variety of sizes of bars, rods, tubes etc., so that it is convenient to use at short notice. For light duty at high speed it serves its purpose very well.

Cloth Gears. — A new material for gears has been introduced within recent years by the General Electric Co.; here cloth pinions are employed in gear transmissions where, because of the noise or for other reasons, the meshing of metallic pinions with metallic gears would be impracticable or undesirable. The advantages claimed for these pinions are the great tooth strength, the noiseless operation, freedom from damage by exposure to dampness, dryness or temperature changes, the elasticity of the teeth, which will absorb shocks liable to break cast iron or brass pinions, and the long life of the gears.

The blanks from which these cloth pinions are made consist of a filler of cotton or similar material which is confined at a pressure of several tons to the square inch between steel side plates which are held together by threaded rivets, or, in the case of very small pinions, by threaded sleeves. After the teeth are cut, the cloth filler is impregnated with oil, and becomes entirely impervious to moisture and unaffected by atmospheric changes. The strength of these cloth pinions is equal to that of any other non-metallic gearing, and as the slight elasticity gives the meshing teeth a good bearing across the full width of the face, together with good shock-absorbing qualities, these gears can safely be used for practically any service within the limits of strength of cast-iron gears. The thickness of the side plates of these pinions varies somewhat with the diameter and pitch, but, in general, it is approximately equal to the thickness of the teeth at the pitch line. The width of the cloth face is equal to the width of the gear with which the pinion is in mesh, plus the total end play of both shafts, so that the side plates will not come in contact with the meshing gear.

Application of Cloth Pinions. — These cloth pinions are intended for use on machine tools such as lathes, planers, drill presses, shears, punches, etc., and are especially recommended for motor-driven tools. They are also used on traveling cranes and for driving looms and spinning frames and for heavy paper and pulp mill machinery, gas engine ignition drives, etc. The invention of the cloth pinion was the result of a search for a pinion to successfully operate a combined punch and shear used in one of the General Electric Co.'s forge shops. This machine was originally fitted with a train of gears consisting of a brass pinion on a motor shaft driving a cast-iron cut gear on a countershaft, which, in turn, drove the main gear through another pinion. A heavy flywheel was mounted on the countershaft and the backlash caused the repeated stripping of the teeth of the cast-iron gear. The brass pinion also rapidly lost its shape and considerable trouble was experienced. The idea of a cloth pinion was then conceived and applied, and a pinion of this material has now been in constant operation for several years. It requires no attention, runs without noise, and shows, as yet, no appreciable signs of wear.

Materials for Racks. — Racks of large size, such as those used for driving the platens of metal planers, are made of iron or steel castings. Smaller ones are made from bar steel stock, either machine steel finished all over, or cold-rolled steel. The latter material does not require other machining than the cutting of the teeth, being accurately finished to certain convenient sizes in the process of rolling. The cutting of the teeth causes the stock to bend, however, necessitating a straightening operation.

The principle of a device used for the purpose of straightening racks made from cold-rolled steel is shown in Fig. 1. This device was designed at the R. K. LeBlond Machine Tool Co., Cincinnati, Ohio. A plain milling machine is used for the purpose. An arbor is mounted in the spindle carrying ordinarily three gears, *A*, *B* and *C*, of the pitches most commonly used. On the table is clamped a channel casting *D*, which is provided with four slots in each side in which may be placed the rollers *E*. The use

of the device is apparent upon inspection. The rollers are dropped into place at such a distance apart as best suits the work in hand, and are brought in line with the proper gear on the arbor, which is then located centrally between the two rollers. The rack is now fed in between the rollers and the gear, and the table is brought up until enough pressure is exerted on the rack to straighten it. The spindle is revolved slowly and the rack feeds through the device and is bent back into shape by the pressure between the rollers and the pinion. The gears *A*, *B* and *C* are so made that they bear on the tops of their teeth as well

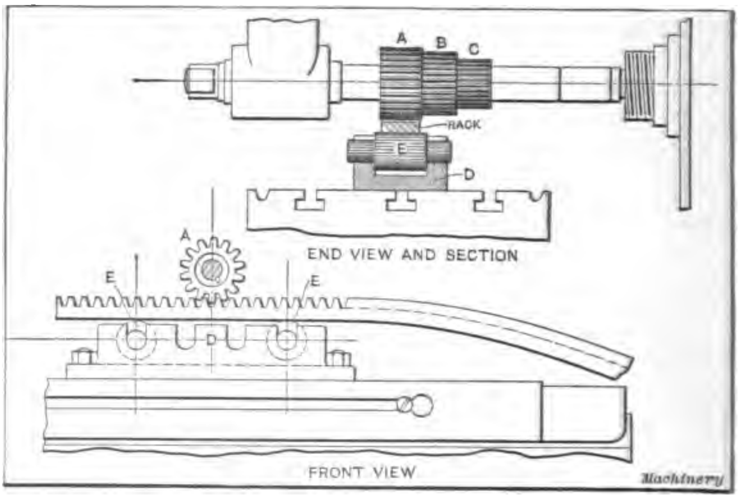


Fig. 1. Straightening Cold-rolled Steel Racks in a Milling Machine

as on each side. This prevents stretching the racks when in mesh with the gears. Were this not so, the wedging action of the gear teeth under heavy pressure would spread the rack teeth, increase the pitch, and, to some degree, lengthen the rack. This device works well for small and medium-sized racks. For larger racks, special machines may be required, but the principle employed remains the same.

Gears for Machine Tool Drives. — In a paper read before the American Society of Mechanical Engineers, December, 1913, Mr. J. Parker, of the Brown & Sharpe Mfg. Co., considered the

six following questions relating to the use of gears for driving machine tools:

1. Under what conditions is it advisable to use cast-iron or steel gears for machine-tool drives?

2. Are the objections to cast iron on the ground of wear or breakage?

3. What tooth pressure is safe for cast-iron gears?

4. What grades of steel give best results and how should they be treated?

5. How hard is it advisable to make steel gears before machining them?

6. Are they to be hardened after machining, and if so, to what scleroscope test?

Conditions Governing Use of Cast-iron and Steel Gears. — There are a number of well-established gear conditions that are common to the majority of machine tools, which, if noted, may prove somewhat of a guide in selecting the proper material for the gears, considered from the standpoints of economy, efficiency and durability. The conditions may be classified as in the accompanying table. The objections to cast iron cover both wear and breakage. If the speed is excessive, say about 500 feet per minute, they are likely to wear quite rapidly; and on slow speeds and heavy pressure breakage will occur, unless they can be made of adequate size, as in the case *E*, where the back-gears are so located in the machine that it is possible to employ large diameters, coarse pitches and wide faces.

Safe Tooth Pressure for Cast-iron Gears. — The question of tooth pressures in cast-iron gears is somewhat problematical. The Brown & Sharpe Mfg. Co. has in successful operation a gear, in the spindle drive of its largest milling machine, made from a hard, close-grained cast iron having a tensile strength of 23,000 pounds per square inch, which, when running at the slowest speed, sustains a pressure on the teeth of 8250 pounds. It is calculated that two teeth are always in contact, which gives 4125 pounds pressure per tooth. The area of cross-section of each tooth is $1\frac{1}{4}$ square inch, giving 3300 pounds per square inch. When the gear runs at the fastest speed the pressure is about 1000 pounds

Gear Conditions Common in Machine Tools

			Material
A	Gears always in mesh, the wear on the teeth being constant.	(*a*) Slow speeds, light duty (*b*) Slow speeds, heavy duty (*c*) Fast speeds, light duty (*d*) Fast speeds, heavy duty.	Cast iron Machine steel Machine steel Machine steel, casehardened.
B	Gears in sets that are removable and interchangeable with each other, distributing the wear over a number of gears.	These are change gears used in thread cutting on lathes, spiral cutting on milling machines, indexing on automatic gear cutters and feed and speed change gears; speeds and pressures are generally moderate.	Cast iron, excepting the smallest, which may be of steel.
C	Gears in sets that are non-removable and partially interchangeable, distributing the wear over a number of gears. Changes made while gears are in motion.*	Used as quick-change feed gears — changes made by levers; speeds and pressures moderate.	Machine steel, casehardened.
D	Gears in sets that are non-removable and partially interchangeable, distributing the wear over a number of gears. Changes made when gears are at rest.†	Used as quick-change speed gears—changes made by levers; high speeds and heavy pressure.	Machine steel, casehardened.
E	Gears that are employed only part of the time the machine is working, and are engaged and disengaged when the machine is stopped.	This condition applies to back-gears for the spindle drive. Gears are made large diameter, coarse pitch and wide face; speeds moderate, and heavy pressure.	Hard, close-grained cast iron.

* If the changes were made when the machine was at rest, the gears would not require hardening, but custom demands that changes be made while the machine is running.

† Although the changes are supposed to be made when gears are at rest, careless workmen will violate this rule, with the possibility of breaking the engaging gears. Some makers use an alloy steel in their spindle train to prevent breakage, but a better way is to provide means whereby it is necessary to stop the machine before throwing in the gears. This applies to the tumbler type of change gearing.

per square inch. It is not known whether the pressure could be increased to any considerable extent, but the gear has been overloaded to at least 30 per cent without injuring it; this was when testing out the machine and the overload was of short duration. It might be said that this gear is not subjected to any sudden shock; if it were, the tooth pressure would have to be less.

Grades of Steel that give Best Results. — For gears that are of small proportions and yet are subjected to heavy duty, it has been found that in cases where the more common steels have failed, excellent results have been obtained by using a 5 per cent nickel steel. This steel casehardens with a very hard surface and still has a strong and tough core, making it an ideal steel to use where the pressure is heavy or the gear is subjected to shock. Experience shows that drop-forgings are more uniform in texture than bar stock.

Heat-treatment of Steel for Gears. — The 5 per cent nickel steel referred to in the preceding paragraph is given an oil treatment and is also annealed before machining. The oil treatment is as follows: Heat to 1550 degrees F. and quench in oil. To anneal, reheat to 1350 degrees F. and cool very slowly. The steel is then ready to machine. After machining, it is carbonized as follows: Pack in any good carbonizing material and cover very carefully to exclude air; place in furnace and heat to 1700 degrees F., and hold long enough to get the desired depth of casing. Care should be taken to have the steel heated entirely through. Ordinarily from three to four hours is sufficient. Then remove the steel from the furnace and cool in the boxes; next remove from the boxes and place in a furnace or bath; reheat to 1550 degrees F. and quench in oil. Again reheat to about 1380 degrees F. and quench in oil or water, according to the size and shape of gear. If the gear is of generous dimensions and free from sharp corners, water is preferable. Small slender gears are quenched in oil, on account of the liability of cracking if water is used. For ordinary gears the scleroscope test should show 80 to 85 points of hardness. If the gears are used as clash gears they should be drawn to 475 degrees F., or about 70 to 75 points of hardness, by scleroscope test, to avoid chipping.

Hardness Advisable for Steel Gears before Machining. — The kinds of steels generally used for gears are of such a nature that they do not call for heat-treatment before machining, but where extra toughness is required to withstand torsion and bending strains, $3\frac{1}{2}$ per cent nickel steel is satisfactory and is heat-treated as follows, after being rough-machined: Place in an open furnace or bath, heat to 1500 degrees F. and quench in oil. It is advisable to experiment with a small quantity in each batch, before subjecting a whole lot to the drawing-out heat, which should begin at about 700 degrees F. If the scleroscope registers between 50 and 58, the correct hardness has been obtained; if higher than 58, the parts should be reheated to a higher temperature than before; if lower than 50, the parts must be rehardened. After this treatment, the pieces are finish-machined. No further hardening is necessary. When machining, slow speeds and feeds must be used.

Hardening after Machining, and the Scleroscope Test. — Practically all alloy and all low-carbon steels are hardened after machining and finished by grinding after hardening. About 0.010 inch on the diameter is left for this operation. All gears should run true, and to obtain this result not only are the holes ground true with the pitch circle, but the hubs are ground on their faces so that they will set square with their shafts when tightened up by nuts. The scleroscope test for 0.30 to 0.35 per cent carbon machine steel is anywhere from 80 to 90, and for 5 per cent nickel steel for ordinary gears, 80 to 85, and for clash gears, 70 to 75. All steels are tested by the file in addition to the scleroscope. The file test by an expert is very reliable and some feel that possibly more confidence can be placed on his judgment than on any testing instrument.

The above notes apply to spur and bevel gears. For worm and worm-wheel drives, the worm should be made of machine steel, casehardened, and the wheel of a hard bronze. Both should run in a bath of oil, especially if working under high speed and heavy duty. Spiral gears should be used only where the duty is light. The material should be the same as for a worm and worm-wheel, and they should also run in oil to avoid cutting.

CHAPTER III

STRENGTH AND DURABILITY OF SPUR GEARING

Strength of Gear Teeth. — There is a great deal of discrepancy between the various rules published by different authorities, largely due to the many varying conditions met with in gearing, these conditions depending upon the materials, the methods of producing the gear teeth, the conditions of service, etc. It might be well said that no rule for the strength of gear teeth is complete unless it states definitely for what class of gears it is intended. For cut gears the Lewis formula, given in the following, is accepted as standard. For cast gears it would seem that the best method would be to use the Lewis formula so modified as to give a factor of safety two or three times that required by this formula, depending upon the severity of the conditions in each case. A general rule for cast gears, used under so many varying conditions, could hardly be satisfactorily devised. The designer's judgment must be depended upon to a considerable extent in the selection of the suitable factor of safety which will be required in excess of that required for cut gears.

As an indication of the uncertainty of the rules given by various authorities, Mr. Lewis mentions in his paper on the "Strength of Gear Teeth," read before the Engineers' Club of Philadelphia, October 15, 1892, and published in the proceedings of the club, January, 1893, that Mr. J. H. Cooper in making an investigation found that there were not less than forty-eight well-established rules in existence for the strength and capacity of gear teeth, these rules being sanctioned by twenty-four different authorities and giving gear teeth varying 500 per cent in ultimate strength.

The Lewis Formula. — The accompanying tables of "Rules and Formulas for the Strength of Gear Teeth," "Factors for Calculating the Strength of Gear Teeth" and "Working Stresses

used in the Lewis Formula for the Strength of Gear Teeth" make it possible to quickly calculate the strength of spur gears. The formulas and factors given are based on the use of the diametral pitch, and the constants Y given in the factor table are valid only when the diametral pitch is used. If the circular pitch is given, it should be transformed into diametral pitch by dividing 3.1416 by the circular pitch. By means of the formulas given, the

Working Stresses Used in the Lewis Formula for the Strength of Gear Teeth

Velocity in Feet per Minute = V	Strength Factors	Safe Working Unit Stress = S, in Pounds per Square Inch					
		Cast Iron		Phosphor Bronze		Steel	
		Ordinary Workmanship	High-grade Workmanship	Ordinary Workmanship	High-grade Workmanship	Ordinary Workmanship	High-grade Workmanship
0	1.000	6000	8000	9000	12,000	15,000	20,000
100	0.857	5150	6850	7700	10,300	12,800	17,100
200	0.750	4500	6000	6750	9,000	11,200	15,000
300	0.666	4000	5350	6000	8,000	10,000	13,300
450	0.571	3400	4550	5150	6,850	8,550	11,400
600	0.500	3000	4000	4500	6,000	7,500	10,000
900	0.400	2400	3200	3600	4,800	6,000	8,000
1200	0.333	2000	2650	3000	4,000	5,000	6,650
1800	0.250	1500	2000	2250	3,000	3,750	5,000
2400	0.200	1200	1600	1800	2,400	3,000	4,000

horsepower which can be transmitted by a gear of a given pitch diameter and diametral pitch, running at a given number of revolutions per minute, can be found by using Formulas (1) to (4) in the order given. The allowable static unit stress for the material in the gear is selected from the first line (velocity = 0) in the table of working stresses; the stress at any given velocity may also be found directly from the table.

The utility of the Lewis formula is due to its simple form, to the fact that it takes into account a greater number of factors than does any other, and to the fact that the effect of each of these factors is rationally expressed in the formula.

Example of Application of Formula. — As an example of the application of the Lewis formula to an actual problem, assume

that it is required to find the horsepower which it is permissible to transmit by a spur gear having 15-inch pitch diameter, 4 diametral pitch, making 100 revolutions per minute, and having a width of face 1½ inch, the teeth being cut according to the 14½-degree involute system. The gear is made of steel and the allowable static unit stress for the material may, therefore, be assumed to be 15,000 pounds per square inch. First insert the values of the revolutions per minute and the pitch diameter in Formula (1) and thus find the velocity in feet per minute at the pitch diameter.

Rules and Formulas for the Strength of Gear Teeth

(Based on the Lewis Formula)

D = pitch diameter of gear in ins.; R = revolutions per minute; V = velocity in ft. per min. at pitch diameter; S_s = allowable static unit stress for material; S = allowable unit stress for material at given velocity; A = width of face in inches; Y = outline factor (see table); P = diametral pitch (if circular pitch is given, divide 3.1416 by circular pitch to obtain diametral pitch); W = maximum safe tangential load in pounds at pitch diameter; H.P. = maximum safe horsepower.

Use rules and Formulas (1) to (4) in the order given.

No.	To Find	Rule	Formula
1	Velocity in feet per min. at the pitch diameter.	Multiply the product of the diameter in inches and the number of revolutions per minute, by 0.262.	$V = 0.262\,DR$
2	Allowable unit stress at given velocity.	Multiply the allowable static stress by 600 and divide the result by the velocity in feet per minute plus 600.	$S = S_s \times \frac{600}{600 + V}$
3	Maximum safe tangential load at pitch diameter.	Multiply together the allowable stress for the given velocity, the width of face, and the tooth outline factor; divide the result by the diametral pitch.	$W = \frac{SAY}{P}$
4	Maximum safe horsepower.	Multiply the safe load at the pitch line by the velocity in feet per minute, and divide the result by 33,000.	$\text{H.P.} = \frac{WV}{33{,}000}$

$$V = 0.262 \times 15 \times 100 = 393 \text{ feet per minute.}$$

This velocity, together with the allowable static unit stress, is then inserted in Formula (2), and the allowable unit stress at the given velocity is found.

$$S = 15{,}000 \times \frac{600}{600 + 393} = 9000 \text{ pounds, approx.}$$

This unit stress is now inserted in Formula (3) together with the width of face, the outline factor Y (which is found from the table to be 0.358 for 60 teeth) and the diametral pitch; in this way, the maximum safe tangential load W is found.

Factors for Calculating Strength of Gear Teeth

No. of Teeth	Outline Factor = Y		No. of Teeth	Outline Factor = Y		No. of Teeth	Outline Factor = Y	
	14½° Involute and Cycloidal	20° Involute		14½° Involute and Cycloidal	20° Involute		14½° Involute and Cycloidal	20° Involute
12	0.210	0.245	20	0.283	0.320	43	0.346	0.396
13	0.220	0.261	21	0.289	0.327	50	0.352	0.408
14	0.226	0.276	23	0.295	0.333	60	0.358	0.421
15	0.236	0.289	25	0.305	0.339	75	0.364	0.434
16	0.242	0.295	27	0.314	0.349	100	0.371	0.446
17	0.251	0.302	30	0.320	0.358	150	0.377	0.459
18	0.261	0.308	34	0.327	0.371	300	0.383	0.471
19	0.273	0.314	38	0.336	0.383	Rack	0.390	0.484

$$W = \frac{9000 \times 1.5 \times 0.358}{4} = 1210 \text{ pounds, approx.}$$

Finally, by inserting the value of W just found and the value V found from Formula (1) in Formula (4), we determine the maximum safe horsepower which can be transmitted by the gear.

$$\text{H. P.} = \frac{1210 \times 393}{33{,}000} = 14.4.$$

General Remarks Relating to the Use of the Tables and Formulas. — It will be noted that two columns of values are given for each material in the table "Working Stresses used in the Lewis Formula for the Strength of Gear Teeth"; as explained, the

first set of values may be used for workmanship of ordinary grade, while the other is permissible with a higher grade. The second column, of strength factors, may be used for finding the allowable working fiber stress to use for any given speed, when the safe static stress is known; to find the fiber stress, multiply the safe static stress by the strength factor. Formula (2) in "Rules and Formulas for the Strength of Gear Teeth" may be used for the same purpose, giving results closely approximating the values in the table.

The variable factor introduced into the problem by the varying shape of teeth of the same pitch, in gears of different numbers of teeth, is taken care of by introducing in the formula the outline factors Y given in the table "Factors for Calculating Strength of Gear Teeth." These factors are given for that arrangement of the formula which applies to diametral pitches.

The formulas in the chart take no account of any such limitation of the width of face for a gear of given pitch as obtains in practice. The strength is made to increase directly with the width of face, without limit. In practice, if the face is too long in proportion to the size of the tooth, we cannot be sure that each tooth of the gear has a full bearing over its whole length on its mating tooth in the other gear. The shafts on which the two are mounted may not be parallel, or, if originally parallel, they may deflect under the strain of the load transmitted. For reasons like this, in ordinary commercial work, the width of face may be considered as well proportioned when it equals, in inches, 8.75 divided by the diametral pitch. As a formula, this gives us $A = \frac{8.75}{P}$.

In cases where accurate workmanship can be depended on, there is a gain in using teeth of wider face and finer pitch than would be allowed by Formulas (1) to (4). There is a gain in efficiency, smoothness of action, and noiselessness, especially at high speeds. In fact, the width of face of a gear may well be made to depend in part on the speed at which it is run, as well as on the pitch, it being taken for granted, of course, that the pitch and width are such as to give the required strength.

Relation between Width of Face and Diametral Pitch. — The rules and formulas for the strength of gear teeth can be used for finding the pitch and width of gear teeth for transmitting a given horsepower, if used in combination with a formula giving the best relation between pitch and width of face. An accepted rule for this relation is as follows: To find a well-proportioned width of face for carefully made gearing, multiply the square root of the pitch line velocity in feet per minute by 0.15, add 9 to the product, and divide the result by the diametral pitch; or, if A = width of face in inches; V = pitch line velocity in feet per minute; and P = diametral pitch, then:

$$A = \frac{0.15\sqrt{V} + 9}{P}.$$

Example: — What should be the pitch and the width of face of a steel pinion, 4 inches in pitch diameter, the teeth shaped according to the 14½-degree involute system, running 750 revolutions per minute, and transmitting 10 horsepower? The workmanship is high grade, and the width of the face is to be proportioned according to the rule and formula given immediately above.

The velocity at the pitch line = $0.262\ DR = 0.262 \times 4 \times 750 = 786$ feet per minute. (See Formula (1) in the table for strength of spur gears.) The allowable running stress for a static stress of 20,000 pounds per square inch is found by Formula (2):

$$S = 20{,}000 \times \frac{600}{600 + 786} = 8660 \text{ pounds per square inch.}$$

The load at the pitch line is equal to

$$\frac{10 \times 33{,}000 \times 12}{\pi \times 4 \times 750} = 420 \text{ pounds.}$$

Assume 5 diametral pitch as a trial pitch for the teeth, then the number of teeth equals $5 \times 4 = 20$. Transposing Formula (3):

$$W = \frac{SAY}{P}, \text{ gives } A = \frac{WP}{SY}$$

Apply this transposed formula and find a trial width of face (factor Y is given in the table of "Factors for Calculating Strength of Gear Teeth"):

$$A = \frac{420 \times 5}{8660 \times 0.283} = 0.9 \text{ inch, approximately.}$$

For 5 pitch, however, according to the formula

$$A = \frac{0.15\sqrt{V} + 9}{P}$$

the width of the face should be

$$A = \frac{0.15\sqrt{786} + 9}{5} = 2.64 \text{ inch.}$$

Thus, the pitch is evidently too coarse. Repeated trials show that with 9 diametral pitch the results from the two formulas for width of face agree fairly well. Thus:

$$\text{Number of teeth} = 9 \times 4 = 36.$$

$$A = \frac{420 \times 9}{8660 \times 0.332} = 1.32 \text{ inch.}$$

$$A = \frac{0.15\sqrt{786} + 9}{9} = 1.47 \text{ inch.}$$

Hence, 9 diametral pitch and $1\frac{3}{8}$-inch width of face are the dimensions to which the gear ought to be made.

Derivation of the Lewis Formula. — The factor Y in the Lewis formula is derived as follows: In Fig. 1 a $14\frac{1}{2}$-degree involute tooth is shown in outline. The pressure F transmitted to the gear tooth at an angle of $14\frac{1}{2}$ degrees to a tangent to the pitch circle may be considered as having two components, one of which is radial and tending to crush the tooth, and one tangential, tending to break off the tooth. The only component we need to consider in connection with the strength of gear teeth is the tangential component W. This component, as shown in Fig. 1, is not considered as acting upon the tooth at its top, but at the point B where the line representing the pressure F intersects the center line of the tooth. The line of pressure, of course, is considered as passing through the corner of the tooth as indicated.

Any parabola having the line BE for an axis and passing through the point B, so that the line representing force W is a tangent to the parabola at this point, encloses a beam of uniform strength. By graphical construction it can now be determined in each case where the point of tangency between the parabola and the contour of the tooth form occurs. The point of tangency determines the weakest section of the tooth, as shown at CD. This weakest section is not necessarily at the very root of the tooth, but may — particularly in gears with a small number of teeth — be located a considerable distance above the root of the tooth; hence the distance L, the length of the beam, is measured not to the bottom of the tooth, but to the line CD.

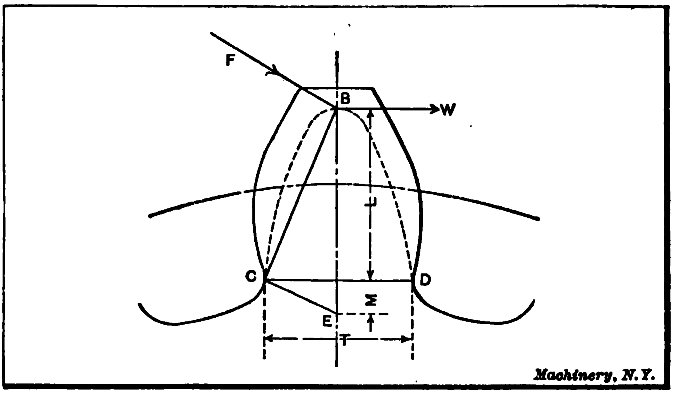


Fig. 1. Diagram used in the Derivation of the Lewis Formula

In order to determine the value Y in the Lewis formula, draw the lines BC and CE, the latter being at right angles to BC and intersecting the center line of the tooth at E. In the following formulas:

S = the fiber stress;
A = the width of the face of the gear;
P = circular pitch of gear;

L, M, T and W denote quantities as indicated in the illustration.

Then we have:

$$WL = \frac{SAT^2}{6}, \text{ or } W = \frac{SAT^2}{6L}$$

But, by similar triangles,

$$M = \frac{T^2}{4L}$$

Hence

$$W = SA \times \frac{2}{3} M = SAP \frac{2M}{3P}$$

where $\frac{2M}{3P} = Y$, the factor in the Lewis formula, for *circular* pitch.

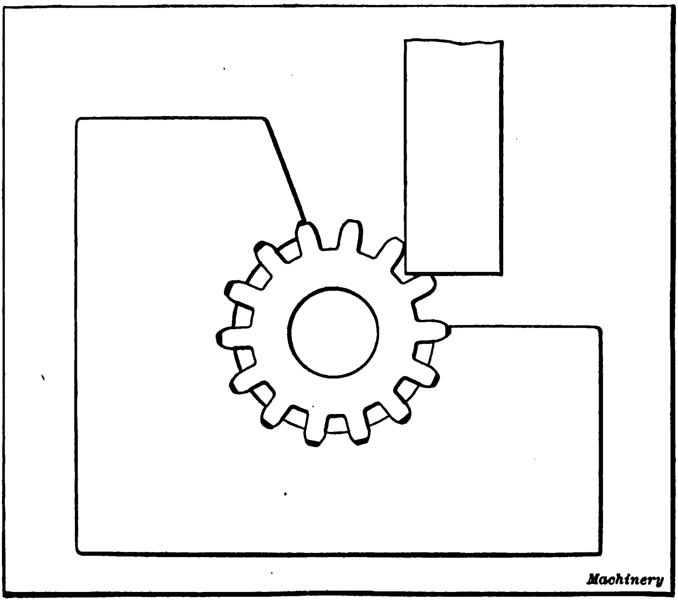


Fig. 2. Method of applying Load to Tooth for Testing Strength

Tests for Strength of Gear Teeth. — In a paper read before the National Machine Tool Builders' Association, October, 1913, Mr. A. C. Gleason gave some information relating to tests of the strength of heat-treated gears. The accompanying table gives the average results obtained from a considerable number of test pieces subjected to the different heat-treatments specified. The test pieces were spur pinions having fourteen teeth of six pitch, 2⅓ inches pitch diameter, 1-inch face width and 1-inch bore. All teeth were of standard depth and shape. During the tests the pinions were held in a special chuck as shown in the accompany-

Ultimate Strength and Elastic Limit of Gear Teeth for Different Steels and Heat-treatments

No. of Test	Heat-treatment	Elastic Limit	Breaking Strain
	0.20 per cent Carbon, Open-hearth Casehardening Steel		
No. 1.	Soft	3,500	....
No. 2.	Casehardened with one tempering heat of 1450° F., 90 scleroscope hardness	8,400	9,000
	Same drawn to 400° F., 85 scleroscope hardness	8,000	9,450
No. 3.	Casehardened with two tempering heats of 1600° F., and 1450° F., 85 scleroscope hardness	8,200	9,675
	Same drawn to 400° F., 80 scleroscope hardness	8,000	9,800
	1½ per cent Nickel, 0.18 per cent Carbon, Natural Alloy Steel		
No. 1.	Soft	4,000	....
No. 2.	Casehardened with one tempering heat, 92 hardness	9,000	10,150
	Same drawn to 400° F., 87 hardness	8,600	10,450
No. 3.	Casehardened with two tempering heats	8,750	10,600
	Same drawn to 400° F., 82 hardness	8,400	10,750
	3½ per cent Nickel, 0.13 per cent Carbon, Open-hearth Nickel Alloy		
No. 1.	Soft	4,000	....
No. 2.	Casehardened with one tempering heat of 1350° F., 90–95 hardness	9,700	11,400
No. 3.	Casehardened with two tempering heats of 1550° F., and 1350° F., 90–95 hardness	9,500	11,650
	Same drawn to 400° F., 85 hardness	9,250	11,950
	5 per cent Nickel, 0.15 per cent Carbon, Open-hearth Alloy Steel		
No. 1.	Soft	4,500	....
No. 2.	Casehardened with one tempering heat of 1350° F., 90 hardness	13,000	13,880
	Same drawn to 400° F., 85 hardness	12,700	14,800
No. 3.	Casehardened with two tempering heats of 1550° F., and 1350° F., 90 hardness	13,000	14,100
	Same drawn to 400° F., 85 hardness	12,800	14,850
	(A) Chrome-nickel Tempering Steel		
No. 1.	Quenching heat 1425° F., drawing temperature 475° F., 65–70 hardness	....	22,450*
No. 2.	Soft	5,000	....
	(B) Chrome-nickel Tempering Steel		
No. 1.	Quenching heat 1480° F., drawing temperature, 525° F., 75–78 hardness	....	17,640*
No. 2.	Quenching heat, 1480° F., drawing temperature, 850° F., 60–65 hardness	....	19,440*
No. 3.	Soft	5,200	....

* No set before breaking.

ing illustrations, the load being applied to the edge of the teeth as indicated in Fig. 2. The initial displacement or elastic limit was determined by following up the load with a vernier tooth caliper which showed when permanent set had occurred (see Fig. 3). All carbonizing was $\frac{1}{32}$ inch deep, and the furnace temperature varied from 1450 to 1500 degrees F., for eight hours, and was held at 1600 degrees for from three to five hours — a total of eleven to thirteen hours.

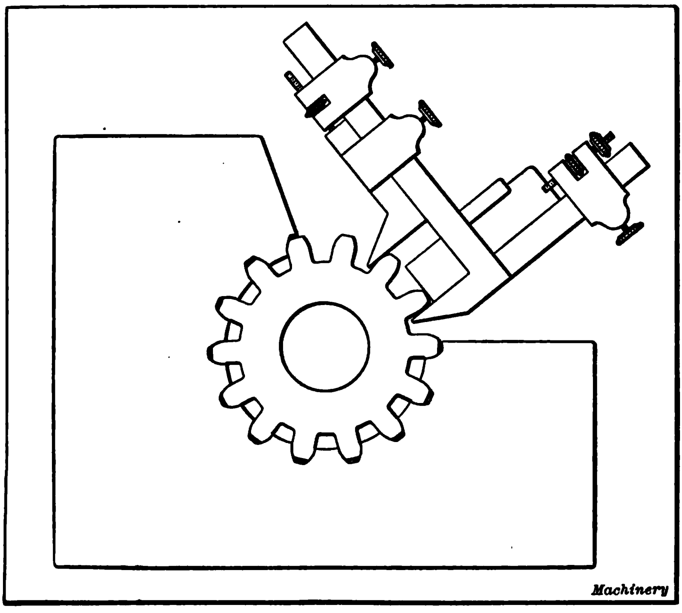


Fig. 3. Gear-tooth Caliper used to determine Permanent Deflection

It will be noted that the increased strength, as a result of the double heat-treatment after carbonizing, is comparatively slight, particularly in the nickel-alloy steels. This is undoubtedly due to the long and low carbonizing heats and the low hardening heats. The various hardened test pieces of the "straight-carbon" steel showed a variation in strength of 10 per cent above and below the records given. This stock was selected especially for casehardening. The ordinary run of machine steel of the same carbon content will vary, in some cases, twice as much as

this. The same may be said of the so-called "natural nickel-alloy steels;" they vary as much as the straight-carbon steels. The higher grades of alloy steels do not vary more than 10 per cent either way, and the average is closer.

Rawhide Gearing. — When rawhide gears were first introduced the rawhide pinion occupied a peculiar position, in that it was required to demonstrate its qualities by a satisfactory performance of service for which cast-iron, steel or bronze pinions had originally been designed. In the electric railway field, for instance, rawhide pinions have thus performed the work originally intended for steel and bronze gears, and successfully competed with the bronze on the basis of mileage cost. Of course, rawhide is not quite as durable as steel, but it is often used in preference to steel, because it will run more quietly and the life of certain other motor parts is increased because of the decrease in vibration. This applied especially to the motors formerly used having high gear velocities and comparatively low tooth loads. Mr. W. H. Diefendorf, chief engineer of the New Process Gear Corporation, Syracuse, N. Y., mentions in one of his published statements that actual performances have thoroughly demonstrated that, at a peripheral velocity of from 1700 to 2000 feet per minute and more, rawhide pinions can be advantageously used in place of cast-iron and bronze pinions, and, occasionally, even in place of steel pinions. The fact that this has been done has naturally led to the belief that rawhide pinions are equal to metal pinions under all conditions. At speeds under 1700 feet, however, owing to the increased tooth loads permissible, by reason of the reduction of shock, it will be found necessary to use rawhide pinions of either larger diameters or of longer face, or both, than would be used for metal pinions. The meshing gears must, of course, be proportioned accordingly.

Allowable Load Per Inch of Face of Rawhide Gears. — The safe working load for a rawhide pinion of the highest grade is 150 pounds per inch width of face for one-inch circular pitch. Other pitches vary in direct proportion, except that the maximum load must not exceed 250 pounds per inch width of face. The reason why the capacity does not increase indefinitely with the increase

of circular pitch, as is within reasonable limits the case with metal gears, is that at a point not far above 250 pounds load per inch width of face the surface of rawhide begins to compress, thereby permitting a distortion of the tooth curve, resulting in friction and causing undue heating. A temperature of 225 degrees F. is more than a rawhide pinion should be subjected to. In exceptional cases, rawhide pinions have worked successfully for two or three months continuously at a load of from 350 to 450 pounds per inch width of face, but when removed from service the sides of the teeth were hardened to a depth of $\frac{3}{4}$ inch or more, and the material resembled hard glue or rosin and had lost all its elasticity.

Formula for Power Transmitted by Rawhide Gears. — In the following formula, given by the New Process Gear Corporation, let

P = circular pitch in inches;
F = width of face of gear in inches;
D = pitch diameter;
N = number of revolutions per minute;
H. P. = horsepower transmitted.

Then,

$$\text{H. P.} = \frac{D \times P \times F \times N}{850}$$

In using this formula, the circular pitch should not be given a higher value than 1.65 inch, in order to limit the total load to 150 pounds per inch width of face for one-inch circular pitch, as previously stated. Of course, rawhide gears of a greater circular pitch than 1.65 inches may be used to advantage, but when calculating the horsepower transmitted by these gears the value for P in the formula should never be given as larger than 1.65.

The denominator 850 is used for the highest grade of rawhide gears. When lower grades of material are used, the factor in the denominator should be increased from 850 to about 1000. It will be noted that in the formula no allowance is made for peripheral velocity and number of teeth. This is because rawhide pinions are intended primarily for high-speed service, where an all-metal drive would be noisy. As a result, the peripheral velocities are high and the cushioning effect of the rawhide teeth

compensates for the usual factor of shock. The number of teeth in rawhide pinions is made from fifteen to eighteen wherever possible.

Example: — As an example of the use of the horsepower formula given, find the horsepower transmitted by a rawhide pinion 6.37 inches pitch diameter, $1\frac{1}{4}$ inch circular pitch, 6 inches width of face and running at 1200 revolutions per minute.

$$\text{H. P.} = \frac{6.37 \times 1.25 \times 6 \times 1200}{850} = 67.$$

Strength of Rawhide Gears with Flanges. — The information relating to the strength of rawhide gears, given in the previous paragraph, relates to pinions having the working face exclusively of rawhide. Pinions are, however, frequently constructed with bronze flanges on the sides having teeth cut through the flanges, these forming part of the working face. In that case, the strength of the pinion would be increased from 10 to 25 per cent, according to the grade of bronze used and the thickness of the flanges.

General Remarks Relating to the Use of Rawhide Gears. — An important consideration in the use of rawhide pinions is that rigid supports are employed. If there is excessive vibration, due either to poor supports of the machines themselves or to insufficient bearings, the disalignment of the meshing teeth that results causes excessive wear. In one extreme case, a pinion mounted on an improperly supported shaft of a motor of 75 horsepower was, after twenty-four hours' continuous running, worn away to about two-thirds of the original tooth thickness. After having provided ample bearings for the shaft and installed a new pinion, this latter was in service for two and one-half years doing good work.

In a train of gears only one should be made from rawhide, the other being of metal. Usually rawhide is employed for the smaller gear or the pinion, because the rawhide is somewhat more expensive than metal. The mating gear should have accurately cut teeth, because the rough surface of cast teeth will wear into the rawhide and irregular spacing will produce a bad strain upon the rawhide teeth. The New Process Gear Corpora-

tion advises that pinions with flanges extending to the periphery of the teeth should be used wherever the duty is severe, as this construction prevents the outer layers of the rawhide from curling over and thus weakening the teeth. Rawhide gears of fairly large size are sometimes made having a cast-iron center or spider, with a rawhide annular ring enclosed on the sides by projecting cast-iron flanges. In this way, the teeth are made from rawhide while the body or framework of the gear is made from cast iron, thus producing a strong combination which is not excessively expensive and has all the advantages of a rawhide gear.

German Rule for Rawhide Gears. — The standard German engineers' handbook, "Hütte," gives a rule which may be translated into the following form for English measurements: *To find the allowable load in pounds at the pitch line for a rawhide pinion, multiply the width of face in inches by from* 180 *to* 360, *and divide the product by the diametral pitch.* It will be seen that this gives much lower permissible loads than does the New Process Gear Corporation's rule, which reduces to a factor of about 470, in place of the 180 to 360 given in "Hütte." In both of these rules the strength is made independent of the velocity at the pitch line, as already referred to. Since decrease of strength with increase of velocity is due to impact, and since rawhide is a substance peculiarly fitted to absorb impact harmlessly, it is logical to assume that the effect of increasing the velocity is negligible. This accounts for the fact that a rawhide gear will be as strong as a cast iron one at high speeds, when it would appear very weak in comparison with it in a static test.

Durability of Gearing. — A pair of gears figured by the rules and formulas in the preceding pages, so that they will be strong enough for the service for which they are to be used, may not be so proportioned as to be commercially durable. By "commercially durable" gears, we mean those which will last well in comparison with the rest of the machine of which they are a part. In some classes of machinery, gears strong enough for their work would certainly be commercially durable. A rack and pinion, for instance, used to raise a sluice gate for a dam, if made strong enough, would evidently wear indefinitely, though they might

rust away. It is plain that all gearing designed for occasional or intermittent use, even under heavy loads, is strong enough to wear well if it is strong enough to bear the load placed upon it. With gearing used for the continuous transmission of power, however, we cannot be sure of this. The gearing of a drive connecting a motor with a printing press, for instance, might conceivably be strong enough and yet not wear as long as the rest of the machine.

The pinion will naturally wear faster than its mate, since each of its teeth is in action a greater number of times per minute. To make the life of the two more nearly alike, it is customary to make them of different materials, as already mentioned, the pinion being made of the more durable one. Thus, a combination of steel pinion and cast-iron gear is common and occasionally conditions are found which warrant the expense of a hardened steel pinion and a phosphor-bronze gear. The use of the better material in the smaller gear of the pair is proper from the standpoint of strength as well as from that of durability. An examination of the Lewis outline constants, as tabulated in the preceding section of this chapter, will show that the teeth of the pinion are always weaker than those of the gear; so it is necessary, if an excess of strength is to be avoided in the gear, to make the pinion of the stronger material; but if the pinion is a little less durable than the gear, it will take most of the wear; and being more cheaply renewed than its larger mate, the mechanism is kept up at a less expense. It is not wise to use soft steel in both members for heavy service at high speed.

Where the velocity ratio is not extreme, but severe service is exacted, as in automobile gearing, the two members may be made of the same material — hardened or, preferably, casehardened alloy steel.

Efficiency of Standard Spur Gears. — The efficiency of two spur gears (or of any other power transmitting mechanism, for that matter) is measured by the percentage they deliver of the power entrusted to them. Thus, if a water wheel delivers 160 horsepower to the driving pinion on the shaft on which it is mounted, and the mating gear on the jack-shaft transmits 140

horsepower to that shaft, the efficiency of the gearing is 140 ÷ 160 = 87½ per cent.

To obtain the maximum of efficiency, attention must be paid to the following considerations:

Form the teeth as near to the perfect theoretical shape as good workmanship will bring them, giving the acting surfaces a fine smooth finish. If the teeth are milled to shape, special cutters should be used, made accurately to shape for the exact number of teeth. The gears must be mounted firmly and accurately in their working position.

Use hard, close-grained materials, preferably different for the two gears. Hardened steel on phosphor-bronze will probably give the best results, though it is difficult to be sure of the exact shape in hardened gears, unless they are finished by grinding after hardening. Soft steel on soft steel is probably the worst combination so far as efficiency is concerned, though it is stronger than cast iron on cast iron.

Provide continuous and copious lubrication, preferably by an oil or grease bath in an enclosed casing. Lubricated gears should always be enclosed if they are exposed to dust or grit in the slightest degree, otherwise they will grind each other away, and might rather run entirely dry.

Use as fine a pitch as possible, without requiring too wide a face to transmit the power required. The smaller the pitch, the greater the efficiency. Anything that tends to shorten the line of contact, confining it to the vicinity of the pitch point, increases the efficiency, as there is more rubbing at the beginning and end of contact than when the teeth are passing the pitch point.

In general, it may be said that there is no method of transmitting power between two parallel shafts that is more efficient than a pair of well designed and constructed gears, working under proper conditions. The highest efficiency, of course, is obtainable only at a considerable expense, so judgment is required to know how far it is wise to carry the possible refinements.

Variation of the Strength of Gear Teeth with the Velocity. — The generally accepted formula for calculating the strength of gear teeth is, as already mentioned, that proposed by Mr. Wil-

fred Lewis. The merit of this formula lies in the great number of variables taken into account as compared with other rules in more or less common use, and in the fact that these variables are rationally considered. The effect of each of them can be calculated with some assurance, with the single exception of the influence of the velocity on the safe stress. In the twenty odd years since the formula was first proposed, the original values for the stress as affected by the velocity have been largely used.

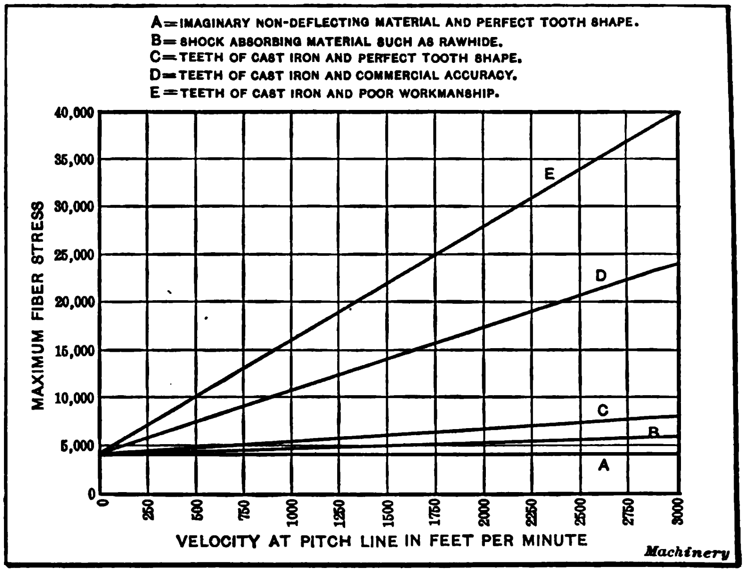


Fig. 4. Hypothetical Diagram showing the Relation of the Velocity to the Fiber Stress

Many designers, however, have felt that these values are rather unsatisfactory, although most of them will agree that they err rather on the side of safety than otherwise. By referring to Mr. Lewis' original paper it will be seen that these values were not given as being definitely determined, but merely as agreeing well with successful cases met with in his own practice. The following is a general analysis of the conditions involved.

Variation in Strength due to Impact. — A variation in the strength of the teeth of a gear, due to a variation in the velocity, can be due, of course, to but one thing — impact. To illustrate

this idea, and to show the cause of the impact, we will study the action of gearing under three different conditions. First, when made of an imaginary material which does not deflect under any strain below the breaking point. Second, with gears of commercial material, such as steel, with teeth of perfect form. Third, gears of commercial material with teeth of commercial accuracy.

1. *Gears of an imaginary undeflectable material.* In Fig. 4 is a diagram in which the horizontal distances give velocity in feet per minute, and vertical distances give stresses in pounds per square inch, starting in this case at 4000, which is assumed to be the maximum fiber stress in the gear we are considering, due to the load at the pitch line, which is supposed to be constant at all speeds. If the teeth of this gear are perfectly formed and well fitted together, so that there is no backlash, if the power is delivered to them steadily and smoothly, and the mechanism they drive runs without shock, any disturbance of the even movement will be impossible, and impact will be entirely absent. In the diagram in Fig. 4, then, there will be no rise of maximum fiber stresses with the velocity, so that the horizontal line A will show the conditions for this imaginary case.

2. *With commercial material and theoretically accurate workmanship.* The conditions in this case are shown in Fig. 5, with all the phenomena greatly exaggerated. The full lines show the conditions under load, while the dotted outlines show the conditions when the load is removed from the driven gear. The teeth A_1, B_1 and A_2, B_2, carrying the load, are deflected by it, as shown. Tooth B, just about to come into contact with tooth A, is on that account shifted from its normal position; it should be located as shown by the dotted lines. If it were in this position, it would come in contact with tooth A under mathematically perfect conditions, and there would be no shock of engagement. As it is, the two come suddenly into action as shown at E, under different conditions than those contemplated by the design, thus the contact takes place in the form of a slight blow, after which the teeth are deflected more and more, until they have taken up their share of the load, as shown later at A_1 and B_1. If the gears are moving very slowly, the deflection takes place very slowly,

and the problem is practically a static one. If the gears are running at a high velocity, the problem becomes essentially a dynamic one, and the stresses induced are greater than with the slow speed.

The increase in stress with the increase in speed for this second case could probably be represented by a line something like *C*, in Fig. 4. The location of this line is purely hypothetical. All we can say about it is that the increase in stress as the speed is increased would be comparatively small, and probably regular. The line has been drawn straight for convenience; we do not know what the real shape is.

3. *With commercial materials and commercial accuracy.* This is, of course, the practical case to consider. A line to show the relation of the velocity to the maximum fiber stress for a given gear would very probably look something like *D* in Fig. 4. This is, in fact, approximately the line which embodies the conclusions of the Lewis tables for a static stress of 4000 pounds. It is considerably higher than line *C*, because impact due to irregular tooth outlines is added to the impact due to the deflection. In all probability the latter is comparatively unimportant as compared to that due to irregularity of outline in gears of only ordinary workmanship.

Deflection and Stresses caused by Impact. — It may be objected that the deflections produced either by the gears coming into mesh out of step, as in case No. 2, or with the added aggravation of poor workmanship, as in case No. 3, are so minute that they could scarcely be considered as a serious factor in the problem. It is true that these deflections are minute — undetectable even, by ordinary means; but this admission does not destroy the argument for laying to this distortion the increase of the stress with the speed. If great loads produce slight deflections, slight deflections likewise produce great stresses, so that the slight bending brought about by the teeth coming into contact at *E* in Fig. 5, under slightly imperfect conditions, may produce great effects proportionately in the fiber stress, and the effects are magnified by the irregularities due to poor workmanship. When we stop to figure out what load per inch of face is re-

quired to deflect a 2-inch circular pitch gear, say 0.003 inch, it is evident that an irregularity in outline of this amount would scarcely be negligible at high speeds, if our hypothesis is correct.

The phenomena of impact are complicated to a high degree. The maximum stresses produced depend on the rapidity of transmission of a wave of stress or deflection produced in the material by the impact. If this wave is propagated slowly, the stresses are high; if rapidly, the stresses are low. The factors entering into the problem are the elasticity of the material, and the mass and shape of the part affected. In very simple cases

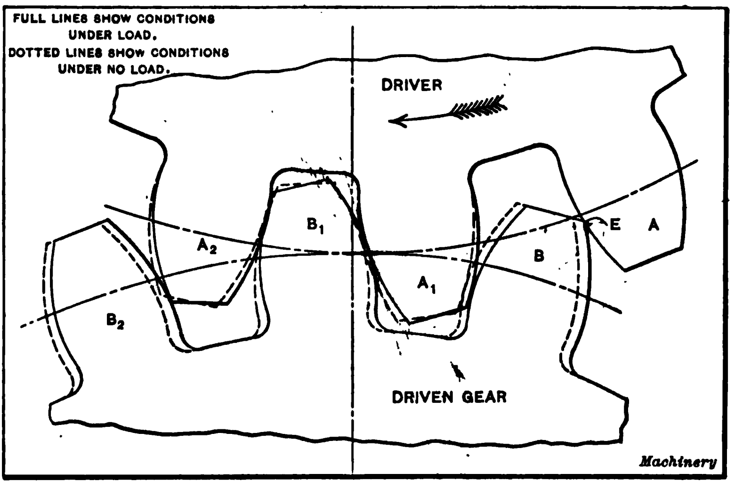


Fig. 5. Action of Gear Teeth under Load, greatly Exaggerated

the problem has been investigated mathematically, but our problem with the gear teeth is so complicated that we must of necessity at once apply to the engineer's court of last resort—experiment.

Practical Considerations Affecting the Effect of Shocks. — It is evident that other variables besides the strength of the material and the velocity at the pitch line enter into the fixing of the line on the diagram of Fig. 4. In addition, the following points will have to be considered.

1. *Accuracy of tooth outlines.* From what has just been said, it is evident that the variation of the stress with the velocity

will be affected by the accuracy of the workmanship involved in forming the tooth of the gear. Investigating the conditions in the case of a second pair of gears, similar to those from which line D was determined, but of a considerably poorer grade of workmanship, we should expect to find results giving a line something like E on the same diagram, giving much higher values for the stresses resulting from the load. It is evident, then, in considering lines C, D and E that workmanship is a variable which should be considered in the experiments, and that a series of tests should be run with two sets of gears of varying workmanship, one of high and the other of only ordinary grade, to make sure that this consideration is really of importance.

2. *Design of gear and mechanism.* Another factor which may affect the increase of the stress with the speed is the design of the rim and spokes of the wheel. It is conceivable that a gear with a very heavy rim and rigid spokes will absorb the shocks due to high velocity less easily than a gear with a light rim and flexible spokes or arms. The whole structure of the machine in which the gearing is carried, so far as its rigidity and massiveness are concerned, should, in fact, affect this matter. The further away from the point of tooth contact the members of the structure are, however, the less effect will they have, so perhaps even the influence of the arms and rim can be neglected. The same consideration affects the design of the mechanism to be used in the tests. It is conceivable that a mechanism involving long shafts and other flexible members might give, for a given set of gears, a line lower down on the diagram of Fig. 4 than would be the case if the construction were very heavy and rigid. The supporting mechanism must not in any case, of course, deflect in such a way as to prevent the teeth from having a full bearing on each other.

3. *The nature of the materials used.* Referring to what has previously been said as to the factors governing impact, it will be seen that the nature of the material used would affect the shape of the curve. It is probable, for instance, that two sets of gears, one made of cast iron and the other of a bronze alloy of the same tensile strength, would show lines of very different shape, owing to the difference in the modulus of elasticity and

the specific weight of these two substances. From this it will be seen that we cannot be sure that the results found to be applicable to cast iron or steel could also be applicable to either a pair of bronze gears or to the case of a bronze gear meshing with a mate of steel or iron. That the nature of the material would have a vital effect on the shape of the curve is still more probable, when we consider the practice followed in the use of such substances as rawhide. This material is particularly fitted to sustain impact and absorb without undue stress the deflection caused by it. Owing to this characteristic, we might expect that the line for a gear of this substance would be practically horizontal, as shown at B in the diagram, approaching A, though governed by entirely different conditions from those producing A. So far as can be learned, this supposition agrees with the practice of the manufacturers of rawhide gears.

It may be found that the points mentioned have so much influence on the question at issue that it would be very difficult to lay down a law governing the variation of stress with velocity, and that the most that can be done is to determine the variation in cases of commercial workmanship and rigid design, using the relation thus established in an empirical formula, with the knowledge that poorer conditions may bring the fiber stresses much higher, while good workmanship and careful design may, on the other hand, bring them much lower. Quite possibly the factors now in use may be found to nearly fill commercial requirements, in which case we must conclude that the criticisms of their being too high have been founded on experience with cases combining the favorable conditions just mentioned.

Practical Considerations Affecting Design. — The fact that the variation of the strength with the velocity is due to impact suggests also a number of points relating to design. Most of these are already well known, and are standard practice, the conclusions being so obvious that simple common sense has suggested them without theoretical analysis being necessary.

1. *Value of accuracy.* It is evident that this theory of impact puts a premium on accuracy in workmanship for gears

that are to run at high speed under a heavy load. It is probable that the strength of a given pair of gears may be cut in two if the tooth outlines are not carefully determined, and if the cutter is not set centrally. This suggests the desirability of a greater sub-division of the standard cutter series for work of this kind. Of course, the gears can always be made heavy enough for the required service, but the extra cost of accurate cutters and careful cutting will be repaid in cases where light weight and compact design are at a premium. In such cases the use of cutters specially designed for each gear is recommended.

2. *Resilience of design and materials.* In high-speed gearing it is evident that the shock due to the impact should be absorbed as quickly and as fully as possible. This suggests the use at abnormally high speeds of rawhide, cloth, etc., for one of the members of the pair of gears. The introduction of spring couplings or similar devices may also be desirable, especially where the other parts of the mechanism are liable to transmit shock to the gearing.

3. *Easing off the points of the tooth.* There has always been a sort of superstition that the points of the tooth should be eased off to make the action smoother. This is done, of course, in standard involute gears, though for another reason, that of avoiding interference with the flanks of the pinions. It can now be seen that there is a solid basis for this practice in all cases where gears are to run at such speeds that severe impact is liable to take place. Referring to Fig. 5, teeth *A* and *B* are taking up the load very suddenly, owing to the fact that they are out of step, due to the deflection of the other teeth momentarily carrying the load. Easing away the points of *A* and *B* would mitigate this sudden reception of the load, allowing the inevitable deflection to take place more slowly, with a consequent gain in the strength of the gear at high speeds. It would have a similar effect in minimizing impact due to inaccuracy of outline. This modification of the outline of the tooth should be very slight, and extend but a short distance, so that, when the load is entirely transferred, the "eased off" portion of the curve will be passed, and the true involute or cycloidal portion begun.

CHAPTER IV

SIMPLIFIED FORMULAS FOR STRENGTH OF GEARS

Need for Simplified Formulas. — It is generally conceded that the Lewis formula for the strength of gear teeth, with its accompanying tables, is the most accurate in form, as the maximum strength of each tooth is determined from its shape. It may be safely used for determining the strength of gears made by modern methods, but its tabulated form makes it difficult to use from the standpoint of the designer. It is well adapted to determine the strength of any given gear or pinion. But the reverse process — that of finding a gear suitable to meet the condition of a given horsepower and revolutions per minute — is not so simple, the trial-and-error method being a lengthy one at best. The following deductions give close and rapid approximations for preliminary work.

Assumptions on which Formulas are Based. — As both the gear and its pinion are usually made of the same material, either cast iron or cast steel, the strength of the pair is determined by the strength of its weakest member, which is the pinion when made of the same metal as the gear. For economical reasons the pinion is usually limited to about 15 teeth, so we may take that number as a convenient base. Circular pitch is used in the calculations, but the circular pitch can finally be transformed into diametral pitch if this is desired.

In a train of gears, the maximum reduction on any pair is usually taken at 4 or 5 to 1, so the number of reductions and ratios may be quickly deduced. Then the problem is usually presented as follows:

Given the horsepower and revolutions per minute of the pinion, what will be the allowable working stress, pitch, face, factor of strength and diameter?

The majority of trade gear lists give the horsepower of gears at

100 R.P.M. with an allowable stress for cast iron of 3000 pounds per square inch; but it is more difficult to transform this horsepower to suit the other conditions, than to proceed independently.

Values of Safe Working Stress. — The values of S, the safe working stress, which Mr. Lewis adopted tentatively, as they gave satisfactory results in practice, were as follows. (See also table in preceding chapter, where these values are slightly modified.)

Let V = speed of teeth in feet per minute and S = safe working stress, then:

For V =	100 (or less)	200	300	600	900	1200	1800	2400
For cast iron:								
S =	8000	6000	4800	4000	3000	2400	2000	1700
For cast steel:								
S =	20,000	15,000	12,000	10,000	7500	6000	5000	4300

When these values are plotted, it will be seen that the curves, though slightly irregular, closely approximate curves of the hyperbolic form. The equations of the curves which most nearly agree with the Lewis values are found to be the following:

$$\text{For cast iron, } S = \frac{88{,}000}{\sqrt{V}}.$$

$$\text{For cast steel, } S = \frac{220{,}000}{\sqrt{V}}.$$

These formulas give the following comparative values:

When V =	100	200	300	600	900	1200	1800	2400
For cast iron:								
S =	8800	6250	5000	3600	2930	2540	2080	1790
For cast steel:								
S =	22,000	15,625	12,500	9000	7325	6350	5200	4475

The agreement with the Lewis assumed values is remarkably close. The new values will probably come much nearer the true ones, as they are in much better line. They are also much more dependable, as the stress suitable for any speed can be easily found from the formula to the fraction of a pound, if desired, on

a true curve; whereas, the use of the tabular values results in the substitution of values which descend by variable steps of from 2000 to 300 pounds at a jump, or if ordinary interpolation is used the result is still inaccurate, as the interpolation necessarily follows a straight line between the two nearest values, and is thus too high. The new curve values also come nearer to the comparative Harkness values as given by Kent.

Derivation of Simplified Formulas. — The face of gears, A, is another variable quantity; but in the manufacturer's standard lists of today the face is usually about three times the pitch, and this may be adopted as close enough for preliminary work. It will be found that the majority of stock gears have either $14\frac{1}{2}$-degree involute or cycloidal teeth, so these styles will be used in these calculations. The factor of strength, Y', in the Lewis tables for a 15-tooth pinion of these types is 0.075. The factor Y' is found from the values of factor Y in the table "Factors for Calculating Strength of Gear Teeth," in the preceding chapter, by dividing the value of Y, as given, by 3.14. We have, therefore, the following data for a 15-tooth cast-iron spur pinion:

Let S = safe working stress, in pounds;
P' = circular pitch, in inches;
A = face, in inches;
Y' = factor of strength;
V = speed of pitch line, in feet per minute.

The Lewis general formula reduces to

$$\text{H.P.} = \frac{SP'AY'V}{33{,}000}$$

From our average determination above, we have:

$$S = \frac{88{,}000}{\sqrt{V}}; \quad A = 3\,P'; \quad Y' = 0.075.$$

Substituting these values in the general formula and reducing, we have for a 15-tooth cast-iron spur pinion:

$$\text{H.P.} = 0.6\,P'^2\sqrt{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By a similar process, we find for a 15-tooth cast-steel spur pinion:

$$\text{H.P.} = 1.5\,P'^2\sqrt{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For a bevel pinion, let

d = small diameter of bevel;
D = large diameter of bevel.

Then $$\text{H.P.} = \frac{SP'AY'V}{33{,}000} \times \frac{d}{D}$$

As $\frac{d}{D}$ usually equals about $\frac{2}{3}$, we can say:

$$\text{H.P.} = \frac{SP'AY'V}{33{,}000} \times \frac{2}{3}$$

and for a 15-tooth cast-iron bevel pinion,

$$\text{H.P.} = 0.4\, P'^2 \sqrt{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For a 15-tooth cast-steel bevel pinion,

$$\text{H.P.} = P'^2 \sqrt{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We now wish to find V in terms of revolutions per minute. For a 15-tooth pinion, approximately:

$$V = \frac{15 \times \text{R.P.M.} \times P'}{12} = 1.25\, \text{R.P.M.} \times P'.$$

Substituting this value in Formula (1) we have:

$$\text{H.P.} = 0.6\, P'^2 \sqrt{1.25\, \text{R.P.M.} \times P'}.$$
$$\text{Squaring, H.P.}^2 = 0.36\, P'^4 (1.25\, \text{R.P.M.} \times P').$$

Reducing, and solving for P', we have for a cast-iron spur pinion:

$$P' = \sqrt[5]{\frac{2.22\ \text{H.P.}^2}{\text{R.P.M}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

A similar substitution and reduction in Formulas (2), (3) and (4) gives the following:

For cast-steel spur pinion, $$P' = \sqrt[5]{\frac{0.36\ \text{H.P.}^2}{\text{R.P.M}}} \quad . \quad . \quad . \quad (6)$$

For cast-iron bevel pinion, $$P' = \sqrt[5]{\frac{5.0\ \text{H.P.}^2}{\text{R.P.M.}}} \quad . \quad . \quad . \quad (7)$$

For cast-steel bevel pinion, $$P' = \sqrt[5]{\frac{0.8\ \text{H.P.}^2}{\text{R.P.M.}}} \quad . \quad . \quad . \quad (8)$$

For rapidly varying loads, or where there is much starting and stopping, it is well to reduce the safe stress to two-thirds that allowed by the above formulas. We then have:

For cast-iron spur pinion, H.P. $= 0.4\,P'^2\sqrt{V};\ P' = \sqrt[5]{\frac{5\,\text{H.P.}^2}{\text{R.P.M.}}}$ (9)

For cast-steel spur pinion, H.P. $= P'^2\sqrt{V};\ P' = \sqrt[5]{\frac{0.8\,\text{H.P.}^2}{\text{R.P.M.}}}$ (10)

For cast-iron bevel pinon, H.P. $= 0.27\,P'^2\sqrt{V};\ P' = \sqrt[5]{\frac{11.0\,\text{H.P.}^2}{\text{R.P.M.}}}$ (11)

For cast-steel bevel pinion, H.P. $= 0.67\,P'^2\sqrt{V};\ P' = \sqrt[5]{\frac{1.8\,\text{H.P.}^2}{\text{R.P.M.}}}$ (12)

The fifth root can be easily determined by logarithms on the slide rule, or from the usual tables, but the values for the common cases are given later.

Corrections for Tooth Numbers. — It now remains to determine the correction for different numbers of teeth. As the teeth of pinions generally range from 12 to 30, we need not go outside these limits. Let N = number of teeth. Plotting the Lewis values for Y' for this case, and determining the nearest curve, we find that the straight line formula:

$$Y' = \frac{2N + 45}{1000}$$

expresses this curve very closely, as will be seen by the following comparative table:

No. of Teeth, N	Y' by Formula	Y' from Lewis' Tables	No. of Teeth, N	Y' by Formula	Y' from Lewis' Tables
12	0.069	0.067	19	0.083	0.087
13	0.071	0.070	20	0.085	0.090
14	0.073	0.072	21	0.087	0.092
15	0.075	0.075	23	0.091	0.094
16	0.077	0.077	25	0.095	0.097
17	0.079	0.080	27	0.099	0.100
18	0.081	0.083	30	0.105	0.102

Therefore, for other teeth, we can multiply the horsepower given in the above formulas by $\frac{2N + 45}{75}$, or more briefly by $0.027\,N + 0.6$.

Correction for Increased Velocity. — We must also correct for the increased velocity of this larger pinion, *i.e.*, multiply the result by $\sqrt{\frac{N}{15}}$ or $0.26\,\sqrt{N}$. The continued product of these last two multipliers might be used, but this does not simplify the calculation. These corrections need seldom be applied for preliminary work.

To Find the Pinion Diameter. — Lastly, to find the diameter of the pinion, approximately:

$$\text{diameter} = \frac{N \times P'}{\pi},$$

or

$$\text{diameter} = 0.318\,NP',$$

or for a 15-tooth pinion,

$$\text{diameter} = 4.77\,P' \quad \ldots\ldots \quad (13)$$

If diametral pitch is desired, it is sufficiently close to say:

$$\text{diametral pitch} = \frac{3}{P'} \quad \ldots\ldots \quad (14)$$

Summary of Formulas. — The following formulas, therefore, Nos. (5) to (14) (as deduced above), give closely enough for all preliminary determinations, the size of pinion required for 15 teeth. P' = circular pitch.

	Stress from Lewis' Tables	Stress ⅔ Lewis' Tables
Cast-iron spur pinion, P' =	$\sqrt[5]{\frac{2.22\ \text{H.P.}^2}{\text{R.P.M.}}}$	$\sqrt[5]{\frac{5.0\ \text{H.P.}^2}{\text{R.P.M.}}}$
Cast-steel spur pinion, P' =	$\sqrt[5]{\frac{0.36\ \text{H.P.}^2}{\text{R.P.M.}}}$	$\sqrt[5]{\frac{0.8\ \text{H.P.}^2}{\text{R.P.M.}}}$
Cast-iron bevel pinion, P' =	$\sqrt[5]{\frac{5.0\ \text{H.P.}^2}{\text{R.P.M.}}}$	$\sqrt[5]{\frac{11.0\ \text{H.P.}^2}{\text{R.P.M.}}}$
Cast-steel bevel pinion, P' =	$\sqrt[5]{\frac{0.8\ \text{H.P.}^2}{\text{R.P.M.}}}$	$\sqrt[5]{\frac{1.8\ \text{H.P.}^2}{\text{R.P.M.}}}$

$$\text{Pitch diameter} = 4.77\,P'$$

$$\text{Diametral pitch} = \frac{3}{P'}.$$

Practically, stock gears are made up to 3 inches circular pitch by $\frac{1}{4}$-inch steps, and a pitch of less than 1 inch is seldom used. The following table will therefore determine the roots for the nearest common pitch:

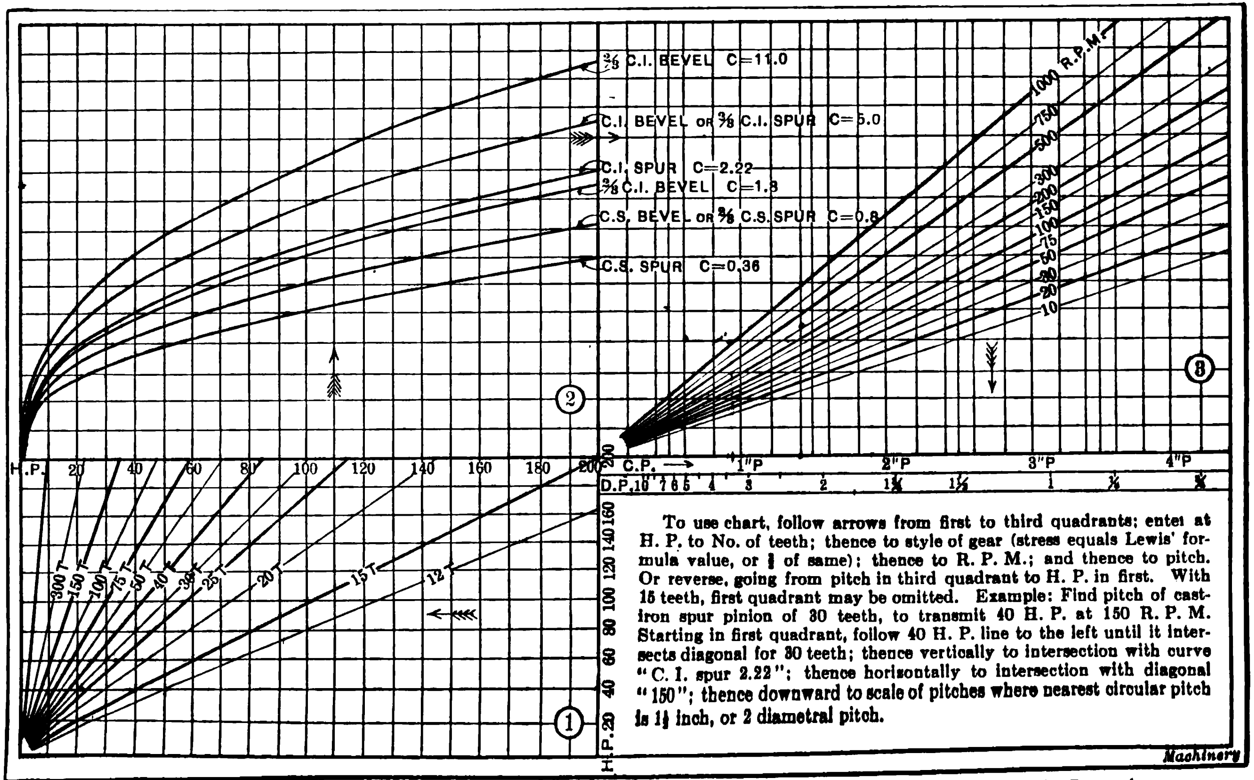
No. or Root	Fifth Power	No. or Root	Fifth Power	No. or Root	Fifth Power
¾	0.24	2	32	3½	525
1	1	2¼	58	4	1024
1¼	3	2½	98	4½	1845
1½	8	2¾	158	5	3125
1¾	16	3	243	6	7776

In case the revolutions per minute of the pinion are less than 80, which is exceptionally slow, care must be taken in applying the formula, or the allowable stress may be exceeded. With a 15-tooth pinion:

80 R.P.M. = 100 feet per minute for 1-inch P'.
40 R.P.M. = 100 feet per minute for 2-inch P'.
27 R.P.M. = 100 feet per minute for 3-inch P'.
20 R.P.M. = 100 feet per minute for 4-inch P'.

Chart for Rapid Solution of Gear Problems. — A simple three quadrant chart has been prepared for the rapid solution of these problems by mere inspection, good for any number of teeth, and for all the different styles, materials, and stresses of gears given by the above formulas, but for occasional preliminary determination, the formulas are sufficient, as their solution is simple.

It will, of course, be understood that the teeth considered in these formulas are those of the usual standard dimensions for cast gears, in which the height of tooth equals seven-tenths of the pitch. What are known as "short-tooth gears," in which the height of tooth equals half the circular pitch, are undoubtedly stronger, but their smaller working face has by many been supposed to cause more rapid wear, and their use is not so common. Although machine-molded cast gears run quietly at low speeds, they should not be used for rim speeds much over 1000 feet per minute. For speeds of from 1000 to 3000 feet per minute cut gears should be substituted.



Three-quadrant Gear Chart for the Solution of Spur and Bevel Gear Problems, based on the Lewis Formula; for 14½ degree Involute and Cycloidal Teeth

For a quick approximation of the diameter of the pinion shaft in inches, the following formula may be used:

$$\text{shaft diameter} = P' + 1.$$

Weight of Gears. — The weight of pinions and gears varies with different makers. Pinions of from 12 to 30 teeth are usually made slightly wider than gears, even if they are not shrouded; and the smaller sizes have solid webs in place of arms. It is found that a formula of the form:

$$\text{weight in pounds} = \text{coefficient} \times P'^2AN,$$

will usually fit the weights.

For many tables, the coefficients of the following values will serve:

$$\text{weight of pinion} = 0.35\, P'^2AN,$$
$$\text{weight of gear} = 0.45\, P'^2AN,$$

or where $A = 3\,P'$,

$$\text{weight of pinion} = P'^3N,$$
$$\text{weight of gear} = 1.35\, P'^3N,$$

or when diameter and P' are known, as $N = \frac{\pi D}{P'}$,

$$\text{weight of pinion} = 3.1\, DP'^2,$$
$$\text{weight of gear} = 4.2\, DP'^2.$$

Price of Gears. — The price of gears varies largely with different manufacturers. The price of cast-tooth spur gears can usually be expressed by a formula of the following form:

$$\text{price} = (\text{coeff.} \times P'N) + (\text{coeff.} \times P').$$

Cut tooth gears usually cost about 20 per cent more than cast tooth gears; and cast-steel gears from 50 to 75 per cent more than cast-iron gears of the same size.

CHAPTER V

THE STUB-TOOTH GEAR

During the last few years, a form of gear tooth, known as the "stub gear-tooth," has been introduced. It has been applied successfully, especially to automobile drives. The features of this form of gear tooth are a shorter addendum and dedendum than used for ordinary standard gears. There are several systems of these teeth in use, but the stub gear-tooth introduced by the Fellows Gear Shaper Company of Springfield, Vt., is by far the most commonly used. The information relating to this class of tooth, given in the following, has been furnished mainly by this company.

Standards for Gear Teeth. — With the constantly increasing use of gears for transmitting power, the question of the correct shape and size of gear teeth becomes of far greater importance than ever before. It is not sufficient merely that a gear be well cut and the teeth properly spaced; the shape and proportions of the tooth itself must be carefully considered. The two most important features to be secured are the nearest approach to a rolling action that it is possible to obtain, and the strongest tooth that will meet this condition. The first feature includes easy running and reduces the friction to the lowest point, thus producing the least wear in action.

We are apt to think of the present standard gear tooth as one of the fixed laws of mechanics, and as representing the highest development possible in this line, just as we consider the question of threads as fixed by the Sellers or United States Standard forms. But this is not the case; for although there is always a perfectly justifiable reluctance about departing from a recognized standard, it is at least good policy to know both sides of the question, and to see if the advantages claimed for a newer form of tooth can be backed up with positive proof.

The so-called standard tooth of today is the involute curve with the $14\frac{1}{2}$-degree pressure angle or angle of obliquity; but its use is not as universal as might be supposed, and the newer form known as the "stub-tooth" with its 20-degree pressure angle has become established to a degree that many do not realize.

The Epicycloidal Tooth Form. — The first gear teeth worthy of the name were of epicycloidal form and measured by circular pitch; and, as the state of the mechanic arts had not advanced to the point of cutting teeth in a machine, the gears were made with cast teeth. With the epicycloidal system it was necessary, in order to avoid the excessive under-cutting of the flank of the tooth, to adopt certain proportions of form and length of tooth, which gave continuous action when the gear of 12 teeth was used. Proportions for the length of tooth and other dimensions, that were originated at this time, when the science of applied mechanics was in its infancy, have been handed down to the present time with little change, regardless of the fact that with the dropping of the epicycloidal form of tooth the conditions have been entirely changed.

The Involute Tooth Form. — The adoption of the involute form of tooth, together with the diametral pitch system of measurement, has been gradual but constant, and it has now practically superseded all other forms. With the epicycloidal system of gear teeth the mean of the angle of obliquity was about 15 degrees, and in determining the proportions of the interchangeable involute gearing, this angle of rack was adopted as the basis of the system. One point that had much weight in deciding upon the angle was the fact that the sine of the angle of $14\frac{1}{2}$ degrees was almost exactly 0.25, a proportion which was easy for the millwright to lay out; and so the angle of $14\frac{1}{2}$ was decided upon. So strongly has this angle become impressed on the mechanic that it has become almost a revered tradition that a greater angle than this cannot be used with any degree of success, because of excess friction and the consequent wear on bearings.

While this idea may have been more or less correct with the proportions of shafts and bearings in use in former days, with cast gears and epicycloidal teeth, it surely has very little weight

under present conditions. The effect of the angle of obliquity on the wear of bearings has been unduly exaggerated; and, as a consequence, the proportions of gearing in general use today are open to great improvement in the three essential points of strength, durability and running qualities.

Length of Tooth. — With an interchangeable set of gears the length of the tooth should be sufficient to give an arc of action, even with the smallest pinions, so that one pair of teeth will be in contact until the next pair is in position to take up the load. There is, therefore, a fixed relation between the length of tooth

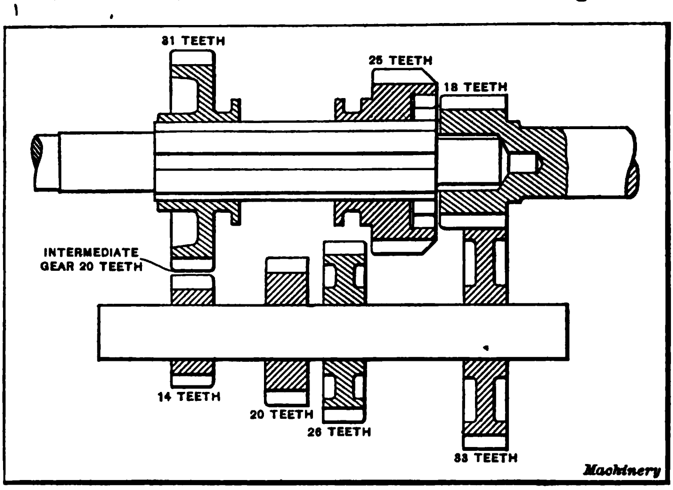


Fig. 1. Automobile Transmission Gearing used as Example in the Study of Stub-tooth Gears

and the angle of obliquity. In ordinary practice the number of teeth of the pinion is limited to twelve, and the length of the tooth ordinarily adopted is such that the action is continuous.

It is, however, a fallacy to argue that the teeth should be as long as possible, with the idea that a gain is made if two or more pairs of teeth are in mesh at once. Conditions are never such that an equal division of the load is possible, and a length of tooth beyond that which is necessary to insure a continuity of action produces undue friction and wear.

Angle of Obliquity, Length of Tooth, Efficiency and Wearing Qualities. — Although the subject of gearing has been investi-

gated and discussed at great length by many mechanics, there is one phase of the subject that has received but scant attention, and this is the correlative effect of the angle of obliquity (or pressure angle) and the length of the tooth, upon the efficiency and wearing qualities of the tooth itself. It can be shown that an excessive sliding action takes place at a certain portion of the tooth action between gears having teeth of the standard or $14\frac{1}{2}$-degree angle, and that by increasing this angle it is possible to so shorten the tooth that only such portions of the curve are

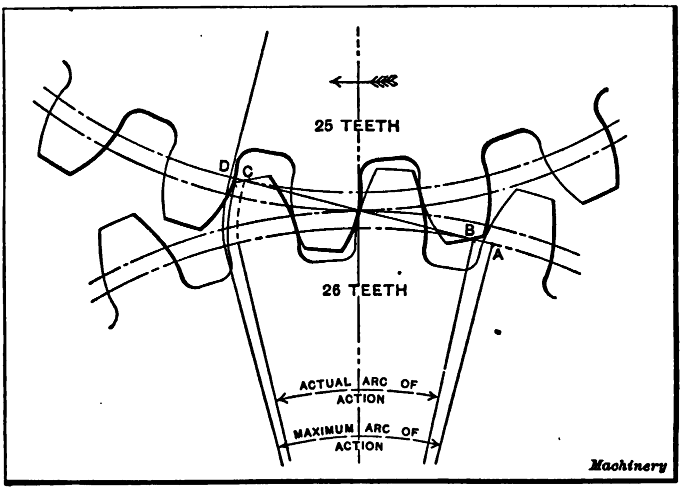


Fig. 2. Tooth Action of Standard Involute Tooth Gears

used as will give nearly a complete rolling action. An increase in the angle of obliquity has often been advocated by others and is advantageous, but its advantages are very limited if a tooth of the standard length is retained.

The use of gears for the transmission of power in automobiles has perhaps called attention to this question more than any other line of work. To show clearly just what this action really is, the accompanying illustrations have been prepared for comparison of the action of the stub-tooth with that of the $14\frac{1}{2}$-degree angle standard type, using as an example the gears of the sliding transmission of an automobile.

Investigation of an Actual Design. — The following gears have been selected as being typical of such a transmission or gear-box.

Low gear	18–33
2nd gear	26–25
3rd gear	20–31
Reverse	14–20–31

These gears are shown assembled in Fig. 1. As the combinations 18–33 and 20–31 are very similar, the second has been omitted from the comparisons and the others will now be considered.

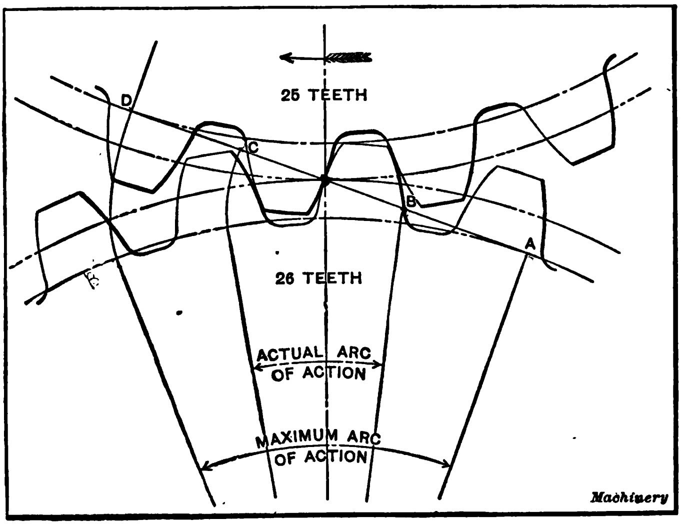


Fig. 3. Tooth Action of Stub-tooth Gears

In Figs. 2 and 3 are shown comparisons of tooth action for gears of both the standard and of the stub-tooth forms, the driver having 25 and the driven 26 teeth. If, in the diagrams, the gears are supposed to rotate in the direction of the arrow, the theoretical action begins at *A* and ends at *D*, the line *AD* being termed the "line of action." It is obvious, however, that the actual action can only begin at *B*, where the outside diameter of the upper gear intersects the line *AD*, ending at the corresponding point *C*. Drawing involutes from these points to the base circle, and continuing the radials to center *O*, the arc

included between lines A and D is seen to be the maximum or greatest possible arc of action, while B and C define the actual arc of action.

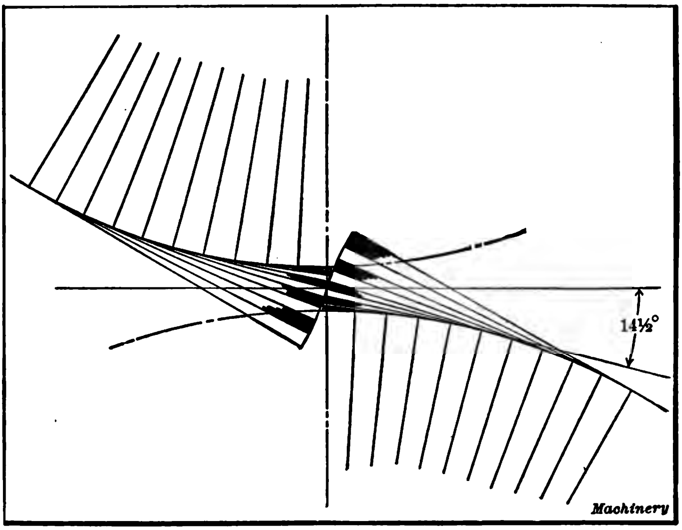


Fig. 4. Analyzing the Tooth Contact of the Gears in Fig. 2

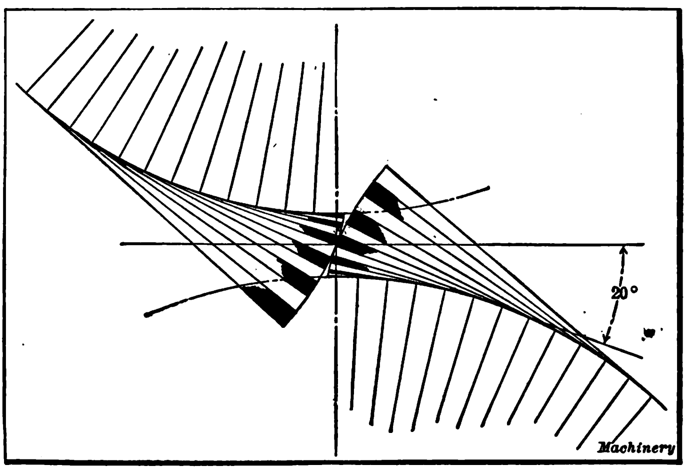


Fig. 5. Analyzing the Tooth Contact of the Stub-tooth Gears in Fig. 3

In Fig. 4 is shown an involute curve of $14\frac{1}{2}$ degrees obliquity from each of the gears in Fig. 2, the curves being of sufficient length to cover the maximum arc of action, and drawn to the

same scale as Figs. 2 and 3. The alternately shaded divisions of the curves show the portion of each that is in contact with its mate during an equal angular movement of the gears.

In Fig. 5 is seen a similar diagram for Fig. 3, with an angle of obliquity of 20 degrees.

In Figs. 6 and 7 we have the same involute curves, developed into straight lines, the points corresponding to the divisions of Figs. 4 and 5 being connected by cross lines.

To one who has labored under the impression that if the involute curves of a pair of gears are correct, the action is nearly a rolling one, a comparison of these diagrams will be both in-

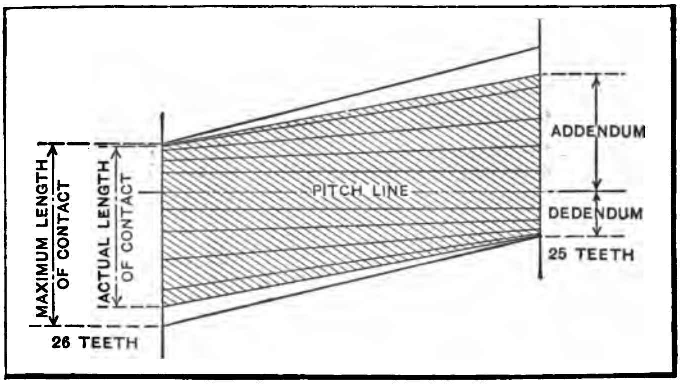


Fig. 6. The Involute Curves in Fig. 4 developed into Straight Lines — Standard Involute Teeth

teresting and instructive. It will be noted that although the divisions of the base circles are equal, those of the involute decrease as the base is approached, showing that the wear is concentrated at this point. When, further, it is considered that the contact is between the flank of this tooth and the point of its mate, it is seen that the condition is far from ideal.

By a comparison of Figs. 6 and 7 we note that the portion of actual contact, denoted by the shaded part, includes in the case of the 14½-degree tooth cross lines that have a considerable angularity, showing an excessive sliding action; while the corresponding lines of 20-degree teeth are nearly parallel, denoting that the action is nearly a rolling one.

Again referring to Figs. 2 and 3 as showing a comparison of

the two systems, we note the two points in which they differ; first, on account of the greater angle of the line of action of the stub-tooth, the maximum arc of action is much increased; second, the ratio of the *actual* to the *maximum* arc of action of the stub-tooth is much less than with the $14\frac{1}{2}$-degree tooth.

This latter point is a very important one, as we thus eliminate contact at both ends of the line of action. When we realize that this is the portion of the action in which the greater part of the

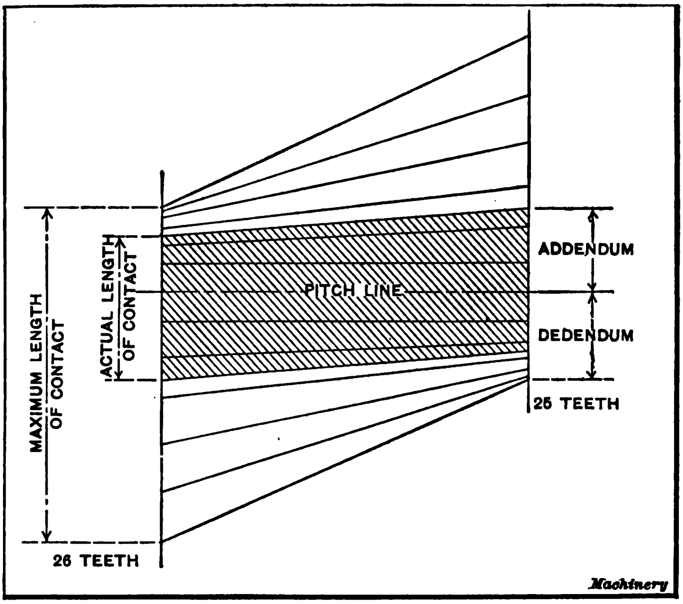


Fig. 7. The Involute Curves in Fig. 5 developed into Straight Lines — Stub Teeth

sliding takes place, with its inevitable wear, we see that it is a good thing to cut out all we can of it. In the stub-tooth the point of the tooth which wears out the flank of its mate is removed, and this reduces the friction while increasing the efficiency. A comparison of Figs. 6 and 7 shows that the action of the stub-tooth is as nearly a rolling one as it is practicable to obtain.

It is of course impossible to entirely eliminate the wear between the teeth of gears working under a load. But if the wear could

be evenly distributed over the entire working face of the tooth, the correct form of tooth would be retained indefinitely and a worn-out gear would, aside from the excessive backlash, run as well as a new gear. And if this wear can be evenly distributed, the durability of any gear will be increased many times.

If the gear combinations 18–33 and 14–20 are analyzed in the same manner as the combination 25–26, it will be found that in these cases, as in the one first considered, the sliding action is

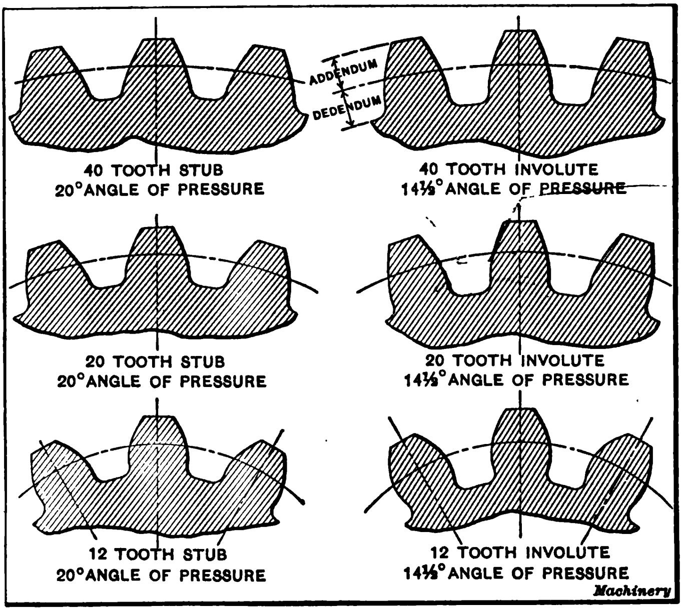


Fig. 8. Comparison between Standard and Stub Type of Gear-teeth

largely eliminated from the stub-tooth gear, due to the combination of the short tooth and the increased pressure angle. It will also be noted that in the case of the $14\frac{1}{2}$-degree tooth, on account of the diameter of the base line of the pinion approaching so closely that of the pitch line, the entire length of the gear tooth does not have contact with the pinion. In order to accommodate this useless length of gear tooth, the tooth of the pinion is materially and unnecessarily weakened. This excess length is about 30 per cent of the addendum in one case and about 40 per cent in the other.

Comparison between Ordinary and Stub Gear-teeth. — A comparison of the regular standard gear tooth and the stub gear-tooth, as shown in Fig. 8, will be of interest. Here are shown, to the left, parts of three stub-tooth gears, and sections of three standard $14\frac{1}{2}$-degree involute gears, with teeth varying in number from 12 to 40. The illustration gives a clear idea of the increase in strength with the increase in pressure angle. The subject of strength of stub gear-teeth will be taken up in subsequent pages.

Summary of Conclusions. — The advantages of the stub-tooth may be stated as follows:

1. Greater strength.
2. Equal arc of rolling contact to the $14\frac{1}{2}$-degree involute.
3. Extreme sliding contact avoided.
4. More even wearing contact.

As a disadvantage, it may be mentioned that we require a greater number of rotary cutters to cover a given range of teeth. For the standard involute system eight cutters are required, whereas the stub-tooth system requires sixteen in order to produce good gears. When a generating machine, such as the Fellows gear shaper, is used, only one cutter is required for each pitch, as the cutting action of this machine is that of a pinion meshing with a gear-wheel.

The stub-tooth system has become thoroughly established in this and foreign countries. Since its introduction in 1899 its use has steadily increased. Fully one-third of the large cutter business of the Fellows Gear Shaper Company is for cutters of the stub-tooth form. Its use is not now, as in the beginning, confined to automobile work. It is used in every line of work where strength, durability and running qualities are demanded.

Dimensions of Stub Gear-teeth. — The stub gear-teeth introduced by the Fellows Gear Shaper Co. (by far the most commonly used) are based on the use of two diametral pitches. One diametral pitch, say 8, is used as the basis for obtaining the dimensions for the addendum and dedendum, while another diametral pitch, say 6, is used for obtaining the dimensions of the thickness of the tooth, the number of teeth, and the pitch diameter. Teeth made

according to this system are designated as $\frac{6}{8}$ pitch, $\frac{12}{14}$ pitch, etc., the numerator in this fraction indicating the pitch determining the thickness of the tooth and the number of teeth, and the denominator, the pitch determining the depth of the tooth. The clearance is made greater than in the ordinary gear-tooth system and equals 0.25 ÷ diametral pitch. The pressure angle is 20 degrees.

Dimensions of Stub Gear-teeth (Fellows Gear Shaper Co.'s System)

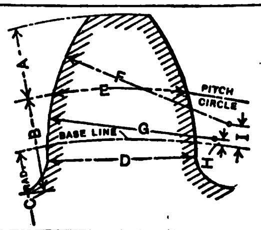
Diametral Pitch	Thickness of Tooth	Addendum	Working Depth	Depth of Space below Pitch Line	Clearance	Whole Depth of Tooth
4/5	0.3927	0.2000	0.4000	0.2500	0.0500	0.4500
5/7	0.3142	0.1429	0.2858	0.1786	0.0357	0.3214
6/8	0.2618	0.1250	0.2500	0.1562	0.0312	0.2812
7/9	0.2244	0.1111	0.2222	0.1389	0.0278	0.2500
8/10	0.1963	0.1000	0.2000	0.1250	0.0250	0.2250
9/11	0.1745	0.0909	0.1818	0.1136	0.0227	0.2045
10/12	0.1571	0.0833	0.1667	0.1041	0.0208	0.1875
12/14	0.1309	0.0714	0.1429	0.0993	0.0179	0.1607

The Nuttall Co.'s System of Stub Gear-teeth. — In a system of stub gear-teeth originated by Mr. C. H. Logue of the R. D. Nuttall Co., the tooth dimensions are based directly upon the circular pitch. The addendum is made equal to 0.250 × the circular pitch, and the dedendum equal to 0.300 × the circular pitch. The pressure angle is retained at 20 degrees. This system was adopted because of disadvantages claimed to exist in the Fellows' system using two diametral pitches, or one diametral pitch to obtain the addendum and dedendum, and another diametral pitch to obtain the thickness of the tooth, and the number of teeth, etc. The chief disadvantages of this system are that the depth of the tooth is not a direct function of the circular pitch, and that this system can be used only in combination with diametral pitch gears. When the stub gear-tooth is made according to the proportions 0.250 and 0.300 times the circular pitch, as just stated, the system can be applied to either diametral, circular or millimeter pitch gears.

Special Tooth Shape for Rolling Mill Gears. — The illustration and table below give data for laying out special 22½-degree

involute gears for rolling mill service. These teeth vary somewhat from the standard shape, and liberal fillets are provided at the root of the teeth to prevent breakage due to sharp corners at these points. The dimensions given in the table are for 1-inch circular pitch. To find the dimensions for any other pitch, multiply the dimension given by the circular pitch. The length

Rolling Mill Gear-teeth



Formulas:

$A = 0.275 \times$ circular pitch;
$B = 0.325 \times$ circular pitch;
$C = 0.462 \times$ pitch diameter;
$E = 0.49 \times$ circular pitch;
$F = 0.251 \times$ pitch diameter;
$G = 0.136 \times$ pitch diameter;
Pressure angle $= 22\frac{1}{2}$ degrees.

No. of Teeth	Pitch Diam.	Base Radius, *C*	Thickness, *D*	Face Radius, *F*	Flank Radius, *G*	Distance, *H*	Distance, *I*	Strength for 1" Pitch, 1" Face, Lbs. at Stress of 1000 Lbs. per Sq. In.
10	3.183	1.470	0.480	0.800	0.433	0.010	0.010	105
11	3.501	1.617	0.496	0.879	0.476	0.014	0.014	115
12	3.820	1.764	0.505	0.959	0.520	0.016	0.016	117
13	4.138	1.912	0.515	1.04	0.563	0.005	0.005	121
14	4.456	2.059	0.526	1.12	0.606	0.010	0.006	126
15	4.775	2.206	0.545	1.20	0.650	0.000	0.000	134
16	5.093	2.353	0.550	1.28	0.693	0.005	0.000	137
17	5.411	2.500	0.570	1.36	0.736	0.006	0.006	146
18	5.730	2.647	0.572	1.44	0.779	0.000	0.000	145
19	6.048	2.794	0.575	1.52	0.823	0.006	0.006	140
20	6.366	2.941	0.580	1.60	0.866	0.014	0.007	136
21	6.684	3.088	0.585	1.68	0.899	0.015	0.000	136

of the face of rolling mill pinions is made about equal to the pitch diameter. The angle of the tooth with a line parallel to the axis of the gear is usually made 30 degrees. The figures given in the column headed "Strength for 1-inch Pitch, 1-inch Face" are based on a working stress of 1000 pounds per square inch. To find the strength of teeth for other dimensions and other working stress, multiply the figures given by: face of gear $\times$ circular pitch $\times \dfrac{\text{working stress}}{1000}$.

Example: — Find allowable pressure on teeth with a working stress of 4000 pounds per square inch, for a 2-inch circular pitch gear of 4-inch face having 20 teeth.

$$136 \times 4 \times 2 \times \frac{4000}{1000} = 4352 \text{ pounds.}$$

Strength of Stub Gear-teeth. — We have so far discussed only the points of efficiency and durability, but there is another advantage of the stub-tooth over the standard form, and one which some might think entitled to first consideration, especially in the transmission of any considerable amounts of power; this is the advantage of greatly increased strength.

Because of the fact that the tooth has been shortened and at the same time widened at its base, there is a very substantial gain in strength, which in some cases amounts to about 75 per cent. The fact that we can sometimes almost double the strength of a tooth is certainly worthy of serious consideration. In Fig. 9 are shown enlarged sections of different combinations of gears, showing the comparative strength of each form by the well-known graphical method of Wilfred Lewis.

In laying out these diagrams, we first draw the normal of the involute AB from the extreme point of the tooth. From the point B, where the normal intersects the center line, erect the parabola BC, with its base tangent to the flank of the tooth and indicating D as the weakest section. Draw DG at right angles to the center line and connect B and D. Then draw DH at right angles to BD, intersecting the center line at H. In this construction GH may be taken as the measure of the strength of the tooth.

A comparison of these diagrams, of both the standard and stub-tooth gears, shows an increase of strength in these cases which is very marked. It should also be noted that the gain in strength is greatest in the case of the smallest pinion, which is a very great advantage; for as a chain is no stronger than its weakest link, so a train of gears is no stronger than the teeth of its smallest pinion. This, in addition to the clearly demonstrated fact of easier running and reduced wear, should command careful consideration for the form of tooth here advocated.

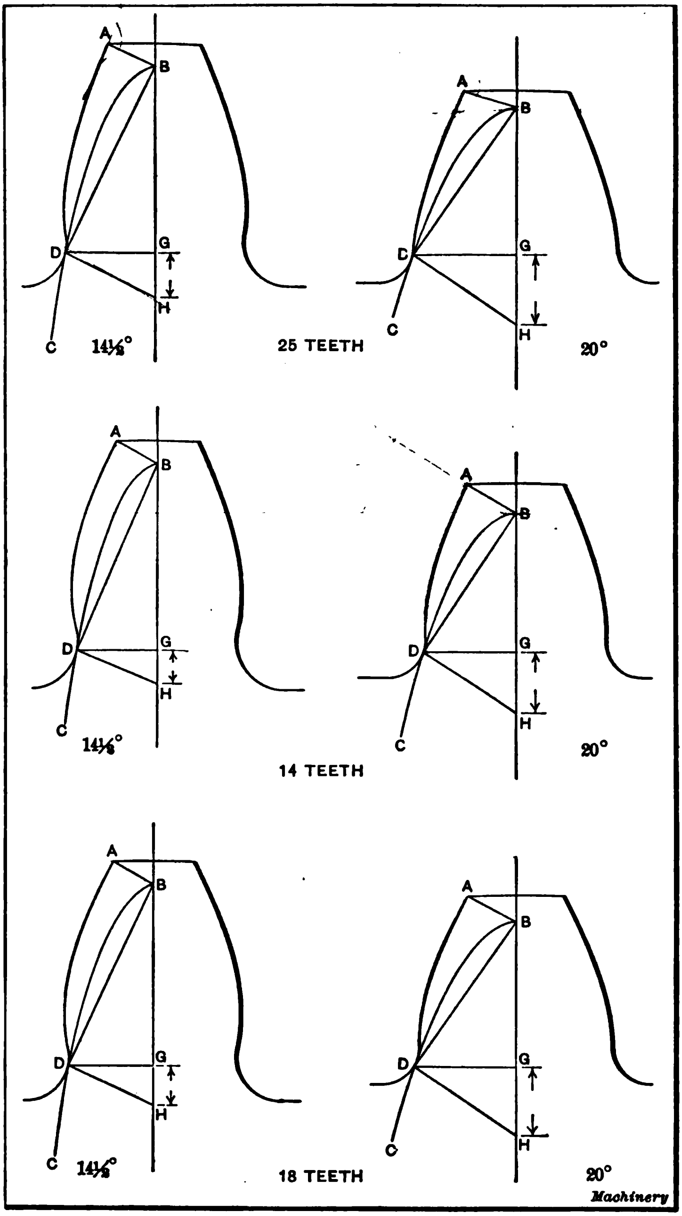


Fig. 9. Illustrations showing the Comparative Strength of Standard and Stub Gear-teeth

Fig. 10 shows a comparison between a pinion having 15 teeth, 6 pitch and $14\frac{1}{2}$-degree pressure angle, and one with 20 teeth, $\frac{8}{10}$ pitch and stub-tooth form of 20-degree pressure angle, both having the same pitch diameter. A comparison of the length of the line *GH* shows that the stub-tooth, although much shorter, has 20 per cent greater strength; and that while the bearing surface *per tooth* is shorter, the *total* area of bearing surface is 6 per cent greater.

This is not given as a suggestion that the pitch be reduced to secure an increase in strength of gears; it merely shows one of the possibilities of the stub-tooth.

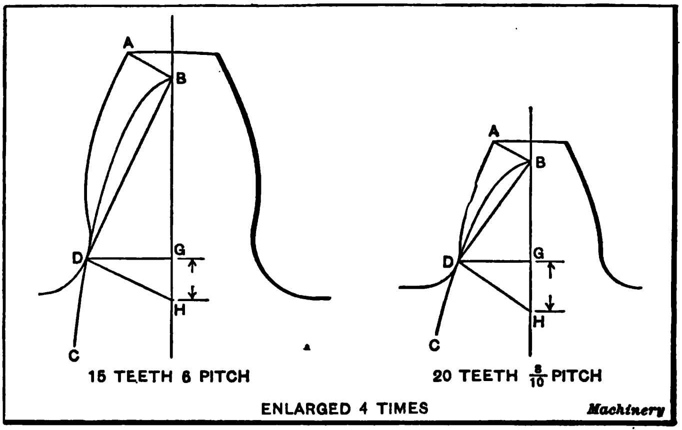


Fig. 10. Comparison of Strength of Two Gears of Equal Pitch Diameter and Unequal Pitch

Some not entirely familiar with this system have made the mistake of thinking that it consists simply of a shorter tooth, while retaining the same pressure angle. They have therefore opposed it on the ground that the arc of action with a small pinion would not be equal to the pitch arc. The action would then not be continuous, because one tooth is out of contact before the next tooth takes up the load. It should be thoroughly understood that the increased angle of obliquity is an essential and vital part of the stub-tooth system; and that with this increased angle, the arc of involute action is even longer than that of the $14\frac{1}{2}$-degree standard tooth.

Analysis of the Strength of Stub Gear-teeth. — An investigation has been made by Mr. L. L. Smith of the strength of stub gear-teeth. This analysis was published in MACHINERY, January, 1914. The following pages contain an abstract of the methods and results of the investigation. Accurate information regarding the strength of these teeth has been lacking, and it was the purpose of this investigation to determine correct values of the factor Y in the Lewis formula for the strength of gear teeth as applied to this system of gearing. The two forms of stub-teeth used in this country — that recommended by the Fellows Gear Shaper Co., 25 Pearl St., Springfield, Vt., and that recommended by the R. D. Nuttall Co., Pittsburg, Pa. — are the ones with which this investigation is concerned.

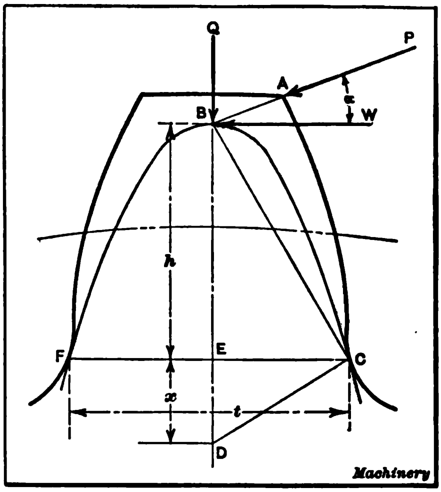


Fig. 11. Diagram illustrating the Derivation of the Lewis Formula

The Strength of Gearing — Lewis Assumptions. — Mr. Wilfred Lewis was the first investigator to take into consideration the form of the tooth profile and the fact that the line of action of the pressure is always perpendicular to the surface. When gears are cut accurately, his formula gives, as already stated, very satisfactory results, and with a few corrections for the number of teeth in contact, etc., it is almost universally used in gear design.

Mr. Lewis assumed that all of the load on the gear was concentrated at the end of one tooth, its line of action being perpendicular to the surface AC as shown in Fig. 11. (See also Chapter III for the derivation of factor Y in the Lewis formula.) The actual force P was resolved into a tangential force W and a radial force Q. The tangential force produces a bending stress in the tooth, while the radial force produces a uniformly dis-

tributed compressive stress. When the material of which the gears are made is stronger in compression than in tension, this radial component is a source of strength, but when the material is of about the same strength in tension and compression it is a source of weakness. For small angles of obliquity, α, this compression does not amount to more than 10 per cent of the total stress and on this account was neglected by Mr. Lewis. The tangential force W he assumed to be equal to the force transmitted at the pitch circle. This last assumption, although perhaps as much as 10 per cent in error for small pinions, gives values of the stress on the safe side and so may be considered accurate enough for practical purposes.

Force Analysis. — To obtain the weakest section of the tooth, Mr. Lewis constructed a parabola CBF (Fig. 11) passing through B, the application point of the tangential force W, and tangent to the tooth profile at C and F. This parabola encloses a cantilever beam of uniform strength and it can be readily seen that the weakest section of the tooth lies along CF. The resisting moment of this section is:

$$M = \frac{Sft^2}{6} \quad \ldots\ldots\ldots \quad (1)$$

where S = allowable fiber stress;
f = face width of the gear.

The bending moment is Wh, hence:

$$Wh = \frac{Sft^2}{6} \quad \ldots\ldots\ldots \quad (2)$$

From the geometry of the figure:

$$h = \frac{t^2}{4x} \quad \ldots\ldots\ldots \quad (3)$$

Then
$$W = \frac{2Sfx}{3} = Sfp' \times \frac{2x}{3p'}$$

where p' = the circular pitch.

Representing the ratio $\frac{2x}{3p'}$ by the symbol Y, we have:

$$W = Sp'fY \quad \ldots\ldots\ldots \quad (4)$$

The value Y is thus seen to be dependent upon the tooth profile and the pitch of the gear. By drawing the tooth profiles for gears with various numbers of teeth, Mr. Lewis was able to calculate the value of Y, for the standard involute system, and found that the following equation very closely fitted the values he obtained.

$$Y = Y_r - \frac{a}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which Y_r = the value of Y for rack teeth;
a = constant depending upon angle of obliquity;
n = number of teeth in gear.

A mathematical analysis of a rack tooth outline shows that the value of Y_r is given by the following equation:

$$Y_r = 0.1273\, p\left(\frac{t}{0.728} - z\right) \quad . \quad . \quad . \quad . \quad (6)$$

in which t = thickness of tooth at pitch line;
z = addendum;
p = diametral pitch.

Stub-tooth Systems. — To meet the demand for gear teeth that are stronger than the standard 14½-degree involute, various methods of increasing their strength may be resorted to. Two of these methods are as follows: (1) shortening the addendum and keeping the standard angle of obliquity; and (2) increasing the angle of obliquity and preserving the standard height of the tooth. The stub-tooth system is a combination of these two methods, in which the height of the tooth is decreased to about 0.8 the standard height and the angle of obliquity increased to 20 degrees.

Advantages of Stub-tooth Systems. — The chief advantage of this system is by many held to be that the teeth will transmit greater loads, and this fact is taken advantage of in the construction of machine tools, hoisting machinery and automobiles, because with the same quality and amount of material, greater strength can be obtained. The stub-tooth gears run smoother than gears with the standard 14½-degree involute teeth, due to the decreased impact, because for a given strength and length

of face, the pitch is smaller and consequently the number of teeth in mesh is greater. Stub-tooth gears are almost universally used by automobile manufacturers, and they report that the action is smoother and the wear less than with standard gears. Mr. Norman Litchfield in the Transactions of the American Society of Mechanical Engineers for 1908 reports that they have given excellent service on the New York subway trains.

Mr. E. R. Fellows in the same volume of the Transactions presents the chart shown in Fig. 12, which is based on actual

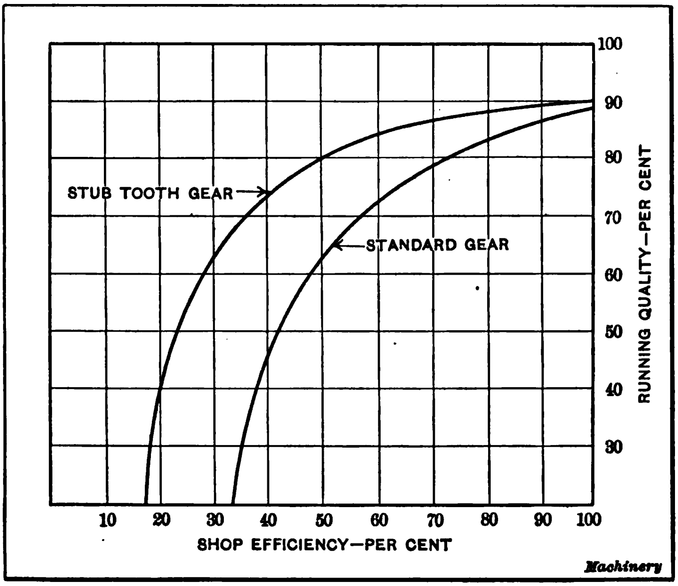


Fig. 12. Diagram showing Relative Efficiency of Stub and Standard Gear Teeth

experience. The curves apply to stub-tooth and standard gears with "shop efficiency" plotted against "running quality." By shop efficiency is meant the relative conditions under which the gears operate and that 100 per cent would only be attained under laboratory conditions, 90 per cent representing first-class commercial conditions and 70 per cent the common conditions in an average shop. By running quality is meant the relative noiselessness of the gears, 100 per cent signifying absolutely noiseless operation and 90 per cent the best condition actually

obtainable. The curves show that under the very best conditions the two gears operate about equally well, but under average commercial conditions the stub-tooth gear is decidedly superior.

Determination of the Strength Factor and Method of Drawing Profiles. — In order to determine the value of Y for a certain gear of a given pitch, it is necessary to lay out a tooth to some magnified scale in the same manner (illustrated by Fig. 11) that Mr. Lewis measured the distance x and computed Y. In this investigation, this has been done for a great many pitches, using a scale of 10 to 1, except for the large pitches where the drawings

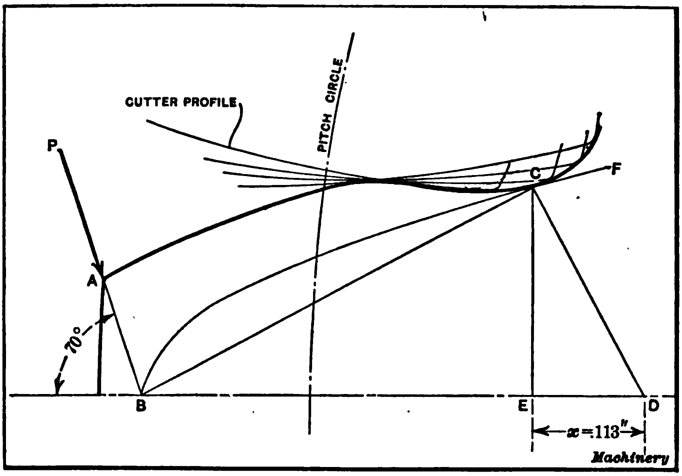


Fig. 13. Method of laying out Gear Teeth to Determine the Values of x and Y

would become too large. For pitches $\frac{5}{7}$ and $\frac{6}{8}$, a scale of 8 to 1 was used and for $\frac{4}{5}$ pitch a scale of 6 to 1. Fig. 13 illustrates the method of laying out the teeth. Before laying out the teeth the profile of the cutter tooth was drawn on tracing paper with the addendum equal to the dedendum of the stub-tooth gear, and the diameter equal to that of a 24-tooth gear. The tooth profiles of gears with various numbers of teeth were then drawn accurately on tracing paper, with the face, from the base circle out, a true involute and the flank and fillet generated by the point of the cutter tooth, as the pitch circles of the gear and cutter were rolled together.

Having thus obtained the profile, the action line AB (Fig. 13) of the force at the end of the tooth was drawn normal to the involute, *i.e.*, at 20 degrees to the vertical. In order to get the parabola BCF passing through B and tangent to the profile, a considerable number of parabolas, differing but slightly from each other, were drawn on cards, so that they could be slipped in under the tracing paper and adjusted to enable the point of tangency C to be determined. The line BC was then drawn and CD and CE drawn perpendicular to BC and BD, respectively. The length x was measured as accurately as possible with a Starrett steel scale and the value of Y computed. Two or more

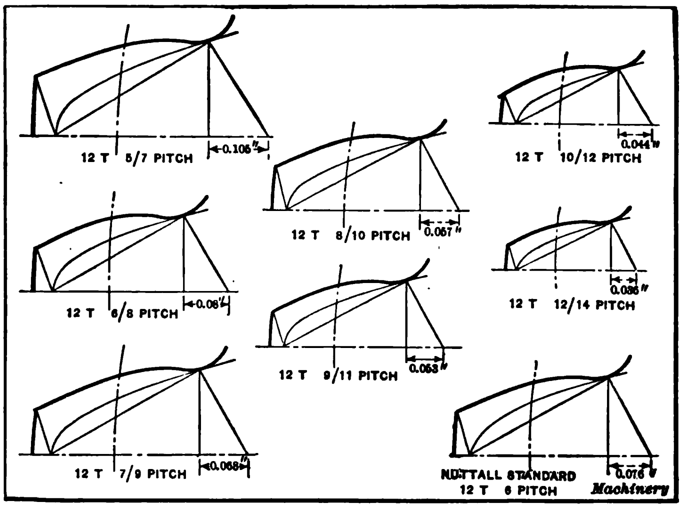


Fig. 14. Tooth Profiles and Values of x for Different Pitches

profiles of the same tooth were sometimes drawn and it was found that the values of x usually checked within 1 or 2 per cent. Fig. 14 shows the tooth profiles and values of x for various pitches.

Method of Plotting. — We have already determined an exact method of computing Y_r in Equation (5):

$$Y = Y_r - \frac{a}{n}$$

If any method could be found for obtaining the value of $\frac{a}{n}$, the problem of determining the value of Y would be very simple.

The use of logarithmic cross-section paper makes the plotting of all such functions as $Y = bx^a$ very simple, because the curve becomes a straight line with a slope a. In Equation (5) $\frac{a}{n}$ is a function of the same character, and when so plotted for different values of n, it should give a straight line with a slope of -1. Advantage was taken of this peculiar property of logarithmic paper by first carefully computing Y and Y_r for five different gears for each pitch, and then plotting the corresponding values of $\frac{a}{n}$. Due to unavoidable errors in calculating Y, the points did not all lie exactly in a straight line, but somewhat as shown in Fig. 15, which gives values of $\frac{a}{n^m}$ for the $\frac{6}{8}$ pitch teeth, and so it was considered best to use five points and draw an average line through them instead of drawing a line through just two determined points.

It was found that instead of the function being $\frac{a}{n}$ it was $\frac{a}{n^m}$ for stub-tooth gears with the slope of the line, *i.e.*, the value of m, lying between 0.72 and 0.93. Values of a and m are given in the table below.

Table of Values of *a* and *m*

Pitch	4/5	5/7	6/8	7/9	8/10	9/11	10/12	12/14	Nuttall System	Standard 20-deg. Involute
a	0.505	0.535	0.605	0.760	0.690	0.490	0.540	0.440	0.592	0.540
m	0.760	0.800	0.830	0.930	0.890	0.780	0.800	0.720	0.820	0.790

The values of Y in the table on the next page were then obtained by simply subtracting the value of $\frac{a}{n^m}$ obtained by means of the proper chart from Y_r. If this method of plotting had not been used, it would have been necessary to lay out gears with from 12 to 200 teeth and measure x on each drawing, a task which would have taken five or six times as long to do.

Conclusions. — One of the interesting facts that was brought out in this investigation was that the function $\frac{a}{n}$ as given by Lewis does not apply to stub-tooth gears, and that a function of this nature to be correct should have an exponent other than

Values of *Y* in Lewis Formula for Stub-tooth Gears

No. of Teeth	Fellows System								Nuttall System
	4/5	5/7	6/8	7/9	8/10	9/11	10/12	12/14	
12	0.096	0.111	0.102	0.100	0.096	0.100	0.093	0.092	0.099
13	0.101	0.115	0.107	0.106	0.101	0.104	0.098	0.096	0.103
14	0.105	0.119	0.112	0.111	0.106	0.108	0.102	0.100	0.108
15	0.108	0.123	0.115	0.115	0.110	0.111	0.105	0.103	0.111
16	0.111	0.126	0.119	0.118	0.113	0.114	0.109	0.106	0.115
17	0.114	0.129	0.122	0.121	0.116	0.116	0.111	0.109	0.117
18	0.117	0.131	0.124	0.124	0.119	0.119	0.114	0.111	0.120
19	0.119	0.133	0.127	0.127	0.122	0.121	0.116	0.113	0.123
20	0.121	0.135	0.129	0.129	0.124	0.123	0.118	0.115	0.125
21	0.123	0.137	0.131	0.131	0.126	0.125	0.120	0.117	0.127
22	0.125	0.139	0.133	0.133	0.128	0.126	0.122	0.118	0.128
23	0.126	0.141	0.134	0.135	0.129	0.128	0.123	0.120	0.130
24	0.128	0.142	0.136	0.136	0.131	0.129	0.125	0.121	0.131
25	0.129	0.143	0.137	0.138	0.133	0.130	0.126	0.123	0.133
26	0.130	0.145	0.139	0.139	0.134	0.132	0.128	0.124	0.134
27	0.132	0.146	0.140	0.140	0.135	0.133	0.129	0.125	0.136
28	0.133	0.147	0.141	0.141	0.136	0.134	0.130	0.126	0.137
29	0.134	0.148	0.142	0.143	0.137	0.135	0.131	0.127	0.138
30	0.135	0.149	0.143	0.144	0.138	0.136	0.132	0.128	0.139
32	0.137	0.150	0.145	0.146	0.140	0.137	0.134	0.130	0.141
35	0.139	0.153	0.147	0.148	0.143	0.139	0.136	0.132	0.143
37	0.140	0.154	0.149	0.149	0.144	0.141	0.138	0.133	0.145
40	0.142	0.156	0.151	0.151	0.146	0.142	0.140	0.135	0.146
45	0.145	0.159	0.154	0.154	0.149	0.145	0.142	0.138	0.149
50	0.147	0.161	0.156	0.156	0.151	0.147	0.144	0.140	0.151
55	0.149	0.162	0.157	0.158	0.152	0.149	0.146	0.141	0.153
60	0.150	0.164	0.159	0.159	0.154	0.150	0.148	0.143	0.154
70	0.153	0.166	0.161	0.161	0.156	0.152	0.150	0.145	0.157
80	0.155	0.168	0.163	0.163	0.158	0.154	0.152	0.147	0.159
100	0.158	0.171	0.166	0.166	0.160	0.156	0.154	0.150	0.161
150	0.162	0.174	0.170	0.169	0.164	0.160	0.158	0.154	0.165
200	0.164	0.176	0.172	0.171	0.166	0.162	0.160	0.156	0.167
Rack	0.173	0.184	0.179	0.176	0.172	0.170	0.168	0.166	0.175

unity in the denominator. In view of this discovery, it was thought strange that for standard teeth this exponent should be exactly unity and the function be $\frac{a}{n}$, or as Lewis gives it for 20-degree involute teeth $\frac{0.912}{n}$. Consequently Lewis values of

$\frac{a}{n}$, derived from his values of Y, were plotted and it was found that the function really is $\frac{0.54}{n^{0.79}}$; but as the function $\frac{0.912}{n}$ is so much simpler to calculate, and is correct within 5 per cent, one might be justified in using it.

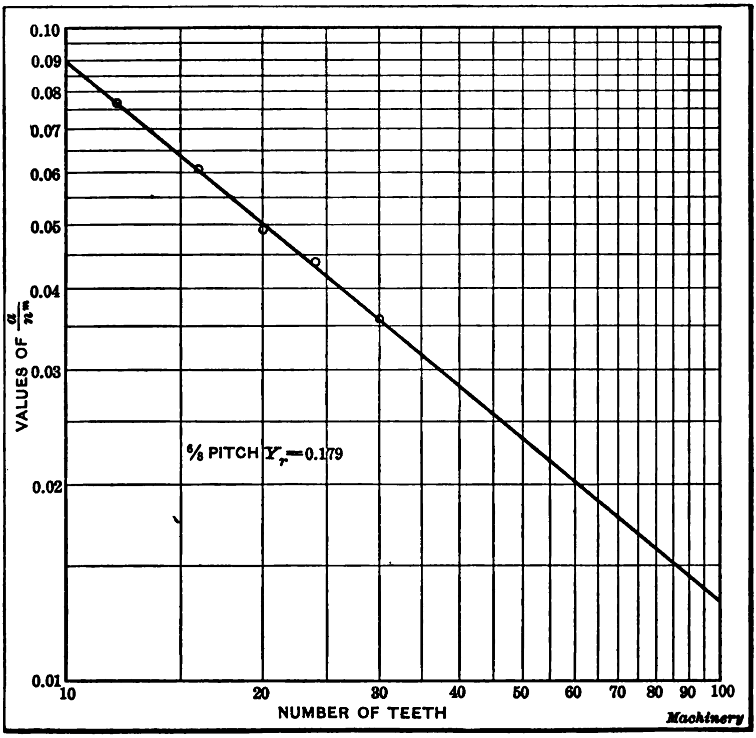


Fig. 15. Diagram showing the Values from which Factor Y is Derived for a 5/8 Pitch Gear

Owing to the varying ratio of the addendum to the circular pitch in the Fellows system, the values of Y are different for each pitch, but in the Nuttall system this ratio is constant, Y being the same for all pitches. The values of Y in the table indicate that stub-tooth pinions with less than 25 teeth are about 25 per cent stronger than the standard 20-degree involute and 40 per cent stronger than the standard $14\frac{1}{2}$-degree involute. For larger gears the difference is in favor of the stub-tooth, but is not quite so marked.

CHAPTER VI

NOISY GEARING

A GREAT deal has been written on the subject of noisy gearing. Many suggestions have been made for its elimination and some improvements have undoubtedly been made. Rawhide and cloth gears, referred to in previous chapters, meet the requirements in many instances; but often metal to metal gearing is required by the conditions of design or application. In the present chapter a method will be explained which tends to produce silent-running metal gearing. This method has been practically applied for several years.

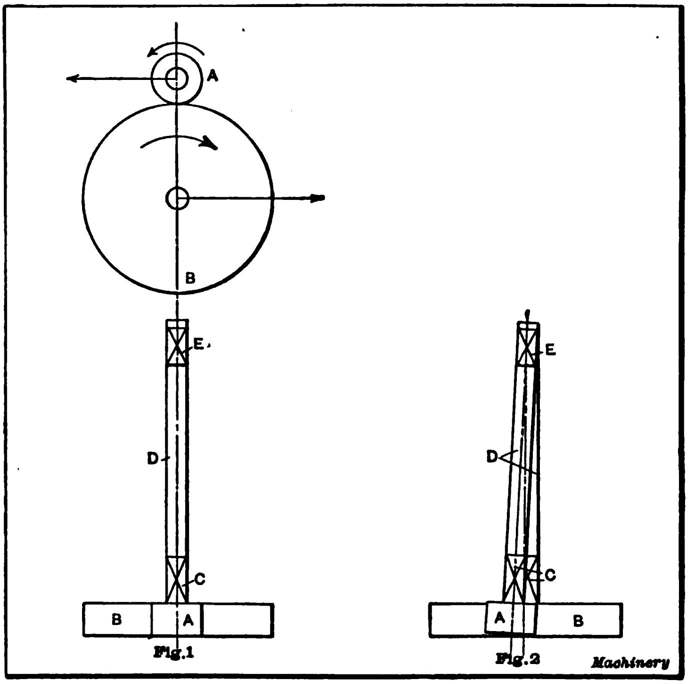
Causes of Noisy Gearing. — All noise in gearing is caused by the vibration of the material in the gear. The source of this vibration is usually a series of blows resulting from one or more of the following causes:

1. — The individual teeth are unequally loaded; that is, the load is borne by more teeth at some periods than at others. In this case it is perhaps incorrect to describe the effect on the teeth as a blow, since it takes the form of an increased compression of the material, which, however, produces vibration.

2. — The blow may be an actual concussion caused by one tooth being disengaged before the next tooth takes up the load. Under present conditions this is a cause seldom met with, but was often found in the past when pinions with too few teeth were used.

3. — Owing to inaccuracies in cutting, the pitch of the teeth may vary around the gear circumference, so that while theoretically two or more teeth should be in contact, only one supports the load. When the latter comes out of contact, the load is transmitted to the next tooth by a sharp blow. This has been a most prolific cause of noisy gearing.

4. — The faulty alignment of the shafts on which the gears are mounted produces a jamming action, causing an objectionable grinding noise. Even if the alignment is perfect when the gears are erected, the shafts are practically certain to become displaced sooner or later owing to uneven wear in the bearings. Figs. 1 and 2 show, diagrammatically, a case of this kind. In Fig. 1 the power is transmitted from pinion *A* to gear *B*. As-



Figs. 1 and 2. Diagrammatical View showing Effect of Disalignment

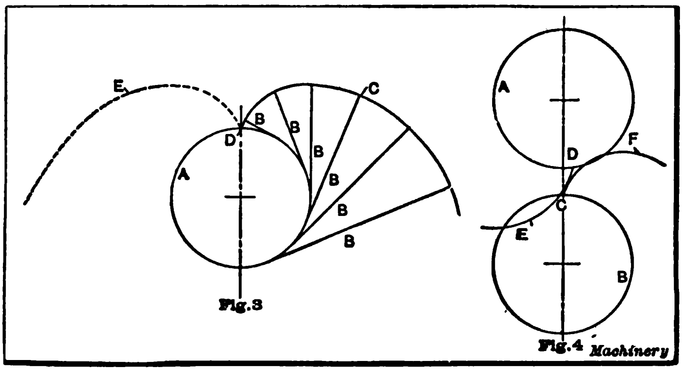
sume that the action between the teeth is perfect and that, therefore, the pressure between the teeth is at all times in exactly the same direction relative to the pitch circles. This pressure tends to force *B* in the direction of the lower horizontal arrow and *A* in the opposite direction. Actual motion is prevented by bearings *C* placed as close to the gears as possible, but, in time, the pressure on the bearings *C* will cause wear; if the bearing surface is inadequate, which is commonly the case,

perceptible wear soon takes place. The result is that the shafts *D* tend to turn bodily about their bearings at *E* where the wear is not likely to be so pronounced. The gears then take a position as indicated in Fig. 2 and the teeth will bear on both sides. This is undoubtedly the reason why gears originally silent in action, gradually become noisy. This defect is difficult to cure entirely, but a great deal may be done by increasing the available bearing surface close to the gears, or by placing another pair of bearings outside of the gears. Of course, the forces acting on the bearings are not horizontal, but slightly inclined, owing to the inclined action on the teeth. For simplicity of illustration, however, the case has been presented as indicated.

5. — Noises are also due to special forms of teeth. For example, with cycloidal teeth, the smallest separation of the shafts destroys the uniformity of transmission and noise results. Since, however, the involute tooth is now practically universally adopted, the present chapter will deal with the latter form of tooth only. The statement that a partial separation of the centers of involute gears does not affect the true working of the teeth is not wholly true; cases may occur when noise will result from this cause. A separation of the centers has the effect of reducing the length of time of contact, and hence it is reasonable to assume a case when two gears have such a number of teeth that one tooth is released from the load at the instant when the next tooth in succession comes into mesh. In this case the conditions mentioned under (2) are met with.

6. — Interference between the teeth themselves is a common cause of noisy gearing. To rightly understand this cause it is necessary to enter briefly into the theory of the shape of the involute tooth. The involute is commonly defined as a curve described by the end of a string as it is unwound from a cylinder, the string being kept taut, so that in every position it may be described as a tangent to the cylinder. In Fig. 3, *A* represents the cylinder and *B* the string in various positions as it is unwound from the periphery of *A*; *C* is the involute described by the end of the string. The circle *A* is known as the "base circle" of the involute, and *D* is called the "source of

the curve." It follows from the definition that the involute is not a closed curve; in other words, it terminates in infinity. It is also evident that it is a curve of two branches because the string may obviously be unwound from the "base circle" in either a clockwise or a counter-clockwise direction, the second branch being indicated in Fig. 3 by the dotted curve *E*. As the involute lies entirely without the circumference of its base circle, the working part of the curve terminates at *D*. It is, therefore, evident that we must so proportion the mating involutes that when the two curves are rolling together, the point of contact between the source of one curve with the other curve shall be the outer termination of the latter.



Figs. 3 and 4. The Involute Curve and Its Application to Gear Teeth

This will be made clearer by referring to Fig. 4. Here *A* and *B* are two base circles with their respective involutes *E* and *F*; *D* is the source of *E*, and *C* of *F*. The two involutes are in contact, and it will be seen at once that the involute *E* must be cut off at the point which has been in contact with *C*, and similarly that *F* must terminate at the point which will come ultimately into contact with *D*. If the involutes are extended beyond these points, the mathematical action still continues but actual contact is impossible, since it entails an overlapping of the curves, causing one tooth to dig into the other. The points *C* and *D* are known as "interference points," and interference or the digging of one tooth into its mate is a

direct result of extending the addendum beyond the circle drawn through the interference point.

Noise Due to Interference. — Evidence that interference actually occurs in practice will usually be found on examining a pair of gears which have been in action for some time and where one of them is a pinion of less than 20 teeth. On examining the pinion teeth, a groove will be found in the face of the teeth, slightly below the imaginary pitch line of the tooth, and the points of the gear teeth will be found to be rounded over and bright. In some cases this groove becomes a keen line, as if it had been drawn with a scriber. Except in the case of equal gears, the groove is usually found only on the teeth of the gear with the lesser number of teeth. The presence of this mark is invariably accompanied by noise, and is a sure sign of there being too few teeth in the pinion. It is proposed to deal more fully with this point later, since it deserves a great deal more attention than it generally receives. The blow which causes vibration and noise in this case takes place between the points of the gear teeth and the flanks of the pinion teeth. On first contact, it is a blow pure and simple, afterward becoming an abrasion, so that from this cause two distinct kinds of noises arise, namely, the ring due to the blow and the grind due to the abrasion. Noisy gears must of necessity be inefficient transmitters of power. Therefore, altogether apart from the question of comfort, noise must be reduced as far as possible if high efficiency is to be attained.

The foregoing paragraphs give the six chief causes of noise in gearing. In the cutting of gearing by the various processes now in use, the limit of refinement in workmanship has practically been reached. It is therefore evident that in order to eliminate the noise of gearing, it is necessary to make a change either in the design of the teeth or in the gears themselves. One method which has been adopted for the reduction of noise is the use of pinions made of rawhide, paper, fiber and other similar materials. Gears made from these materials, however, do not remove the cause of the trouble, but merely allay it, and the very causes which produce the noise in regular gearing in

most cases end by destroying the pinion when made from less wear-resisting materials. The materials mentioned, by their very nature, do not always promise good wearing resistance — with the exception probably of fiber. Besides, other troubles are introduced: rawhide is easily injured by oil, and the same applies to paper pinions; consequently lubrication is difficult, it becoming necessary to use solid lubricants, which are somewhat difficult to apply. Fiber is peculiarly susceptible to moisture, which causes it to swell and often to jam. These materials for gearing cannot, therefore, be considered a permanent aid for securing efficiency of transmission.

Analysis of the Conditions. — It has already been shown that for any two gears in mesh, owing to the nature of the involute curve, interference points exist, and that if circles are drawn through these points, the points of the teeth must not be prolonged beyond these circles. This being so, we are led to believe that, given a ratio, there should be one pair of diameters which will give the best results, and one pair only. It is proposed to prove that such is actually the case, and to proceed to establish formulas which will enable us to fix the correct number of teeth in the larger gear of a pair for any ratio. The present chapter has been written particularly with the object of bringing this important matter before the practical man.

In Fig. 5 is shown, diagrammatically, a pair of gears, and the number of teeth has been purposely chosen very low in order to render the argument more obvious. The tooth profiles are assumed to be involutes and hence the line of contact is a straight line AB. The base circle (that is the circle from which each involute is generated) is marked for each gear. Now, the limit line for each gear is found by striking from the gear center an arc passing through the point of tangency of the line of contact AB and the base circle of the other gear (*i.e.*, the source of the working involute). These points are D and E, respectively, and the limit circles are shown passing through them. It will at once be seen that, as far as the pinion is concerned, no interference is to be anticipated, since the addendum circle lies well within the limit circle, but a considerable shorten-

ing of the addendum of the gear teeth is necessary if interference is to be avoided. The addendum is usually taken as a function of the pitch, the value most commonly used being:

$$\text{Addendum} = \frac{1}{\text{diametral pitch}}$$

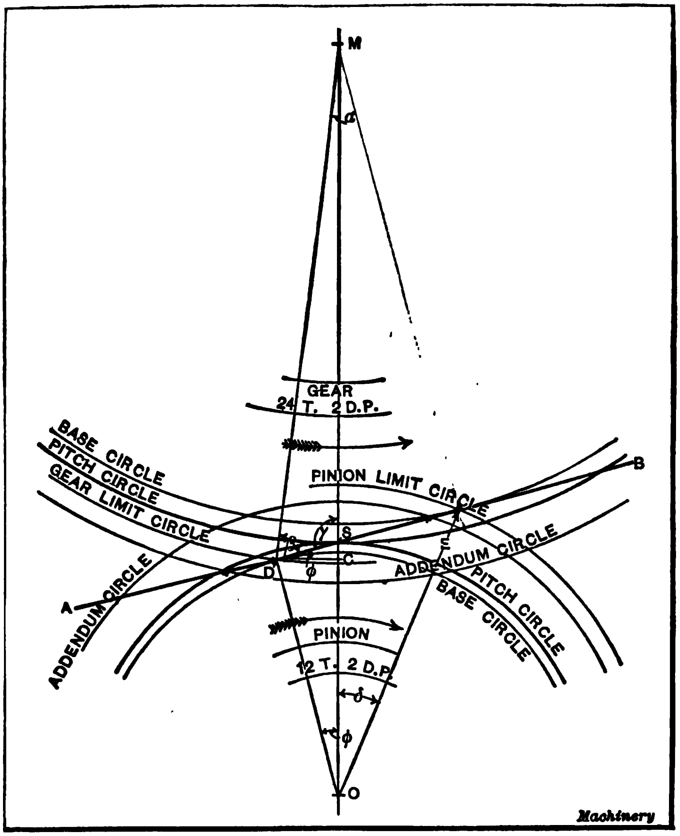


Fig. 5. Diagrammatical Lay-out for the Derivation of Formulas

It will, therefore, be seen that if standard teeth are to be used, it is necessary to redesign the gears, adjusting the number of teeth and pitch so that the addendum circle and limit circle shall at least coincide. The best conditions are secured when these two circles coincide because the maximum arc of contact without interference is then obtained. Further, it is

obvious that since the height of the teeth in the larger gear is dependent on the point of tangency of the line of contact and the base circle of the pinion, any variation in the ratio of the train by altering the pitch diameter of the pinion, and consequently also its base circle diameter, will entail an alteration in the tooth height for the gear; in other words, there is for every ratio one pair of gears, and one only, which will give the best all around efficiency.

The requirements are filled when the first point of the contact of the gear (the intersection between the line of contact AB and the addendum circle) is so located that a radius from the center of the pinion, through this point, makes a right angle with the line of contact. In that case, the addendum circle and the limit circle of the gear will coincide.

In Fig. 5 let angle SDO be a right angle, and let the angle SDC of the involute be called ϕ. The line CD is perpendicular to line MO, the line through the gear centers. Then, angle SOD = angle $SDC = \phi$.

Let $SD = a$, and $SC = x$. Then, $\frac{x}{a} = \sin \phi$. . . . (1)

Let n = number of teeth in pinion;
N = number of teeth in gear;
P = diametral pitch.

Then, $SO = \frac{n}{2P}$, and $\frac{SD}{SO} = \frac{a}{n \div 2P} = \sin \phi$.

Hence, $a = \frac{n}{2P} \sin \phi$. . . . . (2)

Substituting in (1) and transposing:

$$x = \frac{n}{2P} \sin^2 \phi. \quad . . . . (3)$$

In triangle CDO we have $\frac{CD}{CO} = \tan \phi$.

But, $CO = SO - SC = \frac{n}{2P} - \frac{n}{2P} \sin^2 \phi = \frac{n}{2P} \cos^2 \phi$.

Hence,

$$\tan \phi = \frac{CD}{\frac{n}{2P} \cos^2 \phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Let angle $CMD = \alpha$; then, $\tan \alpha = \frac{CD}{CM} = \frac{CD}{SM + SC}$.

But $$SM = \frac{N}{2P}.$$

Hence, $$\tan \alpha = \frac{CD}{\frac{N}{2P} + \frac{n}{2P} \sin^2 \phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Dividing (4) by (5):

$$\frac{\tan \phi}{\tan \alpha} = \frac{N + n \sin^2 \phi}{n \cos^2 \phi} = \frac{N}{n \cos^2 \phi} + \tan^2 \phi.$$

Hence, $$\cot \alpha = \frac{N}{n \sin \phi \cos \phi} + \tan \phi. \quad . \quad . \quad . \quad . \quad (6)$$

from which α can be determined.

We have further, $\alpha + \beta + \gamma = 180$ degrees. But $\gamma = 90° + \phi$.

Therefore, $$\beta = 90° - \phi - \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

from which β can now be found. Further,

$$\frac{DM}{\sin \gamma} = \frac{SM}{\sin \beta}$$

But DM = half the outside diameter of the gear $= \frac{N + 2}{2P}$

$$SM = \frac{N}{2P} \text{ and } \sin \gamma = \cos \phi.$$

Hence, $$\frac{N + 2}{2P \cos \phi} = \frac{N}{2P \sin \beta}$$

$$\sin \beta = \frac{N}{N + 2} \cos \phi. \quad . \quad . \quad . \quad . \quad (8)$$

As β is known from Equation (7), N can be obtained by solving (8). The value of N thus found is the smallest number of teeth permissible in the larger gear if interference is to be entirely eliminated.

Charts for Finding Number of Teeth in Large Gear. — The foregoing solution appears cumbersome, but in applying it to practice the only equations used are (6), (7) and (8), which are easy to solve. The curves corresponding to the equations for $14\frac{1}{2}$- and 20-degree angles of involute are given in Figs. 6 and 7. One curve in each chart was arrived at by solving Equation (8) and was plotted with values of sin β as abscissas and corresponding values of N as ordinates. From Equation (6) values of cot α were then found corresponding to given values of $\frac{N}{n}$ (the ratio). Knowing α and ϕ, values of β were then obtained from Equation (7) corresponding to the given values of $\frac{N}{n}$. Corresponding values of sin β were then found from a table of sines, and the second curve was plotted with $\frac{N}{n}$ as ordinates, and the values of sin β as abscissas. The dotted lines on each chart indicate the course to be traced in using the diagram. The most usual problem will be, given a ratio, to find the most suitable number of teeth for the larger gear. In solving this problem, first find on the left-hand side of the chart a figure denoting the given ratio, and trace horizontally to meet curve marked "Curve I." From this point trace vertically to meet "Curve II," and then again horizontally to the right-hand side of the chart, where the correct number of the teeth for the larger gear will be found. Thus in the example chosen it was required to find the correct number of teeth in the larger gear of a pair having a ratio of 5 to 2. This ratio is first expressed in terms of unity, *viz.*, $2\frac{1}{2}$ to 1, and $2\frac{1}{2}$ is found on the left-hand side of the chart. The dotted line is then followed horizontally to Curve I, and then vertically to Curve II, and finally horizontally to the right side of the chart. In Fig. 6, for $14\frac{1}{2}$-degree involute, the number of teeth is found to be 65, and in Fig. 7, for 20-degree involute, 38. The pinions then will have 26 and 15.4 teeth, respectively. Since fractional teeth are impossible, these values for the pinions become 26 and 16, and, therefore, the corresponding gear teeth values are 65 and 40.

For the moment, since $14\frac{1}{2}$ degrees has been practically universally adopted as the standard angle of involute, our attention

will be confined to Fig. 6. Two things are particularly to be noticed:

1.—As the ratio increases, the number of teeth in the pinion must also increase. Thus with a ratio of 1 to 1 (equal gears)

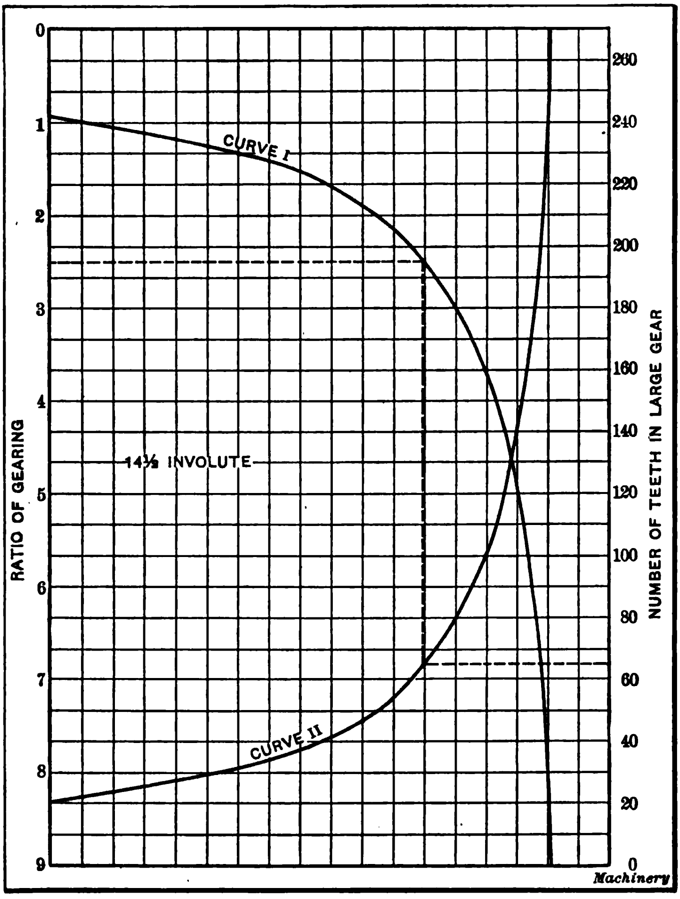


Fig. 6. Chart for finding Smallest Number of Teeth in Large Gear for a given Ratio, with 14½-degree Pressure Angle

both gears should have 22 teeth; with a ratio of 4 to 1 the pinion must have $120 \div 4 = 30$ teeth; and with a ratio of 8 to 1 the value must be increased to $256 \div 8 = 32$ teeth.

2.—The number of teeth in common use for pinions is, as a general rule, far too small, especially for the high ratios.

Possible Methods for Avoiding Interference. — The significance of these two points is very great. A reference to recent specifications for gearing will show that to a certain extent this principle has been grasped, but has not been carried sufficiently far. Probably the greatest field at the present day for gearing is in the transmission of power from electric motors, and it is here that the tendency to increase the number of teeth in the gears is most evident. Large gears with teeth of fine pitch and wide faces are used, but it is doubtful whether the slight advance which has been made in this direction has had any very appreciable effect in reducing the noise from the gearing. There are two reasons why the difficulty cannot be wholly dealt with by adopting finer pitches in order to increase the number of teeth:

1. — From consideration of strength, if fine pitches are used, the gear face must be correspondingly increased. Until quite recently face widths used to be from $2\frac{1}{2}$ to 3 times the circular pitch, but now it is not at all uncommon to find 5 or 6 times the circular pitch used. With such relatively wide faces extreme accuracy in erection is necessary in order to secure an equal bearing all along the tooth face. Equal accuracy in the cutting is also most important. If this accuracy does not obtain, the whole load is thrown on the corner of a tooth, and since the pitch is small, breakage is extremely likely to occur. Wear in the bearings will have the effect described under cause (4) at the beginning of this chapter.

2. — High ratios are, at any rate with electric motors, a necessity, if first cost of installation is to be kept down. Now, if the correct number of teeth is used in the pinion, the number of teeth in the gear becomes so great that difficulties are experienced in the cutting. Even on the hobbing machine, the time taken by the hob in traversing the circumference of the gear is considerable, and on this account there is strong reason to believe that local heating of the blank is introduced with consequent errors of pitch. Such errors may undoubtedly be reduced to a minimum, but the cost of production is thereby considerably increased.

These two objections have obviously been raised by the practical man, who alone has set the limit for the number of teeth which are practically advisable. Fine pitches do not "look right," but we recognize, of course, that silence is important; if it is only to be obtained by the adoption of finer and still finer pitches, the question of practicability arises. Consequently, if it can be shown that we may, by following certain simple rules, return once more to the coarser pitches and secure even greater

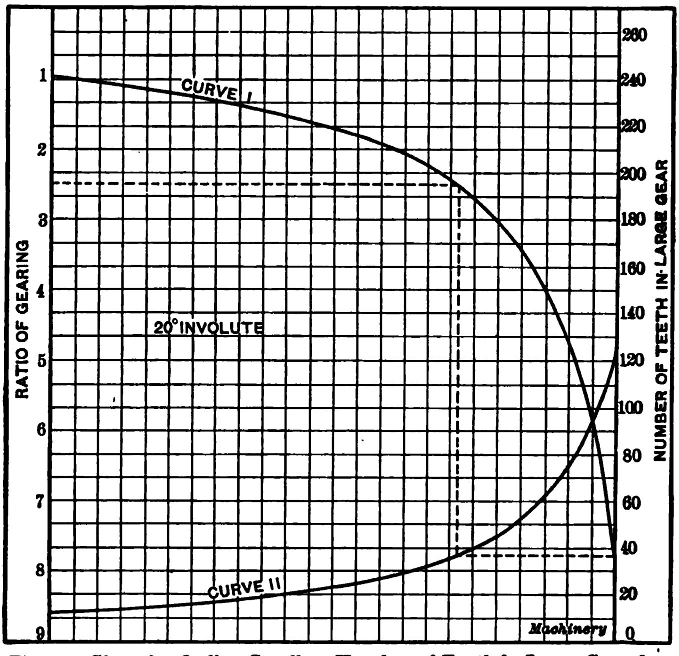


Fig. 7. Chart for finding Smallest Number of Teeth in Large Gear for a given Ratio, with 20-degree Pressure Angle

degrees of silence than has so far been obtained by the use of the finer pitches, a great deal has been done to successfully solve this problem.

It is evident from Fig. 7 that one way of reducing the number of teeth required for efficiency is to increase the angle of the involute. Thus with a 20-degree involute, a 16-tooth pinion may be used with a 5 to 2 ratio, as compared with a 26-tooth pinion with a 14½-degree involute. This looks promising, and

in some cases has been adopted, but it is an unfortunate fact that involutes of a given angle of obliquity will work only with others having the same angle; consequently 20-degree involutes will not work with the standard 14½-degree involutes. It, therefore, will be seen that confusion is likely to result from varying the angles, and in any case, the greater advantage of interchangeability is sacrificed. Another great objection to the adoption of

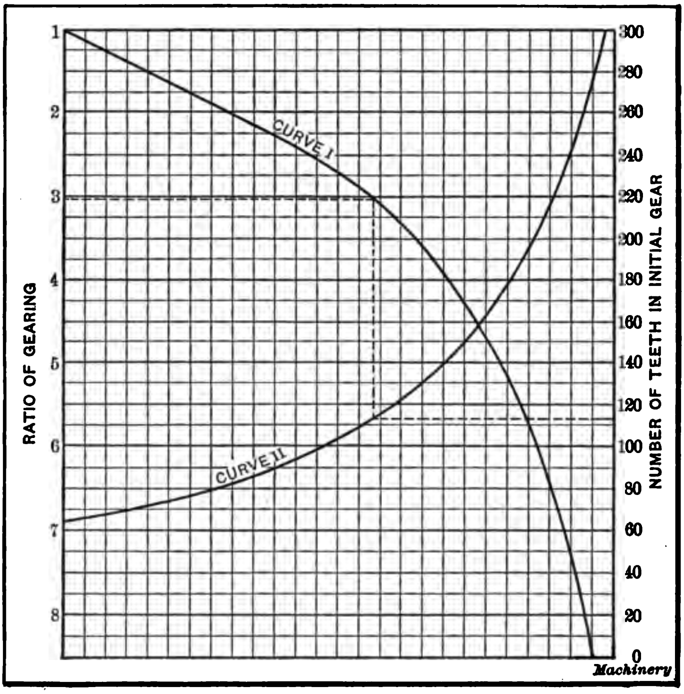


Fig. 8. Chart for finding Smallest Number of Teeth in Large Gear for Internal Gearing, with 14½-degree Pressure Angle

increased pressure angles is that, owing to the greater inclination of the path of contact, the pressure tending to force the gear teeth out of mesh is greatly increased; in other words, gears with teeth of high obliquity crowd on their centers to a considerably greater degree than those with smaller angles. Many designers, therefore, believe that this method of arriving at the required result is not advisable.

Proposed Method — The Shortened Addendum. — A method which has been used in everyday practice with marked success, for several years, may be explained as follows:

Interference, when present, is due to the addendum of the teeth in the larger gear being prolonged beyond the limit circle; therefore, it seems logical to reduce the addendum by the amount it projects beyond this circle. It is, of course, necessary to see whether by so doing any objectionable features are introduced. A point which appears at once is that by shortening the teeth, the duration of contact between two teeth is apparently shortened. If the duration of contact is decreased, the important question to consider is whether the shortened duration of contact is such as to cause one tooth to come out of mesh before the next one is engaged. Referring to Fig. 5 it has been shown that

$$\cot \alpha = \frac{N}{n \sin \phi \cos \phi} + \tan \phi. \quad . \quad . \quad . \quad . \quad (6)$$

By the same process it may be shown that if the angle SOE is called δ, then $\cot \delta = \dfrac{n}{N \sin \phi \cos \phi} + \tan \phi$.

Now, in Fig. 5, $n \div N = \frac{1}{2}$, and ϕ is $14\frac{1}{2}$ degrees. Hence,

$$\cot \delta = 2.3206, \text{ and } \delta = 23 \text{ degrees } 19 \text{ minutes.}$$

Angle $SOD = \phi = 14$ degrees 30 minutes. Hence, angle $DOE \doteq 37$ degrees 49 minutes.

This angle may be taken as a measure of the angular duration of contact. Around the circumference of the pinion there are twelve teeth and twelve spaces. It therefore follows that

$$\frac{37.82}{360} = \frac{x}{12}$$

where x equals the number of teeth always in mesh. In the present case x equals 1.26. Hence, it will be seen that even in the extreme case chosen one tooth will not come out of mesh before the next one is engaged.

Life of Gears. — The question of whether the life of the teeth is shortened should also be considered. In this connection it is

necessary to point out that the action between teeth of true involute profile is only a true rolling action when the teeth pass the pitch point S, Fig. 5. At any other points in contact there is more or less slipping between them, the maximum being reached at the points of first and last contact and the minimum at the pitch point. When the teeth first make contact they approach each other obliquely, and even if interference is absent the slipping is very great and causes wear. If interference is present, the effect is greatly aggravated. It, therefore, would seem that by removing the point of the tooth, the life of the gear is actually prolonged and this deduction is amply borne out in practice.

When it becomes necessary to use a smaller number of teeth than that indicated by the chart, interference may be avoided by reducing the addendum of the teeth in the larger gear to what it would be if the number of teeth found from the chart were used.

For instance, suppose that the number of teeth given by the chart is 180, and that the required diametral pitch is 6. The pitch diameter would be 30 inches and the addendum $\frac{1}{6}$ inch. Suppose also that 120 teeth is the largest number permissible. The diameter being as before 30 inches, the diametral pitch would have to be 4, the standard addendum being $\frac{1}{4}$ inch. With this gear, interference would occur, but if according to the rule the addendum were reduced to $\frac{1}{6}$ inch, no trouble would be met with. The actual reduction in diameter is given by the formula:

$$A = 2\left(\frac{N - T}{P \times N}\right)$$

where A = amount by which the over-all diameter of the gear is reduced;

N = number of teeth, found from chart;

T = number of teeth actually used;

P = diametral pitch actually used.

In the above example:

$$A = 2\left(\frac{180 - 120}{4 \times 180}\right) = \frac{1}{6}$$

which agrees with what has previously been said.

General Requirements. — When carrying out this method in practice difficulties may be anticipated from cutting the teeth if the gear has been previously reduced in diameter; but as the reduction in diameter is a known quantity it can easily be allowed for when calculating the depth of the cut. If difficulties are experienced, the gears may be left with standard outside diameters until after the teeth are cut, when they may be reduced the required amount in the hobbing machine itself.

When giving this method a trial, it is not advisable that experiments be made on gears that have already been in use, because if wear has taken place the results may be misleading. The probability is that a great improvement will be noticed in the running. In cases where it has been tried, no instance has ever been found where the noise did not decrease, except if wear had taken place to any extent, when other conditions are met with, and the trial may prove misleading. To get a fair idea of the results obtained by this method, a pair of gears with a ratio of 3 or 4 to 1 should be made, the pinion having 12 or 14 teeth, and with all dimensions standard. Another pair should then be made precisely similar, except that the diameter of the larger gear should be corrected as suggested; the two pairs should then be carefully erected and run side by side and the difference noted.

It may be pointed out that since the addendum of the gear tooth has been reduced, the flank space of the pinion may be to a corresponding extent filled in, thus shortening the pinion tooth and thereby reducing the under-cut and greatly increasing the strength of the tooth. This can only be done by using a special hob or cutter with shortened teeth, and unless a large number of duplicate gears are to be cut, the expense is not warranted.

It should be pointed out that if the gear ratio is one to one (equal gears), and the number of teeth to be used is less than the number found from the chart, it will be necessary to shorten the teeth of both gears. This being so, we are led to suppose that possibly with other ratios it may become necessary to reduce the height of the pinion teeth, if a low number of teeth

is used. Such is the case in practice, and it may be demonstrated by a construction for the pinion similar to that given in Fig. 5 for the gear. This correction of the pinion may be necessary with ratios from 1 to 1 up to 3 to 2, but since the ratios between these limits are less often used, it has not been thought necessary to give specific formulas.

Application to Rack and Pinion. — The principle may be applied to rack systems in which case it can be shown that to avoid interference:

$$\frac{n-2}{2}\tan^2\phi = 1$$

in which n = the minimum number of teeth in the pinion;
ϕ = the pressure angle in degrees.

Solving this equation for $\phi = 14\frac{1}{2}$ degrees, we obtain $n = 31.86$ or 32 teeth. A smaller number of teeth may be used, provided the rack teeth are shortened in a manner similar to that described for external gears.

If x = the amount to be cut off the rack teeth;
N = the number of teeth to be used in the pinion;
ϕ = the pressure angle;
P = the diametral pitch;

then it may be shown that

$$x = \frac{2 - N\sin^2\phi}{2P}$$

It will be noticed that if N is made $= 31.86$, and $\phi = 14\frac{1}{2}$ degrees, the numerator of the fraction vanishes, whence $x = 0$, which agrees with what has previously been said.

Application to Worm Gearing. — It is obvious that when dealing with the rack and pinion we have also disposed of the worm and worm-wheel. It follows that if interference is to be avoided, a worm-wheel should never have less than 32 teeth. Incidentally it may be suggested that this probably explains why worm-wheels with a small number of teeth frequently run hot in spite of the fact that they are only lightly loaded.

Application to Internal Gear. — One other case remains to be dealt with, *viz.*, the internal gear. It has so frequently been

shown that there is a certain minimum allowable difference between the numbers of teeth in the pinion and the mating internal gear that it is only necessary to mention it in passing, and to say that the interference which occurs if this rule is infringed upon is entirely separate from the interference dealt with throughout this chapter.

The argument already presented for spur gears may be equally well applied to internal gears. As with the spur gear, a similar line of argument results in three formulas which naturally bear a strong resemblance to Formulas (6), (7) and (8).

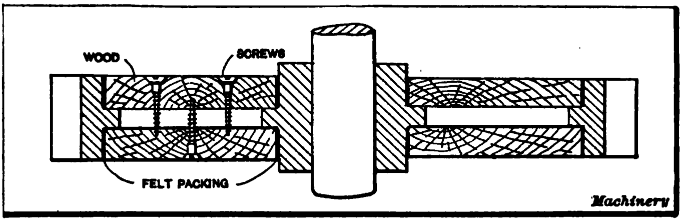


Fig. 9. A Method for Eliminating the Noise in Crane Gearing

$$\cot \alpha = \frac{E}{\sin \phi \cos \phi} - \tan \phi \quad \ldots\ldots \quad (9)$$

$$\beta = 90^\circ + \phi - \alpha \quad \ldots\ldots\ldots \quad (10)$$

$$\sin \beta = \frac{N}{N-2} \cos \phi \quad \ldots\ldots\ldots \quad (11)$$

where E = the ratio of the train;
ϕ = the pressure angle;
N = the number of teeth in the internal gear which gives the best all around results for that particular ratio.

The chart shown in Fig. 8 combines these formulas and puts them into a convenient form for use. A smaller number of teeth than the number found from the chart may be used, and interference is avoided as before, if the internal teeth are shortened. It must be noted that shortening the teeth in this case has the effect of increasing the internal diameter of the gear blank, the increase in bore being equal to $2\left(\frac{N-T}{PN}\right)$

in which N = the charted number of teeth in the gear;
T = the actual number of teeth used;
P = the actual diametral pitch.

Application to Bevel Gears. — In conclusion it may be pointed out that a similar line of reasoning may be applied to bevel gears, which in extreme cases may need correction for interference. It is hardly necessary to state that the actual numbers of teeth in the gears must not be used in applying the formulas, but that for these values must be substituted the developed numbers of teeth obtained in a manner well known to all designers.

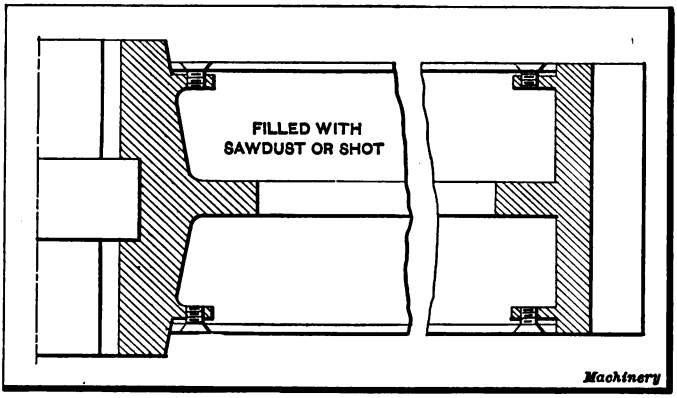


Fig. 10. Another Method to prevent Noisy Gearing

Make-shift Methods for Avoiding Noisy Gearing. — A little kink for eliminating the noise of gearing is shown in Fig. 9. Trouble was experienced from the excessive noise made by the gearing of a crane placed in such a position that the noise was highly objectionable. Several methods were tried to eliminate it. Grease and oils of all kinds were used with but temporary success. Finally the following method was tried: The annular space between the hub and the rim was packed with wood. This wood butted tightly up against felt pads as shown in the engraving. The pieces of wood were secured to each other by ordinary wood screws, care being taken not to have the heads project. Good hardwood should be used, and rubber might be used to advantage instead of felt, except for exposed outdoor

work. This method eliminated the objectionable noise from the gearing.

The method shown in Fig. 10 is even more efficient, but it is also far more expensive. In using this method gears of less than 18 inches in diameter are fitted with two sheets of tin which enclose the space between the hub and the rim of the gear. This space is then filled with sawdust or with No. 4 shot, the idea being to eliminate vibration by this means. In some cases, it has been found advantageous to use a mixture of shot and sawdust. The sheets of tin are fastened to the rim and hub with a number of small screws, as shown in the illustration. When the diameter of the gears exceeds 18 inches, wooden rings are used in place of the tin, the method of attachment being similar in either case. A felt packing is used to prevent the sawdust from leaking out. This arrangement has the further advantage of closing the space between the spokes of a wheel, thus making it impossible for a workman to get his arms or tools caught by the rotating wheel.

CHAPTER VII

DESIGN OF SPUR GEARING

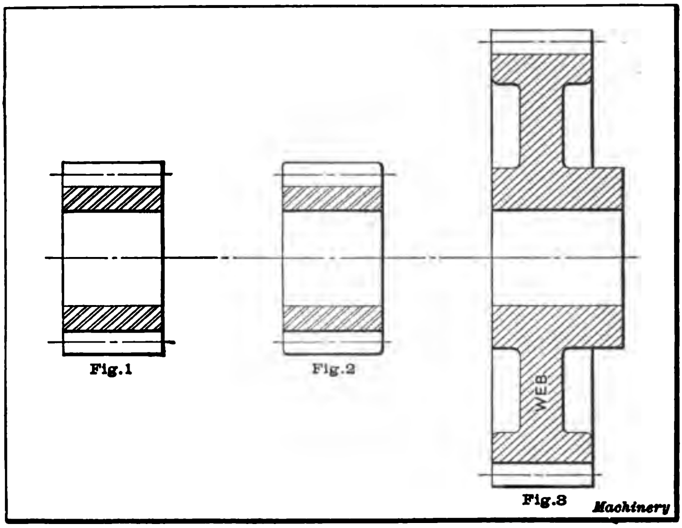
Small and Medium-sized Gears. — Typical designs of gears for different materials and uses are shown in Figs. 1 to 6. The one shown in Fig. 1 is the usual type for a steel, cast-iron, brass, bronze or fiber pinion. All of its proportions are determined by the gear calculations and the diameter of the shaft on which it is mounted, so there is little to be said about the design. When made from steel, it is generally formed from bar stock in the lathe or screw machine; for other metals, cast blanks are mostly used. It is the practice of some firms, notably the Brown & Sharpe Mfg. Co. of Providence, R. I., to round the corners of all pinion and gear blanks, large and small, as shown in Fig. 2. This practice has the advantage of making a gear more easy to handle, and less liable to injury in case it is accidentally dropped; it gives it a neater appearance as well.

In Fig. 3 is shown a design used for gears having a greater number of teeth than those ordinarily known as "pinions." The weight has been lightened by recessing the sides to form the web shown, connecting the rim and the hub. Gears of this shape are rarely cut from bar stock as the removal of the metal to form the web is too wasteful. The usual practice for this design is to make the blank from a casting or a drop forging.

Design of Large Gears. — As the number of teeth for the gear becomes still larger, the increasing weight of the gear may be lightened by cutting out relieving spaces in the web, or by abandoning the web entirely, and using arms for supporting the rim. This scheme, shown in Fig. 4, with arms of oval section, is the one best adapted for small and medium-sized gear blanks, and is often used on the largest work as well. It is the handsomest of all designs of gear wheels, when it is in harmony with the rest of the machine to which it belongs. It requires somewhat

more metal for the same strength than do the two designs next shown. It is very easily molded. Suitable dimensions for gears of various sizes made in this way are tabulated below the illustration.

For the largest gears, made of steel, cast-iron or bronze castings, arms of + or H-section are largely used. Dimensions for gears of these types are given in Figs. 5 and 6. In these designs, the metal is so distributed as to give a high degree of rigidity for the weight. These two forms, particularly that in Fig. 6, are more



Figs. 1, 2 and 3. Different Types of Spur Gears

difficult to mold than those previously shown. The latter form is better for gears whose faces are very wide in proportion to their pitch than either of the two in Figs. 4 and 5.

The tabular dimensions given for the various forms of gears are to be considered as suggestive rather than authoritative. The tables have been in constant use for some years, however, and have proved to be very satisfactory. "Draft," for removing the patterns from the sand in molding, is not shown in any of the illustrations. It should be provided liberally, and should be added to the dimensions given, rather than taken off.

Dimensions of Spur Gears

Dimensions of Spur Gears with Oval Arms

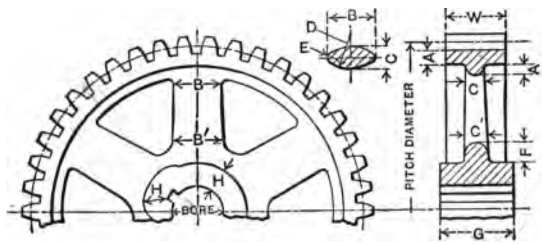


Fig. 4

P = diametral pitch, P' = circular pitch

$A = 1.57 \div P = 0.5\ P'$;
$B = 6.28 \div P = 2.0\ P'$;
$C = 3.14 \div P = P'$;
$D = 4.71 \div P = 1.5\ P'$;
$E = 0.79 \div P = 0.25\ P'$;

$F = 2.00 \div P = 0.65\ P'$;
$G = W + 0.025$ pitch diameter;
$H = 0.44 \times$ bore;
$B' = B + \frac{3}{4}$ inch per foot;
$C' = C + \frac{3}{4}$ inch per foot.

Dimensions of Spur Gears with Ribbed Arms or Arms of H-section

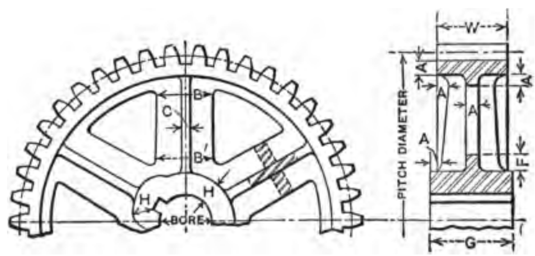


Fig. 5

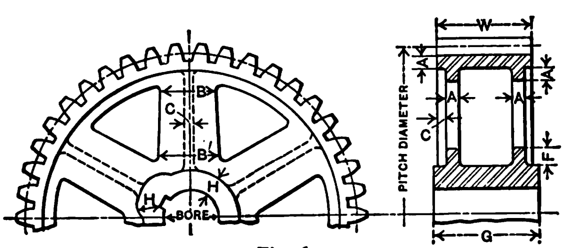


Fig. 6

P = diametral pitch, P' = circular pitch

$A = 1.57 \div P = 0.5\ P'$;
$B = 7.85 \div P = 2.5\ P'$;
$C = 0.94 \div P = 0.3\ P'$;
$F = 2.20 \div P = 0.7\ P'$;

$G = W + 0.025$ pitch diameter;
$H = 0.44 \times$ bore;
$B' = B + \frac{3}{4}$ inch per foot.

The Governing Conditions in the Design of Gearing. — The problem of gear design is one of materials and dimensions. The considerations on which the designer bases his choice of materials and dimensions are those of strength, durability, efficiency, smoothness of action, noiselessness and cost. The gear cannot attain perfection in all these particulars, as some of them are mutually hostile; the item of cost, especially, has to be sacrificed to make a gain in any other direction. The problem of design is thus one of compromise, and the designer has only his judgment to rely on in determining the relative importance of the various considerations.

It is possible, however, to lay down a few simple rules along this line. The prime consideration is that of strength. If the teeth of the gear are not strong enough to transmit the pressure they are calculated on to bear, the gear will break, and the other virtues it may possess in the way of cheapness, noiselessness, etc., will be of no avail. As has already been stated in a preceding chapter, in gearing subjected to occasional use only, the durability is sufficient for all practical purposes if the strength is sufficient; but there is a possibility that gearing transmitting power at high speed may wear out before it breaks. Where gearing is used, as in automatic machinery, primarily to obtain certain desired movements in the mechanism, without requiring the transmission of any great amount of power, the question of efficiency is not of prime importance.

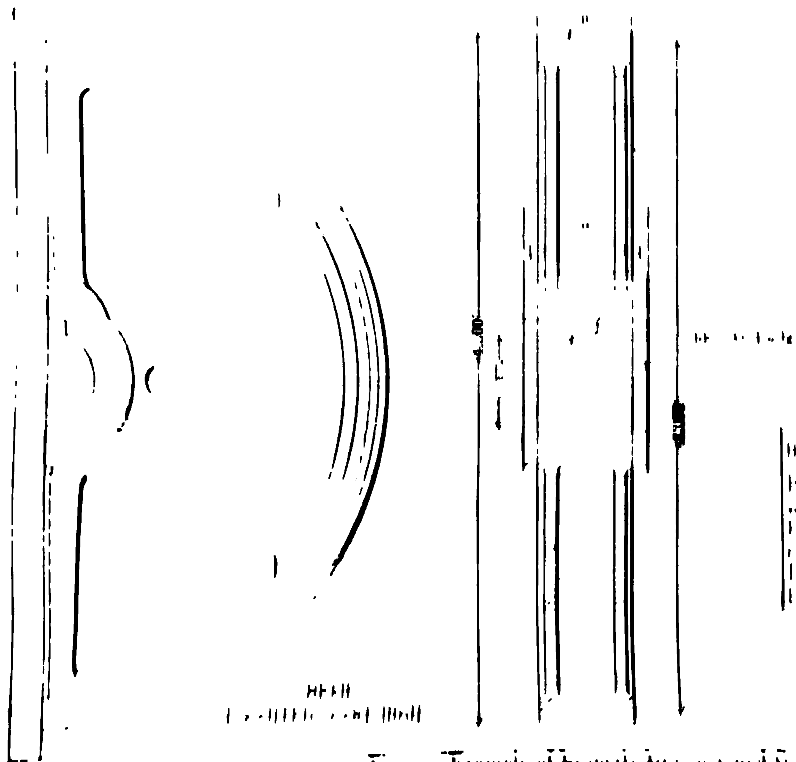
On the other hand, when the main object of a pair of gears is to receive so many horsepower from one shaft, and transmit as nearly as possible the same amount to another shaft, the question of efficiency becomes one worthy of the most careful consideration. As to smoothness of action, if the requirements for efficiency and silence have been met the gears will run smoothly, without shock or vibration. Noiselessness, as a problem in design, is largely a matter of the selection of materials, always supposing that the teeth of the gears are formed to correct tooth curves. The matter of cost is in part an engineering problem, and in part a commercial one. It is an engineering problem in so far as it bears on the problem of obtaining the greatest perfec-

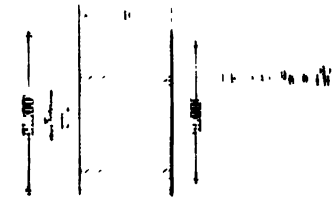
tion in the other particulars enumerated, with a given expenditure; and it is a commercial problem when it comes to determining how great a degree of refinement it is advisable to ask the purchaser of the finished machine to pay for.

A Model Spur Gear Drawing. — After designing a pair of spur gears with all the care that theoretical knowledge and practical experience can suggest, there still remains the important task of recording the results thus obtained on a drawing, in such a form that they will be intelligible to an intelligent workman. This drawing should plainly set forth every point of information needed for the successful completion of the work. In aiming at this mark, the student should study the model drawing shown in Fig. 7. The arrangement of this drawing, and the amount and kind of information shown on it, are based on the drawing-room practice of the Brown & Sharpe Mfg. Co., Providence, R. I. Some changes and additions, however, have been made. The design of the wheel in Fig. 7 is the same as that shown in Fig. 6. As stated, this design is not so easily molded as that in Fig. 5, but it is the most suitable form for gears of a comparatively wide face. No pattern dimensions are given. A drawing for machine shop use should not be confused by a maze of dimensions which are not used by the machinist. The pattern-maker can be taken care of by a separate drawing, or by a special blueprint with his dimensions put on in yellow pencil.

Dimensions on Gear Drawings. — The dimensions given are, perhaps, figured somewhat closer than is required on work of this size. The important dimensions, such as the outside diameter and the center distance, on which depend the proper fitting of the teeth of the gear, are a little too large to be measured with the vernier caliper, but they should surely be accurate within 0.005 inch — a limit easily attainable by a skillful workman.

Where a definite amount of allowable variation from the exact size can be determined on, it is customary in interchangeable manufacturing to give the dimensions with maximum and minimum limits. In work of the kind shown in Fig. 7, however, where the machine is "built" rather than "manufactured," it is not usual to give limits. The diameter of the hole in the hub



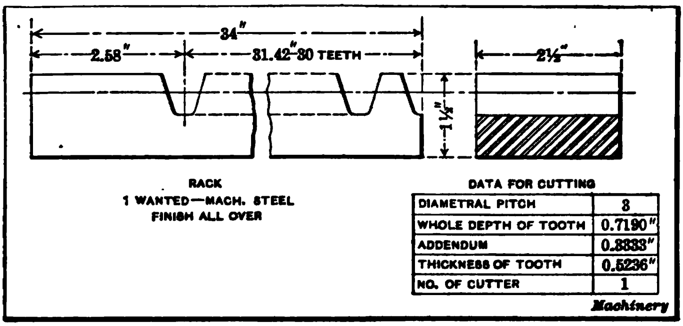


	GEAR	PINION
NUMBER OF TEETH	[illegible]	[illegible]
DIAMETRAL PITCH	[illegible]	[illegible]
WHOLE DEPTH OF TOOTH	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
NUMBER OF CUTTER	[illegible]	[illegible]
CENTER DISTANCE	[illegible]	

Fig. 4 Example of Properly Dimensioned Gear Drawing

of the gear is given as "6″ Std." on the drawing. This means that the hole is to be bored and reamed until it will make a good push fit for a standard plug gage of the size given. It will be noted that all the dimensions needed by the workman who turns the blank are appended to the figure, while those needed by the workman who cuts the teeth are given in tabular form.

The side view of the gear on the left is needed only for showing the number and dimensions of the arms to the patternmaker. For pinions and webbed gears it may be omitted. It is not



DIAMETRAL PITCH	8
WHOLE DEPTH OF TOOTH	0.7190″
ADDENDUM	0.3333″
THICKNESS OF TOOTH	0.5236″
NO. OF CUTTER	1

Fig. 8. Example of Properly Dimensioned Rack Drawing

necessary in standard gearing to show the shape of the teeth, so the side view is given as showing the blank before the teeth are cut. The pitch and bottom circles are represented by broken and dotted circles, respectively. The shape and kind of teeth (whether involute or cycloidal) is taken care of by the cutter called for — specified by its proper number if it is involute, and by its letter if it is cycloidal.

Rack Drawing. — In Fig. 8 is shown a model drawing of a rack, which is self-explanatory. Here, as in the previous case, the blank dimensions are shown attached to the figure of the rack, while the cutting dimensions are tabulated.

CHAPTER VIII

PRINCIPLES OF METHODS FOR CUTTING SPUR GEAR TEETH

THERE is no form of machine tool which has called for more ingenuity in design than the gear-cutting machine. The methods by which gears may be cut are so numerous, the requirements are so varied, the possible application of ingenious geometrical principles through the mechanism used are so nearly limitless, that a wonderful variety in design and construction has been evolved, affording a field of study which is unparalleled in its interest to the machine designer.

The earliest form of gear-cutting machinery to attain anything like its present state of development was the automatic spur-gear machine using a milling cutter to shape the tooth. Later came a period in which various forms of bevel gear cutting machinery were evolved, the demand being stimulated by the necessities of the chainless bicycle business. More recently, the requirements of the automobile have resulted in another period of inventive activity, which has resulted in the development of new machines and processes for gears of all kinds, though the bulk of the attention has been given to the spur and bevel forms.

The Classification of Gear-cutting Machinery. — Gear-cutting machinery may be classified, first, according to its *product*. There are four main divisions in this classification, separating from each other the machines designed for cutting spur, spiral, bevel and worm gearing, respectively. The cutting of internal gears and racks is analogous to the cutting of spur gears, and is included with it. Twisted or herringbone gears having parallel axes are, in general, cut in the same way as spiral gears, though, as gears, they belong to a different class. Some machines are so designed as to be capable of cutting more than one form of gear,

but it is only done by making certain adjustments or using certain attachments which, for the time being, convert them into machines of other types. The best example of a machine which covers all the divisions of this classification is the universal milling machine, which may be arranged to cut the teeth in any one of the four forms mentioned.

The second classification of gear-cutting machinery depends on the *principle of action* involved. The five methods we will

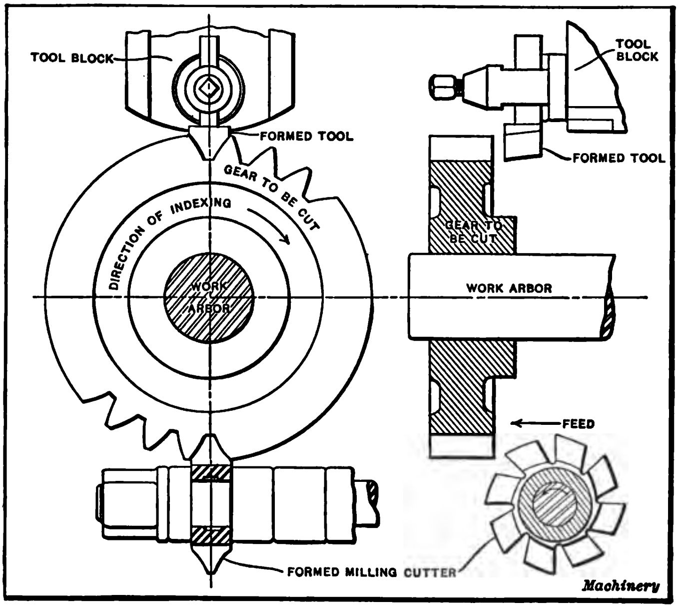


Fig. 1. The Formed Tool Principle of Action as Exemplified by a Shaper Tool and a Milling Cutter

consider are — the formed tool, templet, odontographic, describing-generating, and molding-generating methods. This classification relates particularly to the way in which the tool is held and guided with reference to the work, to produce the desired form for the tooth surfaces.

The third method of classification relates to the *nature of the operation*. The four operations we will consider are — forming the tooth by impression, by planing or shaping, by milling, and by grinding or abrasion.

The Formed Tool Principle. — This, the simplest and most obvious way of forming a gear tooth, is illustrated in Fig. 1. The gear to be cut is held firmly on a work arbor which, in turn, is firmly supported in the machine in such a way that it can be *indexed* (or rotated through an angular distance corresponding to one tooth) from time to time as occasion requires. In the upper part of the cut is shown a planer or shaper toolpost, carrying a formed tool having outlines accurately corresponding to the shape of a space between two of the teeth it is desired to form. It is evident that this formed tool, when mounted in the toolpost of the planer or shaper, may be fed down into the work to the proper depth, in which case, being set centrally, it will reproduce its outline in the work. The work may then be indexed, and the operation repeated to form another tooth space. With the work indexed in the direction shown in the cut, four tooth spaces, or three complete teeth have been formed. A formed milling cutter may be used instead of the planer or shaper tool. This is shown at work on the under side of the blank. It reproduces its outline in the work in the same way as does the planer tool, being rotated in the direction indicated, and fed through the work at the same time.

The Templet Principle. — This method of cutting gears is shown in Fig. 2. As in the previous case, the work is held on the table of the shaper. A templet holder is also mounted on the shaper table, carrying a templet, having a surface formed to the exact outline desired for the finished tooth. The tool-block is disconnected from the feed-screw and weighted so that it falls of its own accord. To its side is clamped the guide shown, which bears on the templet. As the table of the shaper is fed to the right, it will be seen that the curved surface of the templet will raise the guide, the tool-block and the tool, in such a fashion that the desired outline will be reproduced on the gear tooth. The upper surfaces of teeth *a*, *b*, *c* and *d* have been formed in turn in this way, the work being indexed for this purpose as in the previous case. With the primitive arrangement shown, it will be necessary to reverse the work on the arbor to form the other side of the teeth. Teeth *d* and *e* had their faces finished in

this way, tooth *d* being thus completely formed. It will be seen that obtaining accurate teeth by this method requires — first, an accurate templet; second, accurate setting of the templet and tool in proper relation to each other; and third, a bearing surface on the guide of exactly the same shape as the cutting edge of the tool. As shown, the gear to be cut has had the tooth spaces roughed out to shape, so that the finishing operation removes a comparatively small amount of metal.

The Odontographic Principle. — In shaping teeth by the odontographic principle, the tool is guided in some way by suit-

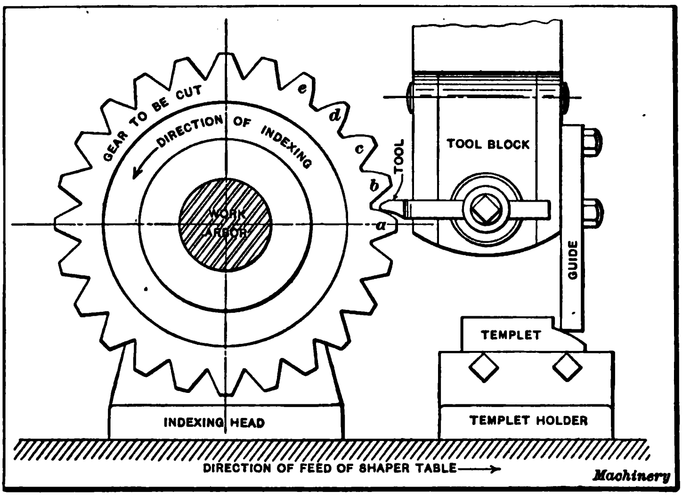


Fig. 2. The Templet Principle as Arranged to be Applied to a Shaper

able mechanism, to closely approximate the desired tooth outline by means of circular arcs, or other easily obtained curves. A simple example is shown in Fig. 3. The gear to be cut is held and indexed as in the two previous cases. The blank has had the teeth roughed out as in the previous case. The gear to be cut has involute teeth. With teeth of this form, in most cases a circular arc may be found which will more or less closely approximate the true outline. Such a circular arc is shown at *xy*, with its center at *o*. The radius tool-holder shown has its center at *o* to agree with that of arc *xy*. The cutting point of the tool used is located on arc *xy*. It will be seen from this, that when

the radius tool is fed from position T_1 to T_2 by the feed worm, its point will follow the desired arc and cut the desired outline for the tooth. By this means, the upper surface of tooth *a* is formed. The same surfaces of teeth *b*, *c* and *d* have previously been cut, as well as the opposite faces of *d* and *e*, tooth *d* being completed. To cut the opposite faces, the work may be reversed on the arbor.

The Describing-generating Principle. — This principle is shown in Fig. 4, applied to the shaping of involute teeth. The

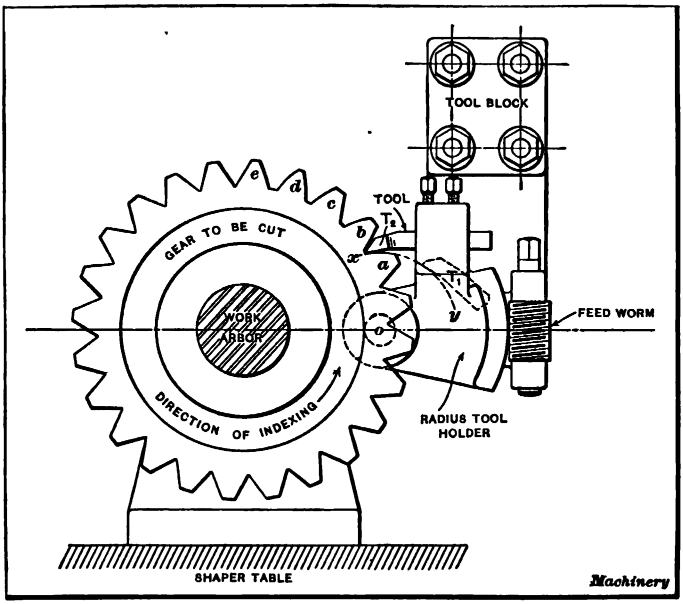


Fig. 3. The Odontographic Principle which Approximately Outlines the Tooth Form by Mechanical Means

cutting of involute teeth only has been hitherto shown in these examples, owing to the fact that in other cases, as in this, it lends itself most readily to the purposes of illustration. The involute, as is well known, is the curve formed by a point in a cord which is being unwrapped from the periphery of a circle. In the cut, the dotted line *xy* shows an involute generated in this fashion from the *base circle* shown. This base circle is formed by the periphery of the rolling disk, which is firmly connected with the gear to be cut through the work arbor on which both are mounted.

Unlike the previous cases considered, the work arbor in this case is free to revolve on centers without being restrained by an indexing mechanism; as in previous cases, the blank has had the teeth roughed out. The tool used is a shaper, as before. To some fixed part of the machine is clamped the tape holder shown. This has fastened to it two thin flexible metallic tapes, M_1 and M_2, the former stretched between screw S_1 on the tape holder

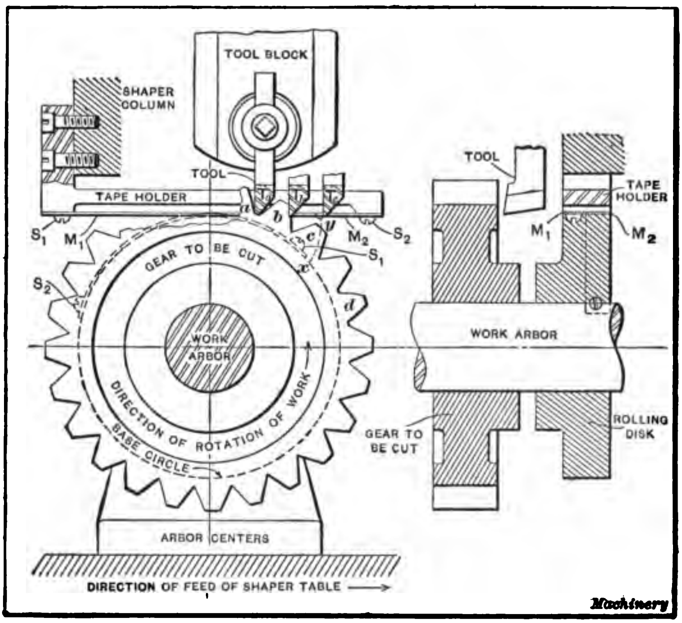


Fig. 4. The Describing-generating Process by which the Point of the Tool is Constrained to Follow the Desired Outline

and the corresponding screw on the rolling disk, while the latter is similarly stretched between screws S_2 and S_2. By this means, it will be seen that when the shaper table is fed in the direction indicated, the unwinding of M_1 and the winding of M_2 will positively roll the disk and the work with it. If, now a tool be placed in the tool-block of the shaper, having a cutting point set at the same height as the middle thickness of the steel tapes, and if the table be fed as shown, the mechanism will constrain the tool point to cut an involute on the side of the tooth of the gear blank.

When the tooth is at c, the tool will be at T_c; when the tooth is at b, the tool, at T_b, will have cut down about half the length of the face, as shown; when the tooth is at a, its outline will have been completed on that side by the tool, at T_a. The way in which the involute is generated will be easily understood, when it is seen that the cutting point of the tool always coincides with a given point y in tape M_2, so that the same involute as is generated by this point in the unwinding tape is reproduced by the tool point.

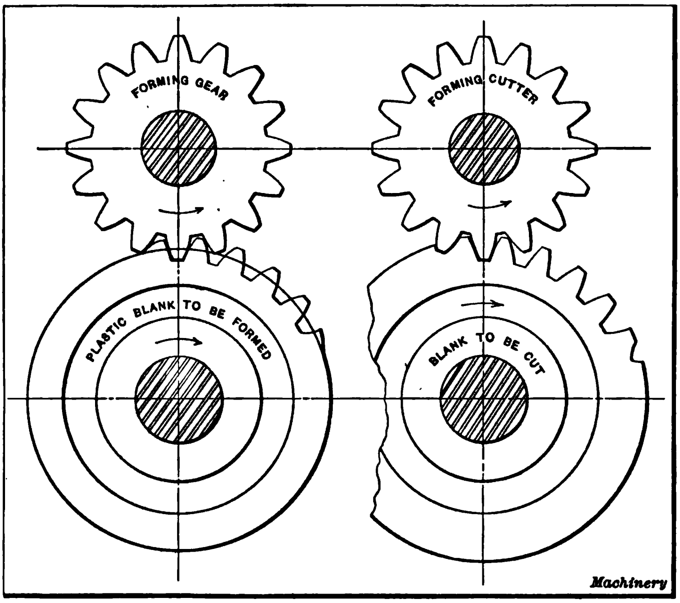


Fig. 5. The Molding-generating Principle Applied to rolling the Proper Tooth Form in a Plastic Gear Blank

Fig. 6. The Same Principle employing a Cutter having a Shaping Action for Cutting Teeth in a Solid Blank

The device is incomplete, as shown, in that no provision is made for indexing. In this case the gear to be cut and the rolling disk have to be indexed with relation to each other, so as to present the different teeth properly for the tool to act upon them. At d is shown a completed tooth.

The Molding-generating Principle. — This method of making gears depends on the fact that in a set of interchangeable gearing a gear formed correctly to run with one of the series will run with

all of the series. The molding process consists in using a completed gear tooth or gear, of proper shape, to form the others. Two examples of this are shown in Figs. 5 and 6. The first case supposes a forming gear, as shown, of correct shape. The blank to be formed is made of some plastic material like wax or clay. The blank and the forming gear are mounted on arbors at the proper distance apart, and rotated together at the proper speed ratio. The teeth of the forming gear, pressing into the plastic blank, will form spaces and press out teeth of the correct shape to mesh with itself, or with any other gear of the same interchangeable series.

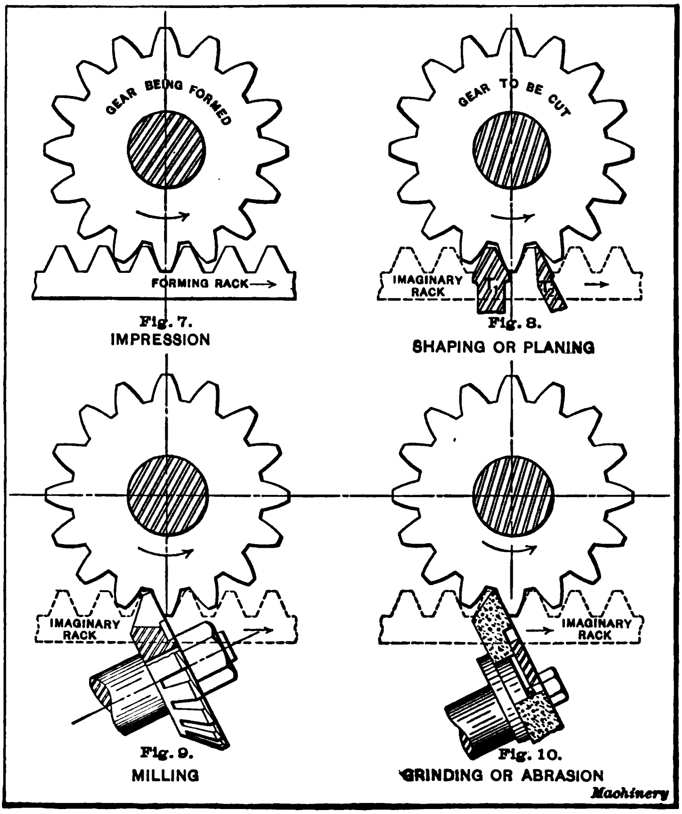
In Fig. 6 the blank is of metal or other non-plastic material, and the forming gear is replaced with a forming cutter, having sharp edges of exactly the same outline. The blank, which in this case is of the full outside diameter of the gear into which it is to be made, is rotated with the cutter, as in Fig. 5. The cutter is reciprocated in the direction of its axis so as to take a series of cuts, to form the tooth spaces as the rotation takes place. The principle is identical with that shown in Fig. 5. Of course, the cutter has to be fed directly in to the proper depth to start with, before the rotating commences.

The Four Methods of Operation. — In classifying gear-cutting methods by the operations involved, we will take for the purpose of illustration the molding-generating method as applied to the spur gear. Later on we will see how the same operations are applied to the cutting of other forms of gears, by other methods. In the four cases shown in Figs. 7 to 10, the molding-generating is done by a rack working in a gear, not by one gear working in another, as in Figs. 5 and 6.

By Impression. — Fig. 5 is an example of this kind, the teeth in the plastic blank being formed by the impression made in it by the forming gear. In Fig. 7 the same thing is shown, except that the forming member is a rack which has shaped the periphery of the gear with which it meshes into correct teeth, as shown.

By Shaping or Planing. — In Fig. 8 but one tooth space of the gear is formed at a time, and instead of using a rack to do

the forming, a tool T_1 may be used having an outline the shape of a rack tooth. This is fed along horizontally, and the gear to be cut is rotated in unison with it, the same way as in Fig. 7. If tool T_1 is given a cutting movement in a shaper, the spaces formed will be of exactly the right shape and identical with those



Figs. 7 to 10. Four Methods of Operation as Applied to the Molding-generating Principle of Action

formed in the previous case. Each of the spaces will have to be formed in the same way one after another, the work being indexed with reference to the imaginary rack, to bring the tool into the proper position for each of them. Instead of forming both sides of a space at one operation, as with tool T_1, a single side tool T_2 may be used, corresponding with one side only of the

rack. In this case only one side of each tooth is finished, so the tool or the work has to be reversed, after which the other sides are completed.

By Milling. — Instead of using a planer or shaper tool to match the side of the imaginary rack tooth, a milling cutter may be used, as shown in Fig. 9. In this case the gear is rotated, and the milling cutter advanced to agree with the advance of the imaginary rack. The cutting face of the cutter must of course be formed on a plane surface, as shown. This arrangement presents some difficulties when the gear to be cut has a wide face, since the circular cutter will cut deeper into the tooth space at the center than it will toward the edges. This deepening of the tooth space at the center does not affect the acting tooth surface, and so is harmless (except possibly in the case of the generation of pinions having a small number of teeth, and involute outlines of low pressure angle, in which case the trouble due to interference is aggravated). The larger the diameter of the cutter, as compared with the face of the gear, the less is the trouble on this score.

By Grinding or Abrasion. — In Fig. 10, the milling cutter of Fig. 9 has been replaced by an emery wheel of similar shape, having a plane face perpendicular to the axis of the wheel spindle. The action on the work is identical with that in the previous case, subject only to the limitations of the grinding process, such as the rapid wearing away of the material of the wheel, involving the necessity for constantly truing it up. Besides this, only a small amount of stock can be removed in a given time, as compared with the rapidity possible with a milling cutter. The process has the advantage that it can be used in hardened work.

As intimated, each of these various operations can be applied to different kinds of gears, acting according to different principles, though many of the possible combinations are impracticable.

Machines for Forming the Teeth of Spur Gears. — As described in the previous section, spur gear-teeth may be formed in any one of five ways — by the formed tool method, the templet method, the odontographic method, the describing-generating method, or the molding-generating method. The extent to which these various schemes have been applied in practical use

varies greatly. The formed tool method is at once the most obvious and the most used of them all. The templet principle has been applied to a limited extent, principally for gears of very large size. No practical application of the odontographic principle has been made in the cutting of spur gears. The only machine that has come to notice involving the describing-generating process was one invented by Mr. Ambrose Swasey, and in use a number of years ago in the shops of the Pratt & Whitney Co. This was not used, however, for making gear teeth, but for making gear-tooth cutters — before the days of the formed cutter, which it was not adapted to making. The molding-generating process in various forms has received a wide application, second only to the formed tool method.

The operations available for the formed tool method are — impression, shaping or planing, milling, and grinding or abrasion. Of these, the impression process is obviously unsuited for practical work. The shaping or planing, and the milling (particularly the latter) have a wide range of application; the machines used for milling, especially, are so well known as to need no further comment. In the case of the operation of grinding or abrasion, only a single machine has ever been built embodying the formed tool principle.

Machines using Formed Shaper or Planer Tools. — The primitive application of the formed tool method is that in which a gear blank is mounted on index centers on the planer or shaper table, and has its teeth cut by a tool having an outline corresponding to the desired tooth space. In this operation the tool is fed by hand to the proper depth and withdrawn. The work is then indexed for a second cut, the tool is fed down again, and the operation is repeated until the gear is finished. This is shown diagrammatically in the upper part of Fig. 1. It is the simplest method of cutting a gear which has to be made immediately, and for which formed milling cutters are not available. It also has its application in the case of gears of unusual size. Under these circumstances, however, the machine used is generally a slotter instead of a planer or shaper. A formed tool is fastened in the toolpost of the machine, while the work is clamped

to the revolving table. The indexing is done by such means as may be provided, usually a worm and worm gear or a master wheel. The Gleason and Newton templet machines also may be, and doubtless often are, used in the same way.

The Molding-generating Milling or Hobbing Machine. — The most widely used process involving the milling operation of

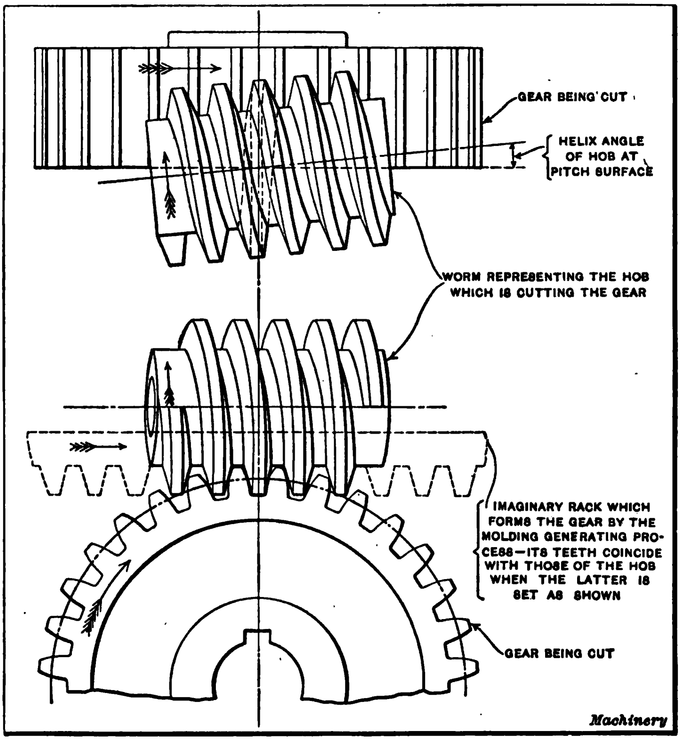


Fig. 11. Diagram illustrating the Principle of the Hobbing Process of Forming Spur Gears

molding-generating is the hobbing process. The principle of this method is shown diagrammatically in Fig. 11. Here we have an imaginary rack meshing with a gear, and molding its teeth in the same way as in Figs. 7 to 10. The teeth of this rack, shown in dotted outline, coincide with the outlines of a hob, shown in full lines, which has been set at such an angle as to make the teeth on its front side parallel with the axis of the gear.

In other words, it has been set at the angle of its helix, measured at the pitch line. It will be seen that the teeth of the hob, when set in this position, correspond with the teeth of the rack. If, now, the hob and blank be rotated at the ratio required by the number of threads in the hob and the number of teeth in the gear, this movement will cause the teeth of the hob to travel lengthwise in exactly the same way as the teeth of the imaginary rack would travel, if in mesh with the gear whose teeth are to be cut. It will thus be seen that the hob fulfills the requirements necessary for molding the teeth of the gear to the proper form. In practice the hob is rotated in the required ratio with the work, and fed gradually through it from one side of the face to the other. When it has passed through once, the work is completed.

Of the great number of machines built during the past few years involving this principle, many are arranged for cutting spiral gears as well as spur gears. Of course, all of the machines capable of cutting spiral gears are capable of cutting spur gears also. The spiral gear-hobbing machine bears about the same relation to the plain spur gear-hobbing machine that the universal does to the plain milling machine. The added adjustments and mechanism required in each case tend to somewhat limit the capacity of the machine in taking heavy cuts, though they add to its usefulness by extending the range of work it is capable of performing.

Requirements of Gear Hobbing Machines. — The requirements of the successful gear hobbing machine are:

First. A frame and mechanism of great rigidity.

Second. Durable and powerful driving mechanism.

Third. Accurate indexing mechanism.

The first requirement is one of great importance, not only in its influence on the heaviness of the cut to be taken and the consequent output of work, but on the matter of accuracy as well. The connection between the hob and the work, through the shafts and gearing, is liable to be so complicated that the irregular cutting action of the hob produces torsional deflections in the connecting parts, leading to serious displacement from the desired relation between the hob and the teeth being cut.

This displacement from the desired position results in teeth of inaccurate shape, weak and noisy at high speeds.

In its effect on the output, rigidity is even more important in the hobbing machine than in the orthodox automatic gear cutter. A heavier cut is taken, since a greater number of teeth are cutting on the work at once. The number of joints between the cutter and the work-supporting table and spindle must, therefore, be reduced to a minimum, and the matter of overhang both for the work and the cutter must be carefully looked out for. The reduction of overhang is hampered at the cutter head by the necessity for a strong drive and an angular adjustment. In the case of the work-supporting parts, it is difficult to bring the cutting point close to the bearing on account of the necessity for plenty of clearance below the work for the hob and its driving gear.

The matter of design of the driving mechanism for the hob and the work is a difficult one. Not only must it be rigid for the sake of accuracy, as previously explained, but careful attention must be given to durability as well. It requires great skill to design a durable mechanism for the purpose within the limitations imposed — in the cutter head by the necessity for reducing the overhang, and in the work table by the high speed required for cutting small gears.

Since the indexing wheel works constantly and under considerable load, both the wheel and worm must be built of such materials as will preserve their accuracy after long continued use. Particular attention should be given to the homogeneity of the material of the index worm-wheel, to make sure that it does not wear faster on one side than on the other.

The field of the hobbing process for cutting spur gears has not yet been definitely determined. In some work it appears to have certain advantages over the usual type of automatic gear-cutting machine, while in other cases it falls behind. It will doubtless require continued use, with a variety of work, and for a considerable length of time, to determine just what cases are best suited for the hobbing machine, and what for the machine with the rotating disk cutter. It is not probable that in the

future either of them will occupy the field to the exclusion of the other.

Designing a Hob for Hobbing Spur Gears. — In explaining the methods used in the design of hobs for spur gears, it is best to assume a practical example. Suppose that the gears are to be cast iron, with 120 teeth, 16 diametral pitch and $\frac{5}{8}$ inch width of face. The pitch diameter, hence, is $7\frac{1}{2}$ inches. The hole in the hob for the spindle is to be $1\frac{1}{4}$ inch in diameter with a $\frac{1}{4}$-square inch keyway, the hob to be run at high speed. Mr. John Edgar, in MACHINERY, November, 1912, gives the following solution of the problem.

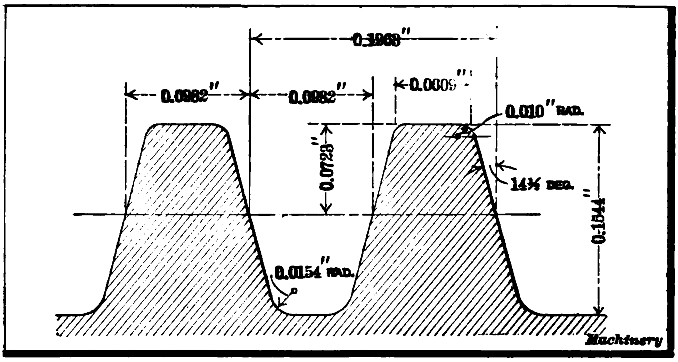


Fig. 12. Standard Hob Tooth Dimensions

Form and Dimensions of Tooth. — The first thing to be settled is the form and dimensions of the tooth or thread section of the hob. If the form is to be the standard shape for the involute system with a $14\frac{1}{2}$-degree pressure angle, the dimensions of the hob tooth would be as shown in Fig. 12. A modification of this shape may in some cases be advisable, and will be referred to later in this chapter. The standard rack-tooth shape with straight sides, as shown in Fig. 12, however, is the easiest to produce, and it is entirely satisfactory, unless gears with a very small number of teeth are to be cut.

The circular pitch corresponding to 16 diametral pitch is 0.1963 inch, which is obtained by dividing 3.1416 by 16. The thickness of the tooth on the pitch line is one-half of the circular pitch, or 0.0982. The height of the tooth above the

pitch line is equal to the reciprocal of the diametral pitch plus the clearance, which latter is equal to 0.1 of the thickness at the pitch line. Hence, the height of the tooth above the pitch line equals 0.0625 + 0.0098 = 0.0723 inch. This distance equals the space in the gear below the pitch line.

The depth of the tooth of the hob below the pitch line is usually made greater than the distance from the pitch line to the top of the tooth. The extra depth should be equal to from one-half to one times the clearance. On small pitches, one times the clearance is not too great an allowance, and, therefore, the depth below the pitch line is made equal to 0.0723 + 0.0098 = 0.0821, making the whole depth of tooth equal to 0.1544. The extra depth at the root of the thread is to allow for a larger radius at the root, so as to prevent cracking in hardening. The radius may then be made equal to two times the clearance, if desired. In the illustration, however, the radius is made equal to 0.1 of the whole depth of the tooth. The top corner of the tooth is rounded off with a corner tool to a radius about equal to the clearance, or say 0.010 inch. This corner is rounded to avoid unsightly steps in the gear tooth flank near the root. Having obtained the hob tooth dimensions, the principal dimensions of the hob may be worked out with relation to the relief, the diameter of the hole and the size of the keyway.

Relief of Hob Tooth. — The proper relief for the tooth is a matter generally decided by experience. We may say that, in general, it should be great enough to give plenty of clearance on the side of the tooth, and on hobs of $14\frac{1}{2}$-degree pressure angle the peripheral relief is, roughly speaking, about four times that on the side. For cutting cast iron with a hob of the pitch in question, a peripheral relief of 0.120 inch will give satisfactory results; for steel, this clearance should be somewhat increased. The amount of relief depends, necessarily, also upon the diameter of the hob.

With a peripheral relief of 0.120 inch, the greatest depth of the tooth space in the hob must be 0.1544 + 0.120 = 0.2744. The gash will be made with a cutter or tool with a 20-degree

included angle, $\frac{3}{32}$ inch thick at the point, and so formed as to produce a gash with a half-circular section at the bottom. The depth of the gash should be $\frac{1}{16}$ inch deeper than the greatest depth of the tooth space, or about $\frac{11}{32}$ inch.

Thickness of Metal at Keyway. — The radius of the hob blank should be equal to $\frac{5}{8} + \frac{1}{8} + \frac{11}{32} +$ the thickness of the stock between the keyway and the bottom of the flute. If we use a 3-inch bar we can turn a hob blank $2\frac{3}{4}$ inches in diameter from

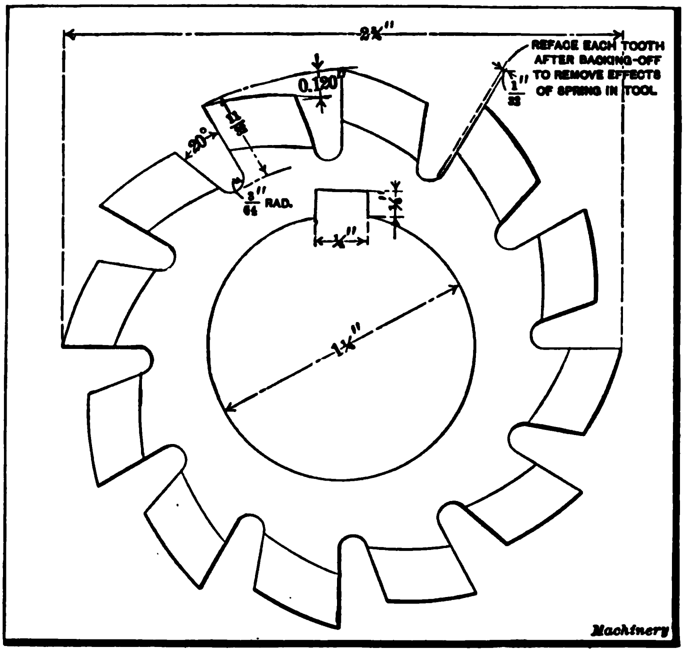


Fig. 13. Hob with Twelve Gashes or Flutes

this, which would allow sufficient stock to be turned from the outer portion of the bar to remove the decarbonized surface. If we make the blank $2\frac{3}{4}$ inches in diameter we have $\frac{9}{32}$ inch of stock over the keyway, which is sufficient.

Number of Flutes. — The number of gashes or flutes depends on many factors. In Fig. 13 is shown an end view of a hob with twelve gashes. This number gives plenty of cutting teeth to form a smooth tooth surface on the gear without showing prominent tooth marks. A larger number of gashes will not, in prac-

tice, give a better tooth form, but simply increases the liability to inaccuracies due to the forming process and to distortion in hardening. This number of gashes also leaves plenty of stock in the teeth, thus insuring a long life to the hob.

Straight or Spiral Flutes. — The question whether the gashes should be parallel with the axis or normal to the thread helix is one that is not easily answered. It must be admitted that when the angle of the thread is great, the cutting action at both sides of the tooth is not equal in a hob with a straight gash; but in cases of hobs for fine pitch gears, where the hobs are of comparatively large diameter, thus producing a small thread angle, the parallel gash is more practical because it is much easier to sharpen the hobs, and the long lead necessary for spiral gashes, in such cases, is not easily obtained with the regular milling machine equipment. However, when it is desired to obtain the very best results from hobbing, especially in cutting steel, the gash should be spiral in all cases when the thread angle is over $2\frac{1}{2}$ or 3 degrees. In our case the thread angle figured at the pitch diameter of the blank is equal to 1 degree 22 minutes; hence, straight flutes are not objectionable.

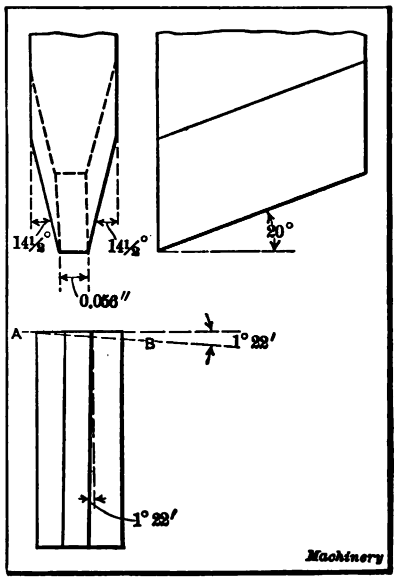


Fig. 14. Threading Tool for Hob

Threading the Hob. — The linear pitch of the hob and the circular pitch of the gear, when considered in action, are to each other as 1 is to the cosine of the thread angle. In the present case they do not differ appreciably and may be considered as equal. In cases where the difference is over 0.0005, the true linear pitch should be used.

The change-gears for the lathe may be figured by the formula:

$$\frac{\text{Gear on lead-screw}}{\text{Gear on stud}} = \frac{\text{lead of lead-screw}}{\text{linear pitch of hob}}$$

On a lathe with a lead-screw of six threads per inch, or a lead of $\frac{1}{6}$ or 0.1667 inch, the gears that would give accurate enough results for the present hob would be 28 teeth on the lead-screw, and 33 teeth on the stud.

Thread Relieving Tool. — In Fig. 14 is shown the hob thread relieving tool. The front of the tool is relieved with a 20-degree rake for clearance. The sides are ground straight at a $14\frac{1}{2}$-degree angle to form the sides of the thread, and are at an angle

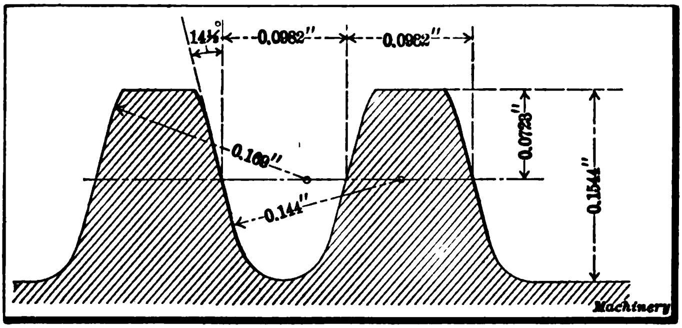


Fig. 15. Special Hob Tooth Dimensions

of 1 degree 22 minutes with the vertical to clear the sides of the thread. A tool made like this can be sharpened by grinding across the top without losing its size or form. If the gashes were made on the spiral, the top of the tool should be ground to the angle of the thread, as shown by the dotted line *AB*. In cases where the angle of the thread is considerable, the angle of the sides of the tool must be corrected to give the proper shape to the hob tooth. (See MACHINERY, May, 1905, or MACHINERY'S Reference Book No. 32, "Formula for Planing Thread Tools.") The point of the tool should be stoned to give the proper radius to the fillet in the bottom of the hob tooth space.

Heat-treatment of Hob. — The best practice in making the hob is to anneal it after it has been bored, turned, gashed and threaded, the annealing taking place before relieving the teeth.

Before hardening, the hob ought to be re-gashed or milled in the groove, removing about $\frac{1}{32}$ inch of stock from the front side of the tooth to eliminate chatter marks and the effect due to the spring in the tool, which always leaves the front edge of the teeth without relief. In hardening, do not attempt to get the hob too hard, as the required high heat and quick cooling would distort the teeth badly.

Modified Tooth Shape in Hob. — In case the 120-tooth gear is to run with a pinion of a small number of teeth and is the driver, as in small hand grinders where gears of this size are often used, it would be advisable to make the tooth shape as shown in Fig. 15. This shape will obviate undercutting in the pinion and relieve the points of the teeth in the gear so as to obtain a free-running combination. This shape is more difficult to produce and requires more care in forming. If the hob is made of high-speed steel, it should run at about 115 revolutions per minute for cutting an ordinary grade of cast iron with a feed of $\frac{1}{16}$ inch per revolution of the blank. The feed may be increased considerably if the gear blank is well supported at the rim. The best combination of speeds and feeds in each case can be found only after considerable experimenting.

Interchangeability of Hobbed and Milled Gears. — There is always an objection to changing existing methods in shop practice when the change necessitates discarding established standards and valuable tools and fixtures. Whether such a change will be profitable or not is a question that requires a close study of the conditions in each case. When the change means an improvement in the quality of the product, the cost of the tools and fixtures should, of course, be a secondary consideration. When the question is mainly one of quantity, the problem must be solved on a cost basis only.

Another factor to be considered, however, is that of interchangeability. This is a most important item in the case of a product in connection with which renewals are constantly being made. Many improvements in design and in methods of manufacture are sacrificed in deference to the demands for interchangeability. In the case of gears, interchangeability is supposed to

be rigidly adhered to, but while we have a standard which is supposed to produce interchangeable gears, there are so many variations of the standard, due to the secret forms established by different manufacturers of cutters, that it is necessary in many cases to adhere to one make of tools if interchangeability is to be maintained in any degree. Many manufacturers have installed the hobbing machine in the desire to reduce the cost of gearing, only to encounter the non-interchangeability of the product of the hobbing machine with the milled tooth gear; this has been the cause of turning many against the hobbing machine, through no fault of the process itself.

Variations from the True Involute Tooth Shape. — The form of the standard tooth, as adopted by the cutter manufacturers, is not the true involute, but an improvised form built around the involute as a basis. The deviation from the involute is necessary for several reasons:

1. The inability of the formed milling cutter to mill an undercut tooth.

2. The necessary alteration in the form of the point of the mating tooth caused by the fullness of the milled tooth below the pitch line.

3. The desire to make the contact of the approach as gradual as possible by a slight easing off of the form at the point of the tooth; this provides against the slight variation in the form of the tooth due to irregularities in the division of the space and to the elasticity of the material.

4. The interference in gears with thirty-two teeth or less when in mesh with those of a greater number of teeth. As the $14\frac{1}{2}$-degree formed gear-cutters are based on the twelve-tooth pinion with radial flanks, a rack tooth to mesh with this radial flank tooth can be made with the straight sides extending only to a point 0.376 inch outward from the pitch line in a rack of one diametral pitch. The remainder of the tooth must be eased off from this point outward, sufficiently to clear the radial flank of the pinion tooth. This rounding off of the rack tooth may be made by using the cycloidal curve from the interference point, with a rolling circle of a diameter equal to that of the twelve-

tooth pinion. A circular arc tangent to the tooth side, drawn from a center on the pitch line at the point of intersection of the normal to the tooth side at the point of interference, will be a near approximation to the cycloidal curve.

The hobs used extensively today are not made to produce teeth in any near approximation to the shape produced by the milling cutter. The only correction that is made, in many cases, is to make the teeth of the hob a trifle fuller at the base or root to ease the approach; even this is done only in a few instances. The difference between the hobbed tooth and that produced by milling is seen in Fig. 16. The hobbed tooth is shown in full; this shape was traced from an actual hobbed tooth, photographed and enlarged. The gear had twenty-one teeth. The hob used was corrected for the "thinning" of the tooth at the point, but in a gear of this diameter the effect would not show to any great extent. The dotted lines are drawn from actual milled tooth curves and show the difference between the two forms of teeth. Attention is called to the fullness of the milled tooth at the root, and the thinning of the tooth at the point. The difference would be greater in the case of a twelve-tooth pinion.

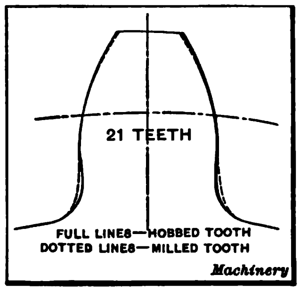


Fig. 16. Comparison between Hobbed and Milled Gear Teeth

The filling-in of the flank of the tooth is not done to any rule based on a proportion to the number of teeth in the gear. The curve selected is made to fill in the space at the root to just clear the corrected rack tooth. Neither is the thinning of the tooth at the point proportional to the diameter in the sense that the curve of the hobbed tooth is. Each form of the cutter system is made and varied to the extent necessary for smooth action, and the curves of the entire system cannot be produced by the hobbing process with a single hob. To accurately reproduce the form of the milled tooth, a special hob would be necessary for each number of teeth. However, a close approximation may be obtained, within a narrow range of teeth, with a hob generated from a

milled tooth. This is being done in the automobile industry with good results. The necessity for interchangeability makes the duplication of the milled tooth imperative when the originals were made with the formed cutter, and the introduction of the hobbing machine, in such cases, depends on the successful duplication of these forms. It is no exceptional thing to see the hobbing process used in conjunction with the automatic gear-cutter in the production of interchangeable transmission and timing gears. The shapes produced by the standard sets of cutters, from a rack to a twelve-tooth pinion, cannot, however, be generated by a single hob, because the shapes are only an approximation of the correct curve. The gears mentioned above as being successfully hobbed are, therefore, when milled, cut with special cutters for each number of teeth, as in this way only can a curve of correct shape be obtained.

As stated, most hobs are of the straight-sided shape, and the tooth hobbed is of pure involute form. In gears of less than thirty-two teeth, the flank is undercut to a considerable extent. This undercutting does not involve any incorrect action in the rolling of the gears, but in the case of the twelve-tooth gear, for example, the involute is cut away at the base line close to the pitch line, giving but a line contact at a point which is subjected to heavy wear. This eventually develops backlash. The teeth of the gears also come into action with a degree of pressure that is continuous throughout the time of contact; this results in a hammering which in time develops into a humming noise.

Special Hobs for Gear Teeth. — To overcome these objections a hob tooth may be developed to generate a curve which will closely resemble that of the formed tooth. Such a hob tooth is shown in Fig. 17. Theoretically, the correction for interference or undercutting should begin at a point located above the pitch line a distance as determined for a twelve-tooth pinion by the expression:

$$0.03133 \times \frac{N}{P}$$

in which N = number of teeth in the smallest gear to be hobbed;
P = diametral pitch of gear.

However, to begin the correction for interference at this point would reduce the length of the true involute and result in too full a tooth, causing noisy gears. Therefore, a compromise is made and the correction is obtained for a minimum of twenty-one teeth. To compensate for the extra fullness of the tooth at the root, the point of the tooth is thinned down in proportion, and this is done by leaving the tooth of the hob full below the pitch line by striking an arc from a center on the pitch line, and also employing a large fillet having a radius equal to $0.45 \div P$ (see Fig. 17). It will be noticed that the radius of the arc at the top of the hob tooth is smaller than the radius at the bottom of the hob tooth. This will thin the tooth of the gear in excess of the amount necessary to clear the flank, easing the action and elimi-

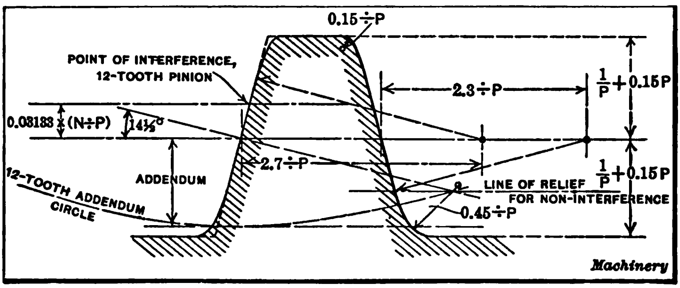


Fig. 17. Hob Tooth designed to generate the Approximate Shape of a 14½-degree Involute Milled Tooth

nating the hammering effect due to the theoretical contact. It will be seen from the illustration that the thinning of the teeth does not affect the twelve-tooth gear to any appreciable extent, but is gradually increased with the number of teeth. The fact that a twelve-tooth gear will mesh without interference at the point of the teeth makes the thinning unnecessary; besides, the small pinions are usually the drivers.

Fig. 18 shows a twenty-degree pressure angle hob tooth with standard addendum and corrections for non-interference. The curve of the tooth begins at a point 0.702 inch from the pitch line, in the case of a one diametral pitch tooth, and is based on non-interference with all teeth from twelve teeth up.

Fig. 19 shows the shape of the hob tooth to reproduce the stub teeth of the gears generated on the Fellows gear shaper. The

particular tooth in the figure is a $\frac{6}{8}$-pitch tooth, and the proportions are given in terms of the pitch numbers so as to be easily applied to the other pitches; thus the height of the tooth above the pitch line is stated as: $\frac{1}{8} + \frac{0.25}{8}$ where 8 is the addendum number of the pitch designation.

The shape of the rack or hob tooth to roll with the gears produced by the gear shaper should be generated from the cutter used. The Fellows cutters have perfect involutes above the base line, with radial flanks, so that the hob tooth would be straight only a distance from the pitch line equal to $0.0585 \times N \div P$, where N is the number of teeth in the cutter; in most cases the cutter would have more than seventeen teeth and the

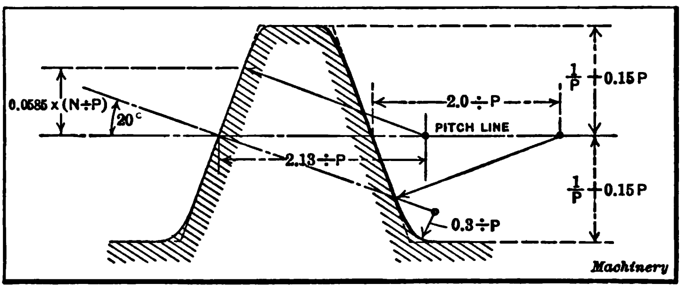


Fig. 18. Hob Tooth for generating a 20-degree Involute Milled Tooth

hob tooth would be straight-sided to the point. In this system the radial flank of cutters with more than seventeen teeth does not affect the shape of the face of the tooth, as the involute portion of the cutter tooth generates a pure involute. The straight side of the hob tooth should extend to the root in such cases.

To reproduce gears of some standard the exact shape of which is not known, the hob-tooth shape can be easily generated from the gear tooth on the milling machine, as will be explained later.

Generating Hob-tooth Shapes. — This can be done on the universal milling machine, or on the plain milling machine if the screw can be connected up with the worm of the dividing head, as in milling spiral work. The spindle of the dividing head is set vertical, and the master gear or templet of the shape it is desired to produce by hobbing is mounted on an arbor in the spindle.

In making the master templets, care should be taken to produce the correct shape; if the templet is not true, the shape generated will not be accurate, of course.

The gearing connecting the feed-screw and the dividing-head must be for a lead equal to the circumference of the pitch circle of the gear from which the hob templet is generated.

To provide a rest on which the tool templet to be laid out may be clamped, a parallel is bolted to the outer arbor support so as to be horizontal and parallel with the milling machine table and at right angles to the machine spindle. The rest may also be in the form of an angle plate clamped to the face of the column, but the former type is the most desirable, as it brings the work in a more accessible position.

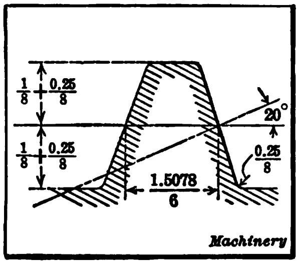


Fig. 19. Hob Tooth for a 5/6-Pitch Fellows System Stub Gear-Tooth

The blank templet should be a piece of sheet steel about one-sixteenth inch thick, one edge of which should be straight and true and the surfaces smooth and bright. The surface to be laid out can be given a coat of copper solution, or, still better, varnished so that the lines may be etched deeper, as the handling in working out the shape tends to obliterate the shallow lines in the thin copper coat. This blank templet can then be clamped to the rest in a convenient position.

There must be plenty of room for the travel of the gear, so as to obtain the proper amount of "roll" to generate the shape desired. The true edge of the plate should be parallel with the rest and the direction of the movement of the milling machine table. Adjust the knee vertically so that the plate will come up under the gear on the dividing head so as to just clear it; the saddle can then be adjusted across to bring the edge of the plate in line with the end of a tooth in the gear when the center line of the tooth is about at right angles to the axis of the feed-screw, as shown in Fig. 20. In this way the templet is set to the proper position for depth. The back-lash should be taken up by turning the screw in the direction in which it is to be used.

Now select a tooth space A, Fig. 20, as the one to be used in the scribing operation, and run the point of a slim, sharp scriber along the outline of the tooth space, scratching the line on the plate; then move the table about one-half turn of the lead-screw and scribe another line, and repeat the operation until the table has been moved through a length equal to three times the circular pitch. When this has been done the lines on the templet will resemble that in Fig. 21. The lines should now be etched in and the plate polished.

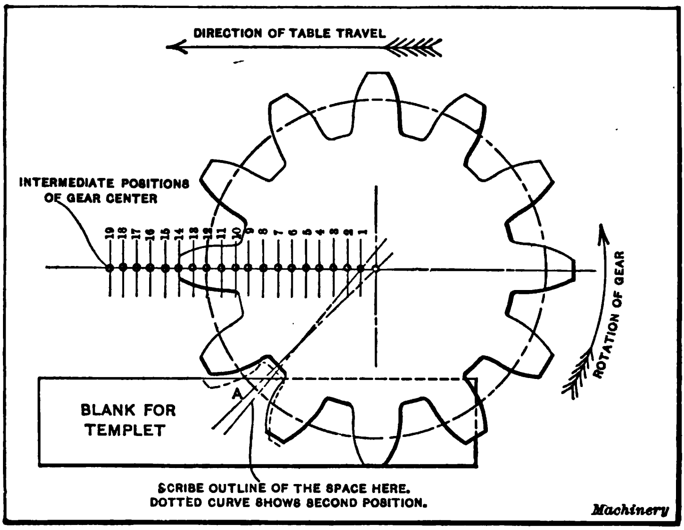


Fig. 20. View showing the Relative Position of Gear and Templet

The combined lines on the plate will be seen to describe the rack tooth shape of the hob teeth in a clear-cut manner, if the operation has been carefully carried out. If the gear tooth from which the lines were taken is theoretically correct, the sides of the outline on the plate will be straight a greater portion of the way from the point of the tooth to the edge of the plate; the lines diverge from the straight line at a point near the edge of the plate, as shown by the dotted lines in Fig. 21. This point will be found to be, in the case of the $14\frac{1}{2}$-degree tooth, at a distance from the pitch line of $0.03133 \times N \div P$, where N is the number of teeth in the gear and P the diametral pitch. If the hob to be

made from this form is to be used for N teeth or less, the shape of the templet will be correct, but if the hob is to cut gears of a larger number of teeth, the straight portion of the tooth must be carried down to the edge of the plate in order that the teeth of the larger gears will not be cut away too much at the points.

In making a templet in this way for any other shape than for gears, it should be cut to the lines on the plate, as no correction can be intelligently made in those cases. Some success has been made in the layout of templets for a hob tooth for gears of a limited range of teeth, by using the space below the pitch line

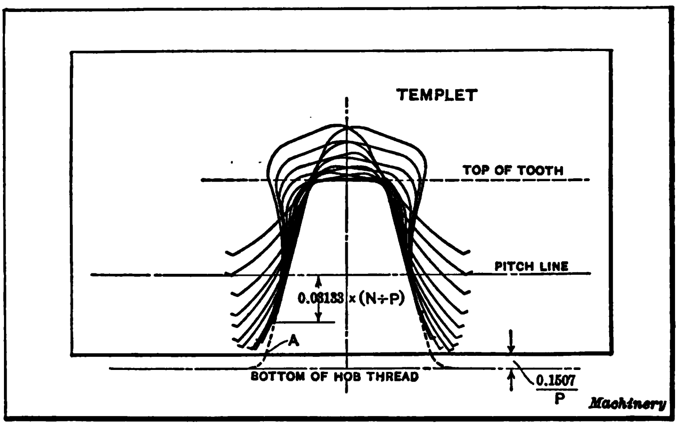


Fig. 21. Lines scribed on the Hob Tooth Templet from a Hobbed Gear

of the smallest gear in the set and the space above the pitch line of the largest gear in the set as the shape in generating the hob tooth templet. This is of value in generating a hob-tooth shape to reproduce a set of gears milled with formed cutters. However, the best and easiest method is to take the smallest gear in the set as the one from which to generate, and prolong the straight portion of the hob tooth to the edge of the plate, easing the side at A, to point the teeth slightly. The teeth of the hob are generally made with an extra clearance at the bottom, as shown. This is a matter on which authorities differ, some preferring to have the hob cut the top of the teeth, to make the teeth of standard length if the blanks should be over size; however,

it is also practiced to make the tooth the same length both above and below the pitch line as in Figs. 17 and 18.

If the form is for a generated gear and results in the straight-sided tool in Fig. 21, all that is necessary is to measure the angle and make a thread tool that will cut a thread of this section. Should the shape turn out to be a compound of curves, as will be the case in reproducing the milled tooth, the templet should be filed out to the lines, making a female gage to which a planing tool is made, the planing tool being a duplicate of the hob-tooth shape. The threading tool is planed up with this tool. In making the thread tool, it is not usual to make it of female shape, that is, like the templet, but pointed as usual, planing the sides with the opposite sides of the planing tool. The proper corrections should be made in the thread tool to correspond to the angle of the thread, and the setting of the tool and the fluting of the hob, whether it is gashed parallel to the axis or normal to the thread helix.

Making a Master Planing Tool for a Hob. — A master planing tool can be made in the following manner, without the use of the scribed line templet. It is necessary to have a universal milling attachment for the milling machine. The spindle of the attachment is set in the horizontal position with the axis parallel with the direction of the table movement. A fly-tool holder is then placed in the spindle, in which the blank planing tool is to be held. This tool should be roughly formed to the shape to which it is to be finished. The top of the tool should be radial, that is, it should be in the plane of the center of the spindle. The gear or other master templet that it is desired to duplicate by hobbing must be hardened and ground to a cutting edge on one face, preferably the top face when mounted in the spindle of the dividing head, so that the pressure of the cut will be downward. The knee should be adjusted to bring the ground face of the gear to the level of the center of the spindle of the attachment. The dividing head and the table screw are connected in the same way as previously described, but in this case the power feed can be used and the saddle can be fed in to depth as needed, care being taken to use the power feed, in generating

the tool, only in one direction, as the backlash in the gears and screw will throw the tool and dividing head out of relative position if used in the opposite direction. As many cuts can be taken as required to obtain a tool of the correct shape.

If the tool is to be used in making more than one threading tool, as might be the case in many instances, the planing tool can be made in the shape of a circular tool which can be ground indefinitely without losing its shape. In this case the fly-tool holder would give place to the standard milling machine arbor. This method is the most accurate way of making the master planing tool, and where the universal milling attachment is available, it should be used when accurate results are desired. It eliminates the human element and the amount of skill required in making the master templet. The inaccuracy of the machine is the only element that is likely to cause error.

One point that is likely to cause difficulty is the relation of the generated tool shape to the thread shape, as it appears in the normal section of the hob tooth. The simple fact is that the master tool shape, as generated by the direct method of making the master planing tool, or the shape as outlined on the hob tooth templet in the first method, is the shape of the cross-section of the hob thread on a plane normal to the hob thread helix. This relation should be kept in mind throughout the process of making the tools and hob. This statement also clears any haziness regarding the question of the lead, as in single-threaded hobs this must be such that the normal pitch of the thread is equal to the circular pitch of the teeth hobbed. In the case of hobs of small thread angles, the normal and axial leads are practically the same, and may be treated as such in cases where the angle is less than 2 degrees and the pitch less than $\frac{1}{2}$ inch; an error of more than 0.00025 inch should not be exceeded in any case. The effect of the error is apparent in the case of a 6 diametral pitch hob 3 inches in diameter, when the axial lead is taken as the circular pitch of the teeth, as it results in an error of more than one-half degree in the pressure angle of the hobbed tooth.

Only in extreme cases should the angle of the hob thread be more than ten degrees. Hobs with greater angles than this are

difficult to make and use. In hobs of long lead the diameter should be proportioned so as to obtain a reasonable angle of thread. However, the extreme in diameters is as bad as the steep angles, and in cases where the two extremes are met a compromise is the only solution.

The method used in laying out the tooth shapes on the drawing-board is an interesting study, but the method of generating the tool as described is the most useful, and can be relied on for accurate results; this is not the case with the drawing-board method which is of value only as a means of getting an approximate shape.

CHAPTER IX

METHODS OF PRODUCTION AND HEAT-TREATMENT OF GEARS

Processes in Production of Automobile Transmission Gears. — One of the most important problems in modern automobile construction, and one which has received a great deal of attention from mechanical engineers during the past few years, is that of the quiet working of the running parts. Next to the engine itself, the gears have proved the greatest offenders in making noise. Therefore, the demand for gears which are accurate, interchangeable and silent, together with the necessity for producing them both rapidly and at a low cost, has caused a great deal of attention to be devoted to the various processes, tools and appliances whereby that demand may be satisfied.

We thus find that there are being placed on the market an increasing number of machine tools, steels and carbonizing materials, each of which claims some advantage over its predecessors, such as increased output, greater simplicity, superior generating features, and better hardening results. It is the intention, however, to deal here chiefly with the processes of manufacturing gear-box gears by means of a complete equipment of gages, tools, jigs, etc., with the object of insuring interchangeability. To a very large extent fitting is thus dispensed with. After the final machining operation has been performed, the parts should be ready to be assembled. When a duplicate part is wanted it can be supplied from stock, as the methods here to be described insure that it will fit into its correct position without trouble. These methods were outlined in a paper read by Mr. W. Betterton before a branch of the Institute of Automobile Engineers.

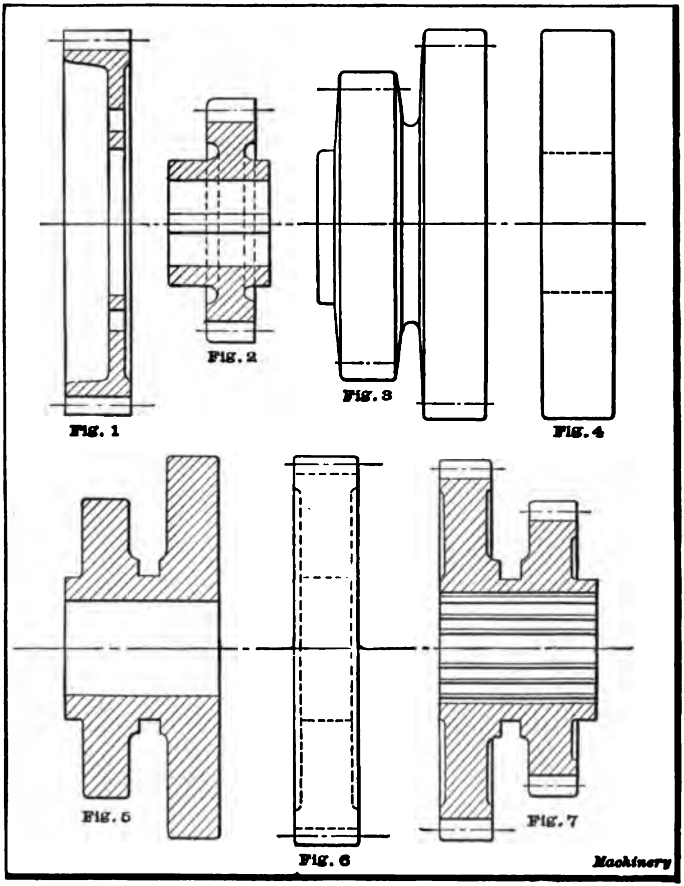
There is probably no part of an automobile that is subjected

to greater use — and abuse — than the gears, especially the gear-box gears. Carrying, as they do, practically all the power developed by the engine, and receiving at the hands of a careless driver the strains imparted by suddenly applied load or by rapid changes, it is absolutely necessary that the gears be made of the highest grade materials, and that the very greatest care and the best workmanship should be bestowed upon them. As the saving in weight is an important factor in the design of the transmission, the gears must be made as small and as light as possible, and yet be sufficiently strong to carry suddenly applied loads with no danger of breaking. Owing to the methods by which the speeds are changed, and the clashing and bruising to which the gears are thus subjected, the transmission mechanism must be made of material which is both hard and tough. Different kinds of steel have been used, and each has been treated by various methods in the attempt to discover the perfect gear material. Although this has not yet been found, so much progress has already been made that the transmission gear of a modern well-made automobile, when carefully handled, will last nearly as long as the car itself.

Steels used for Automobile Gears. — Of the various kinds of steel which have hitherto been employed, nickel, nickel-chromium and chrome-vanadium steels seem to have more advocates than any others. In most factories the gears are casehardened, and it is this class of gear which will be dealt with in the following. Gears treated in this way have been taken out of cars which have been run many thousands of miles, and in some instances the original tool-marks on the face of the teeth were still visible.

The processes in the manufacture of low-carbon nickel-steel casehardened gears, such as the finished gears shown by Figs. 1, 2 and 8, will be described in the following. The various processes will be explained in the order in which they are performed. The composition of nickel steel, suitable for high-speed gears, is as follows: Carbon, 0.20 per cent; manganese, 0.65 per cent; silicon, not exceeding 0.20 per cent; phosphorus, not exceeding 0.04 per cent; sulphur, not exceeding 0.04 per cent; and nickel, 3.50 per cent. Steel with 3.50 per cent nickel

rolls and forges well, and, when hardened, the ratio of the elastic limit to the ultimate strength is very great. The influence of nickel on steel is that it increases the tensile strength and the elastic limit.



Figs. 1 to 7. Automobile Transmission Gears at Various Stages of Completion

Nickel steel of the composition just mentioned should have an elastic limit, after treatment, of about 30 tons per square inch. The influence of silicon on the results of quenching is similar in many ways to that of carbon. It is dependent on the co-existing amount of carbon and manganese, and it is difficult to obtain

silicon in steel without the presence of manganese. Silicon appears to increase the tensile strength and diminish the ductility; but for various reasons it is generally considered objectionable.

Phosphorus is the least desirable element in steel, but up to one per cent it appears to increase the tensile strength. Sulphur tends to produce hot-shortness and difficulty in working, but in the presence of manganese the effect is diminished.

The Gear Blanks. — Blanks for gears of the type shown in Fig. 1 should be cut from the bar, since it has been proved that steel is not improved by drop forging, although some steels are less sensitive to injury than others. An investigation of drop-forged and bar-cut nickel-steel gears, details of which were given in a paper read by Mr. John A. Mathews before the Franklin Institute, showed that under static tests the bar-cut gears were fully 25 per cent stronger, and also that the resistance to shock was greater. The gears shown in Fig. 8 should be made from a drop forging, as shown by Fig. 3, although when only small quantities are required, it would not pay to make dies. In this case ordinary forgings should be considered. Gear blanks should be annealed previous to machining.

The reasons for leaving so much extra metal will be explained in the order in which they concern the various operations necessary in the attempt to get a perfect gear — an end which, it is needless to say, is seldom, if ever, attained. In the case of a gear as shown in Fig. 2, made from a bar, it is not necessary, for reasons which will be seen later, to leave any extra metal. Much of the trouble due to distortion in the heat-treatment is caused by the forging operations being done at too low a temperature, in which case the metal does not have a chance to flow properly, but is merely forced into shape by the die. This sets up internal strains that will be released when the part is annealed.

Rough-turning. — The first operation is to rough-turn the part all over for the purpose of removing the outer skin, previous to the second annealing, leaving a one-sixteenth inch case on the parts required to be hardened, such as the top diameter of the gear, and the sides of the teeth. For the bore a one-eighth inch allowance is required, in gears where the hole is to be a running

fit, or castellated, and has to be hard. A one-quarter-inch allowance, however, is necessary when the gear has to be bolted to a center, or to another gear, in which case the bore need not be hard, as it is only used for locating the gear centrally. In rough-turning, allowance must be made for the extra metal, the gears, after machining, appearing as shown in Figs. 4 and 5.

Annealing. — When making gears it is, of course, necessary to have the steel carefully and uniformly annealed. The process of annealing is one of great importance, and is better performed in a specially designed sealed furnace, constructed as a muffle, so that the required heat is obtained uniformly by radiation, without any flame to impinge on the steel. In addition to softening the steel, and making it easy to machine, annealing has the effect of bringing it to a more homogeneous condition by eliminating the molecular strains which are set up by rolling, hammering and stamping. Hence, when the steel is heated preparatory to hardening, equal expansion should follow, and also equal contraction when cooled.

It will thus be seen that should the steel not be annealed uniformly throughout, the risks of warping when hardening are considerably increased. The object of rough-machining is to break down the scale preparatory to the second annealing, and as the strains set up by rough-machining are released by the second annealing, the metal is then in as normal a condition as possible. At the present time there are many compounds used for annealing. A few years ago, the ashes from the forge were considered sufficient for properly annealing steel, but to-day many special preparations are manufactured and sold for the purpose.

The more common materials used are powdered charcoal, charred leather and hydro-carbonated bone-black. These same materials are used for carbonizing, but after having been used once they are of very little use for that purpose. However, their use for annealing has the additional merit of economy, because they can be used repeatedly, adding each time a little that has only been used for the carbonizing process. Air-slaked lime may also be used for this process. The piece to be annealed is usually packed in a wrought-iron box, using one

of the previously mentioned materials, or combinations of them, for the packing. The whole is then heated to the proper temperature, which is about 1760 degrees F. In the case of the gears in question, this temperature should be maintained for one hour. The box may then be set aside with the cover on in order to cool down to atmospheric temperature, or it may cool off with the furnace. It should be noted that the annealing temperature ought always to be higher than that for carbonizing. For all kinds of steel and for all grades of annealing, the slow-cooling furnace gives the best results, because the temperature can easily be raised to the right point, kept there as long as necessary, and then regulated to cool down as slowly as desired. Gas, oil, or electric furnaces are, of course, the easiest to regulate.

Finish-boring and Broaching. — Gears to be broached, as shown in Fig. 8, should be finish-bored with an allowance of 0.015 inch for grinding after hardening, and faced on one end true with the bore to take the thrust of the broaching on a Lapointe or similar broaching machine. The broaches should be made of carbon steel, oil hardened, tempered and ground, and should be specially treated. It is necessary to treat the steel with some carbonaceous material until it will harden in oil, as it is well known that steel hardened in oil is less likely to spring than if hardened in water. The tendency for steel to crack is almost eliminated and it has a maximum of toughness, unless, of course, the steel has been improperly treated in the fire. The special treatment consists essentially in supplying the surface of the steel with an additional amount of carbon by some material that will not injure the steel. No form of bone should be used on tool steel for this process, as bone contains a high percentage of phosphorus, and the effect of this is to make the steel weak and brittle. Charred leather gives the best results.

The over-all length of the roughing broaches, suitable for the gears shown in Fig. 8, would be about 48 inches, with the last four teeth parallel on each broach. The pitch depends upon the length of the work being broached, but may be about $\frac{5}{8}$ inch,

as an average. The teeth should never be cut spiral, as this tends to twist the broach while in operation and, consequently, the castellation would not be true. To produce an accurate castellated hole, as shown in Fig. 8, in nickel steel, it is necessary to use two roughing broaches and one finishing broach. The latter will be about 36 inches long, and will remove only from 0.001 to 0.002 inch of material. There should be a parallel portion of about 10 inches at the end of the third or finishing broach. On the first broach, the pilot should be round and of the same diameter as the hole in the gear to be broached. On

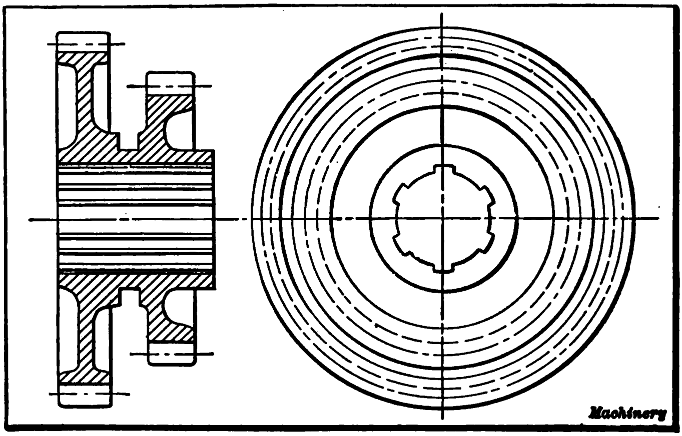


Fig. 8. Completed Gear, shown in Progress of Evolution in Figs. 3, 5 and 7

the following broaches, the pilot should be a sliding fit in the hole produced by the previous broach, and should, at the same time, be located from the castellations.

The finishing broach should have a piece at the rear end, about 3 inches long, of the exact size of the castellated shaft on which the gear is to be fitted when finished. This will act as a burnisher. The broaches are pulled through the work, which is quite a good feature, as it tends to keep them straight while in operation. The cutting portion being so long enables the operation to be performed without previous rough-slotting, which is required when the gears are drifted on the power press. After broaching, the gears should be turned.

Finish-turning. — Castellated gears should be turned on a mandrel, locating from the castellations. The parts required to be hard, such as the top diameter of the gears, which should have an allowance of 0.005 inch for gear-cutting purposes, and the sides of the teeth and fork groove are then finished, leaving the gear as shown in Fig. 5. In the case of the gear in Fig. 1, the top diameter should be finish-turned with an allowance of 0.005 inch, and the sides of the teeth and the bore with an allowance of 0.125 inch, so that the latter can be bored out again after carbonizing, which would leave the hole soft, as it is not required to be hard. The finish-turning leaves the gear as shown in Fig. 4. Gears such as shown in Fig. 2 can be finish-turned and bored complete at this operation.

Cutting the Teeth — Roughing Operation. — One of the most important operations is the cutting of the teeth after annealing. The method to be described is at present in practice and gives very successful results. The teeth should be roughed out on the hobbing machine, and, where possible, several gears should be placed on the work-arbor at one setting. A great deal of time is saved by placing several gears on the work-arbor, which should always be steadied at the top. Suppose six are to be cut at one setting; this would mean that the hob would only have to travel into and clear the work once instead of six times, which would be necessary if they were cut singly. When putting several gears on the arbor, they must be faced exceedingly true, or the arbor will be bent. This is especially true when the hole is small and the gears large in diameter. Plenty of lubricant should be applied in this operation, oil being most commonly used. The hob should be made of high-speed steel, a six-pitch hob being 3 inches in diameter, and a five-pitch hob, $3\frac{1}{2}$ inches in diameter. On account of the accuracy required, single-threaded hobs are preferable. For the roughing operation a cutting speed of sixty feet per minute of the hob, and 0.020 inch feed per revolution, are considered good practice for a nickel-steel six-pitch gear.

Cutting the Teeth — Finishing Operation. — For the finishing operation at least 0.010 inch should be allowed for a six-pitch

gear and other pitches in proportion. If the finishing cut is merely a scraping cut and not enough stock is removed to let the cutter get a real chip, the cutter may glaze over the work, especially if the cutter and the work-arbor are not held rigidly. The gears should be finished one at a time, excepting plate gears, in which case several can be placed on the arbor at one setting. For this purpose a gear shaper should be used, because the cutter can be made far more accurate than a hob or a rotary cutter. The teeth of this cutter can be ground after hardening, and this corrects any inaccuracies that may have crept in. On the cutter-arbor, at the back of the gear-cutter, should be placed a round disk made of high-speed steel, hardened, ground and backed off, which will act as a shaving tool and will take off the 0.005 inch left on the top diameter, as previously stated. This will make the outside diameter true with the pitch line. The teeth should be cut about 0.001 inch thin at the pitch line to produce a running fit. The gears should now be tested for center distance, for which purpose a plate with two pins for the bores, set at the correct center distance, should be used.

Rounding the Ends of the Teeth. — The tooth-rounding should be performed on an automatic tooth-rounding machine. Several gears should be mounted on the same arbor, and the teeth of the wheels may be rounded in succession at one setting. No doubt most readers are quite familiar with the reason for this operation, which, therefore, requires little explanation, unless it be to say that it is done to facilitate the changing of the gears, and also to prevent the teeth from being chipped when engaging, which would occur if the corners were left square. The sides of the teeth which are not rounded should be fraised before carbonizing, which is the next operation.

The Carbonizing Process. — In carbonizing, great care should be taken, as, to a very great extent, the life of the gear depends on this process. The result of the process is determined by four factors, namely: the nature of the steel; the nature of the carbonizing materials; the temperature of the carbonizing furnace; and the time taken by the process. The carbonizers in general use at the present time are animal charcoal, hydro-carbonated

bone-black, charred leather, and a few other compositions sold under various names. Owing to the various conditions under which the operation is carried out, experience must largely guide the operator. Theoretically, the perfect carbonizer should be a simple form of carbon, and charred leather gives very satisfactory results. Care should be taken to avoid poorly charred leather, or that made from old boots, belting, etc. Good charred leather should contain about 88 per cent of carbonizing matter.

As it is essential that the core of the gears should be left soft in order to withstand the high speed and sudden shocks to which they are subjected, the carbon content in the core should be low. For this reason preference is given to 0.20 per cent carbon steel. The carbonizing pots are made from both cast and wrought iron; the former are cheaper in first cost, but the latter bear reheating so many times that they are really cheaper in the end. The carbonizer having been thoroughly dried and reduced to a fine powder, a layer of not less than 1½ inch in depth is placed in the carbonizing pot and well pressed down. Upon this are placed the articles to be treated. Care must be taken to have sufficient space all around each piece so as to prevent them from touching each other or the walls of the pot. About 1½ inch is sufficient. Another layer of carbonizing material is then put in and well pressed down, care being taken not to displace any of the gears. The process is then continued until the pot is full, finishing with a layer of about 1½ inch at the top.

The object in view is to make the contents of the pot as compact as possible, consistent with a sufficiency of carbonizer in contact with the gears. The more solidly the pot is packed, the more complete the exclusion of air. The lid is then put on, and the joint luted with clay all around. The pot should be placed in a furnace similar to that used for annealing, and heated to about 1700 degrees F., which heat should be maintained constant for from six to ten hours. The length of time occupied is regulated by the depth of casing required, which should be about three-sixty-fourths inch, and also by the dimensions of the gears. At the close of the carbonizing period, the pot is withdrawn and put in a dry place where it is allowed to cool to atmospheric

temperature. It is then opened, the articles are taken out, and the process is completed by brushing to remove all adhering matter.

It may be noted here that the case should only be deep enough to resist wear and battering. If the case is so deep as to form a considerable part of the cross-section of the teeth, the teeth may break unless the case is considerably tempered.

Turning Operations Preceding Hardening. — The next operation is to turn out the carbon from the parts required to be soft. In the case of the plate gear shown in Fig. 1, which is to be bolted to another gear or center, as previously stated, the hole need not be hard. For this reason, it was left with an allowance of $\frac{1}{8}$ inch at the previous turning operation. It should now be bored with an allowance of 0.015 inch for grinding after hardening, and faced down on both sides. This refers to both plate and castellated gears as shown in Figs. 6 and 7. The boring operations should be performed while the gear is held in a collet-chuck, locating from the top of the teeth, which were trued up with the pitch line by the shaving tool used when cutting the teeth, as previously explained. This method has been found to be more efficient than locating with balls or rollers on the pitch line. The penetration of carbon being only about three-sixty-fourths inch it is now removed from the parts which have just been turned. Consequently these parts will not be hardened. The excess metal is left as shown in Figs. 6 and 7 until after hardening, in order to prevent warping, which would undoubtedly happen if the gears were finished as shown by Figs. 1 and 8.

Casehardening and Oil Tempering. — Steel of the composition mentioned can be hardened as follows: Heat from 1450 to 1525 degrees F. and quench in water; reheat from about 1400 to 1450 degrees F. and quench in water. The reheating must be conducted at the lowest possible temperature at which the steel will harden. It will be found that this is sometimes as low as 1300 degrees F. Then, as a safeguard, reheat to a temperature of between 250 and 500 degrees F., in accordance with the requirements of the case, and cool slowly in oil. Parts of intricate shape, such as the gears dealt with, having sudden changes of thickness,

sharp corners and the like, should always be tempered or drawn in order to relieve internal strains.

Another method of procedure is as follows: Heat from 1450 to 1525 degrees F., and quench in hot brine. Reheat from 1450 to 1525 degrees F., and quench in oil. The temperature need not be drawn when the gears are quenched in oil. The final quenching should be done at the lowest temperature at which the piece will harden, as stated in the first method.

A small gas muffle should be used for hardening. A properly constructed gas muffle can be regulated with the greatest nicety, and in hardening this is most important. When steel is gradually heated, there is a certain point at which a great molecular change takes place, and perfect hardness can be obtained only by quenching at this critical point. This would lie between 1300 and 1450 degrees F. When steel is cooled, whether slowly or not, it bears in its structure a condition representative of the highest temperature to which it was last subjected. From this it will be quite clear that in casehardening, as in all methods of hardening, the steel must be quenched on a rising heat.

Steel which is overheated previous to the final quenching is very brittle and liable to fracture easily, and although quenched and subsequently hardened, the metal has little or no cohesion, and rapidly wears away. Steel so hardened breaks with a very coarse crystalline fracture, in which the limits of the case are badly defined. If quenching takes place below the critical point previously mentioned, the steel is not sufficiently hard. If above, though full hardness may be obtained, strength and tenacity are lost, in part or completely, according to the degree of heat by which the critical temperature is exceeded.

It may be asked why it is not sufficient, when the pieces are heated at the first reheating to about 1500 degrees F., to place them in another furnace and reduce to the critical temperature and quench, instead of quenching twice. The answer is that the high temperature has already created a coarse crystalline condition in the steel, and that, until it has been cooled down below the critical point, and reheated to the critical temperature, a suitable molecular condition cannot be obtained.

As a further means of illustrating what is meant by the critical point, Fig. 9 shows a curve plotted from results obtained by a recording pyrometer, in which the decalescent and recalescent, or critical points are shown. From this it will be seen that the

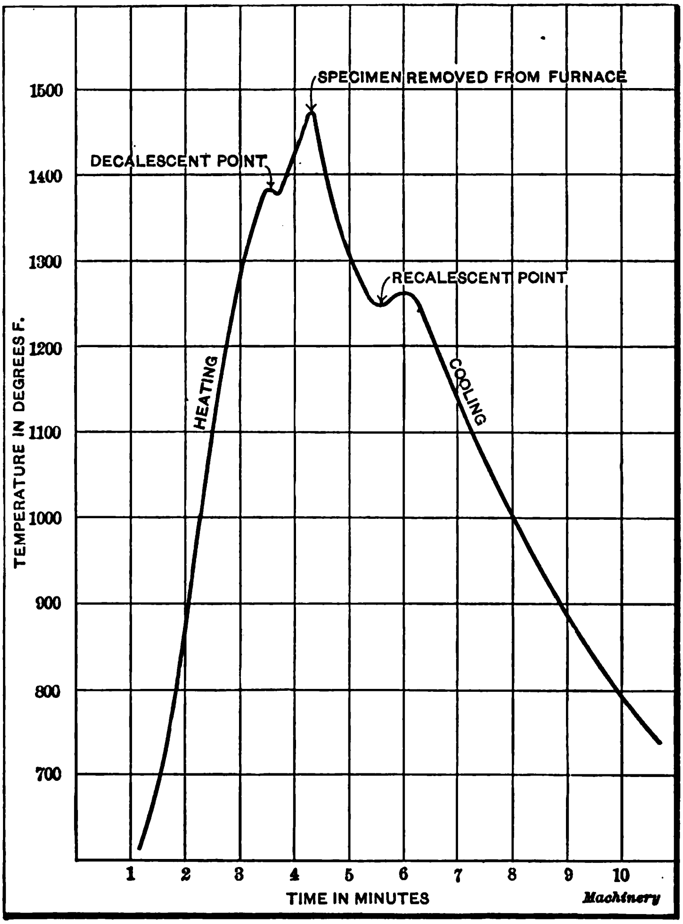


Fig. 9. Curve showing the Decalescent and Recalescent Points for Low-carbon Nickel Steel

absorption of heat occurred at about 1375 degrees F. on the rising temperature; and the evolution of heat at 1250 degrees F. on the falling temperature, which was allowed to fall slowly. The relation of these critical points to hardening is that it cannot

take place unless a temperature sufficient to produce the first action is reached in order to change the pearlite carbon to hardening carbon; and also unless it is cooled with sufficient rapidity to eliminate the second action. The temperature would, of course, be gaged with a pyrometer.

Sandblasting. — The sandblasting, which serves to scour off any roughness or stains which have been left on the surface during the heat-treatment, etc., is best conducted in a building separated from the remainder of the shop. The sand should be kept in a bin in one corner, and sucked up by a centrifugal blower and forced by air pressure through a blowpipe which terminates in a nozzle. The sand being forced by the air at a high velocity may be directed at all parts of the piece to be cleaned. This is one of the most efficient methods of polishing and cleaning the gears, and does not injure the hard surface in any way.

Hardness Testing. — The gears should now be tested for hardness. The scleroscope appears to be the best instrument for doing this. If a gear shows a considerable drop in hardness, a file should be used to determine whether the cause is due to the piece not being hard, or to crystallization. If the part can be scratched with a file, it shows that it is not hard enough. If, however, the file will not bite on the spot where the scleroscope reads low, then it is positively known that it has been overheated and is crystallized. A good method of testing the teeth is the drop test. By this method a ten-pound weight with a 56-inch drop is directed at one tooth. The number of blows necessary to break the tooth should be noted. This test would only be applied occasionally, say, on one out of each batch of gears. A gear which has given satisfaction should be tested, and the result used as a standard for future comparison.

Final Turning Operations — Grinding. — The metal left for supporting the gears while undergoing heat-treatment is now removed. This operation should be carried out with the gear held in a collet-chuck, and care should be taken that the outside diameter runs true before starting. The hole, having been bored after carbonizing, is now soft. The metal can be turned out on both sides, or on one side, as the case may be, leaving 0.005 inch

on the face of the web for grinding. This operation should come next, and is most essential in order to get a true running gear when in position. The castellated gears which were finish-bored with an allowance of 0.015 inch before carbonizing are now hard, this being necessary as they have to be a sliding fit on the shaft. The part of the bore which bears against the lands of the castellated shaft is all that can be ground. Unfortunately no method has yet been devised for grinding the castellations themselves, and these, therefore, have to be lapped separately by hand — which is most unsatisfactory. While grinding the bore, the gear should be held in a collet-chuck from the top diameter — hence the truing of the top diameter with the pitch line in the gear-cutting process. Gears ground in this way should be perfectly true, and are now ready to be tested for center distance and true running.

Drilling. — It should be clearly understood that the web of the gear shown in Fig. 1 is now soft. This brings us to the final machining operation, that is, the drilling. There are many advantages gained in leaving this operation until the last. The bore is now the correct size, this being essential for locating the drilling jig. Furthermore, in the grinding operation, the bore is totally ignored, the top diameter being the most important. The hole has been ground true with the latter, so that if the holes had been drilled previous to the hardening, they would not be concentric with the bore, and would not match with the gear or center to which it has to be bolted. Another advantage is that the holes are now soft and can be reamed with the piece to which the gear is to be bolted, it being understood that these would also be left soft in the parts in which the holes are required. All these are important points, and this is the reason for leaving the drilling until last.

"Running-in." — The gears should now be bolted to their respective parts and finally "run in" before being placed in the car. This should be performed in a special case, and the "running-in" done under belt power. The bearings, in these special cases, are set at the proper center distance, so as to accommodate the various gears of a train, thus wearing in the gears so that

all those for similar parts are absolutely interchangeable. The case is made oil-tight, and a mixture of finely powdered emery and lubricating oil is forced through an opening in the top, so that this grinding material will come in contact with all the teeth in mesh in the train. The grinding is continued until each tooth has been worn perfectly smooth, and to an accurate fit with the teeth of the other gears with which it comes into mesh. As a further means of thoroughly running in the gears of the transmission to a perfect fit, the motor, transmission and driving shaft are installed in the chassis, and the motor is run while the various speeds of the transmission are thrown into mesh. During this run an electric dynamotor, by means of which a variable load may be applied, is connected to the end of the driving shaft. The gears should now be as perfect as, with the best practice yet known, it is possible to get them.

It may be objected here that the leaving of extra metal for heat-treatment is rather costly. This is a question upon which there is room for considerable difference of opinion. It seems that if the method of leaving extra metal is used with discretion, and only in the case of very intricate gears, the most satisfactory results should be produced.

Heat-treated Gears in Machine Tools. — The requirements in machine-tool construction differ somewhat from those in the automobile trade. In the following paragraphs these requirements and the means used for meeting them are reviewed, as stated in a paper read before the National Machine Tool Builders' Association by Mr. A. C. Gleason. The methods described are those in use in the Gleason Works, Rochester, N. Y. The points which are of interest to the average machine tool builder will be considered. The methods of heat-treatment described here are those best suited to the requirements at the Gleason Works and it is not claimed that they represent general practice. In fact, it is rather difficult to say what is the general practice, as the heat-treatment of steel gears is largely a matter of individual requirements. Methods vary considerably even among manufacturers of a certain class of automobile gears which are of a standard shape and designed for one purpose, so that hardened

gears for machine tools may well be considered a distinct proposition. Looking at this subject from the viewpoint of the machine tool builder, the principal points to be considered are:

1. The advantages to be gained by the use of heat-treated gears.

2. The selection of steel to suit the purpose for which the gears are intended, and the design to suit hardening conditions.

3. The methods of hardening and the necessary equipment and materials.

4. The cost of heat-treated gears.

Advantages of Heat-treated Gears. — The advantages in the use of heat-treated gears properly made are greatly increased strength and hard tooth surfaces which resist wear. These points are certainly of vital importance in modern machine tool design, and it seems inevitable that hardened gears will very soon be in general use in this class of machinery. The failure of soft metal gears to stand the wear and tear in machine tools is too frequently the cause of breakdowns, and in spite of the rapid development in design in other ways such gears are still commonly used. They are a serious source of weakness and the logical remedy is heat-treated gears. Heat-treated gears with their increased strength and ability to withstand wear offer almost unlimited opportunity for compact design. For example, the automobile transmission suggests what can be accomplished with such gears in making machine tools more convenient in operation and more durable.

Steel to be Used. — The question of the steel to be used depends not only upon the purpose for which the gears are intended but also upon the design. When the gears are subjected to severe shock or heavy overload at times, a steel which will show the greatest tensile strength possible, without sacrificing toughness, is plainly the most desirable; but steel which will show these qualities has certain limitations for use in machine tool construction, and it will be interesting to note the value of a straight hardening steel as compared with the more commonly used casehardening steels. Straight hardening steels for gears are invariably alloy steels. Chrome-nickel alloy steels of several

different makes show an increased tensile strength of 150 per cent after heat-treatment. The analysis varies considerably in different makes, requiring a corresponding difference in the heat-treatment, but manufacturers making a specialty of alloy steels now furnish them carefully graded with instructions for hardening which can generally be relied upon.

There are frequent exceptions, however, to the rules for hardening any kind of gear steel, and the only safe method is to experiment with every change in design. It is not sufficient to cut a piece off the bar and harden it, regardless of the shape of the gear being made, as the sample piece should be practically the same as the gear in order to produce the same effect. Take, for example, a pinion solid on a shaft. The teeth will chill much more quickly than the solid section under them, and in order to avoid shrinkage strains special heat-treatment is required to suit this shape and the quality of steel that is used. If a sample pinion of this size were made separate from the shaft, the effect of the same heat-treatment would be radically different owing to the fact that the center would cool almost as rapidly as the teeth.

The use of as few grades of steel as possible is advisable. If the steel is selected to suit exactly each different shape of gear and the purpose for which it is intended, it would necessitate a large variety for the varied requirements of the machine tool builder. There are some steels which are more adaptable to different conditions in hardening than others, and considering the comparatively small quantities and the variety of sizes and shapes in gears for machine tools, it is well to keep this in mind. "Fool-proof" methods in the heat-treatment of gears are far from possible, but by the selection of a few grades of steel which show good average results, the work can be greatly simplified.

Water or Oil-hardened vs. Casehardened Gears. — On account of the great strength and toughness of chrome-nickel straight hardening steel and the fact that it hardens clear through, it is well adapted for automobile transmission gears and any similar purpose in the construction of machine tools. Gears of this kind do not chip on the edges of the teeth and will stand al-

most unlimited hardship in sliding in and out of mesh. Chrome-nickel straight hardening steels tend to keep their shape in hardening better than the low-carbon casehardening steels, but they do warp somewhat and they cannot be straightened without sacrificing the hardness almost entirely. Gears made of this steel do not show as hard a bearing on the surface of the teeth as those made of casehardened steel. The cost of machining alloy straight hardening steels will average at least twice as much as that of casehardening steels, and the cost of the steel itself varies from fourteen to sixteen cents per pound, as compared with less than half that price for high-grade alloy casehardening steels.

The chief advantage in the use of casehardened gears is the file-hard tooth surfaces. The easy machining qualities and low cost of the stock are also important advantages, but the superior wearing qualities of casehardened gears make them the best at any price for average conditions in machine tool construction. In designing a machine tool there may often be occasion to make the gears as small as possible, and it then becomes a question as to the choice of straight hardening steel or alloy casehardening steel. A 5 per cent nickel low-carbon casehardening steel will give the best results possible if strength and wearing qualities are considered, although if strength is the main consideration, straight hardening steels, as previously explained, should be the choice.

Of the alloy casehardening steels, three principal grades of nickel steel have become standard to a large degree in the trade at the present time. These are 5 per cent open-hearth nickel alloy; $3\frac{1}{2}$ per cent open-hearth nickel; and 1 to $1\frac{1}{2}$ per cent nickel natural alloy. The principal characteristics of these steels are a higher tensile strength than straight-carbon steel, and a correspondingly higher strength after casehardening. The carbon case has a close bond with the core and is less likely to chip than the ordinary machinery steel. In fact, a number of manufacturers of the higher grade automobiles use the 5 per cent casehardening nickel steel in their change speed transmissions, where straight hardening steels might be regarded as more favorable.

When it comes to substituting casehardened gears for soft steel or cast-iron gears, in machine tools, it will generally be found that a straight-carbon casehardening steel will answer every requirement, and unless the original gears have been considerably overloaded the more expensive alloy steel gears would be an extravagance. In straight-carbon steels for casehardening, 0.15 to 0.25 per cent carbon is to be recommended. A lower degree of carbon than this is likely to produce a laminated case which will crack or chip under heavy pressure. Nickel alloy steel for carbonizing should not have over 0.20 per cent carbon; 0.10 to 0.20 per cent carbon is considered the best for this purpose. The nickel alloy has practically the same effect in hardening as an increase in carbon of about 0.10 per cent. A 0.25 per cent carbon nickel alloy steel, casehardened, will generally harden clear through very much the same as a straight hardening steel.

The teeth of soft steel gears in machine tools seldom break or strip until they have worn quite thin, and long experience goes to show that durability is generally of more importance than excessive strength in gears of this kind. Casehardened steels show an increased tensile strength ranging from 30 per cent in standard carbon steel, to 75 per cent in high-grade nickel alloy steel. The straight-carbon casehardening steels are not as uniform in analysis as the alloy steels and, therefore, it is not always possible to obtain the same degree of hardness in the case as with the higher grade steels. This is particularly noticeable in gears of heavy section hardened in oil, but it rarely happens that the hardness will drop as low as that of straight hardening steels.

Selective or Local Hardening of Gears. — A strong point in favor of casehardening steels for machine tool gears is that certain parts can readily be kept soft where subsequent fitting is found desirable. Hub projections, web surfaces, etc., can be copper-plated or enamelled with various preparations so as to exclude the carbon in the carbonizing heat, leaving these parts soft for final machining operations. Present-day design of casehardened gears for machine tools often makes it necessary to

keep certain parts soft in order to avoid shrinkage strains in the hardening process. In a gear with light hub projections, as shown in Fig. 10, the carbon case would extend almost through the thin section of the hub at the keyseat, making it quite likely to crack on account of the uneven shrinkage in hardening. The making of gears for hardening of even section throughout is advisable, but where it is necessary to use light hub projections like the one shown, they can be made safely by the method of selective hardening.

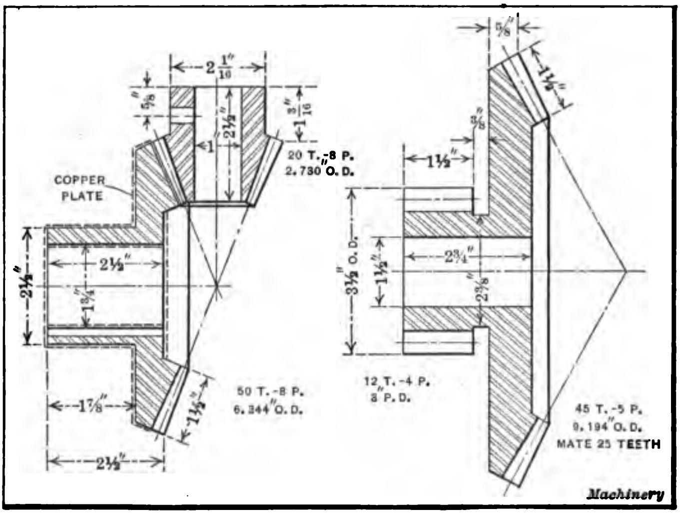


Fig. 10. Gear with Light Hub kept Soft by Copper Plating

Fig. 11. Gear having Small Pinion Integral with Hub

In selective hardening, copper-plating the surfaces which are to be kept soft is the most satisfactory method. Generally the parts to be plated can be immersed in the solution, leaving the teeth clear, or the blank can be copper-plated all over, so that when the teeth are cut, the bearing surfaces will be free to carbonize. It should be borne in mind that wherever this plated surface is marred, carbon will enter and leave a hard spot after the gears are hardened.

Some Points in Gear Design. — Points for criticism in the design of the pair of gears shown in Fig. 10 are the extremely long face in proportion to the cone distance and the long backing or

overhang of the pinion on its bearing. The chances are all against getting a full bearing of the teeth throughout the length of face, after the gears are hardened and the bores are ground to size. It is advisable to cut down the length of face to not more than one-third of the cone distance and to use a coarser pitch with a smaller number of teeth for the same diameters.

Another condition which it is necessary to guard against in gears of this kind occurs where a small pinion is made solid with the gear in place of the light hub, as shown in Fig. 11. There would be still greater risk on account of shrinkage strains in hardening this piece. The pinion section would be weak when worked out of the center of bar steel and it would be far better to make it separate from the gear.

All established rules for the horsepower transmitted by gears are based on the use of soft steel or cast iron. They usually allow a stress for steel of two and one-half times that of cast iron. This may be correct as far as strength is concerned, but it certainly is not right if wear is to be taken into account. Gears of a good mixture of cast iron, showing 35 to 40 on the scleroscope test for hardness, will withstand wear fully as well as open-hearth cast-steel gears of the same size. This brings up the subject of wearing qualities of casehardened steel gears as compared with soft gears. A number of electric motor drives have been equipped with casehardened gears, making them very much smaller than the soft steel gears formerly used, with most satisfactory results, and judging by these records, a stress of four times the usual standard allowed for cast iron in standard horsepower rules such as the Lewis formula, is permissible.

For example, in a 30 horsepower electric drive in connection with a positive pressure blower, the original soft steel gears, computed according to the standard rule for horsepower, required a pair of gears having, respectively, 49 and 16 teeth, of 3 diametral pitch, $3\frac{1}{4}$-inch face; the casehardened steel gears which have been in use now for several years have 49 and 16 teeth, of 4 diametral pitch and $2\frac{1}{4}$-inch face. The rule making the strength of casehardened steel gears four times greater than cast iron is conservative, as the gears serving as a basis for the

calculations are considerably larger than automobile bevel driving gears and transmit more power. As evidence of the advantage of casehardened gears over hardened alloy steel gears, may be cited the standard automobile practice in bevel driving gears, where automobile manufacturers use only casehardened steels for such bevel driving gears at the present time. Their requirements, as is well known, call for the greatest possible combined durability and strength.

The quenching of casehardened gears at the lowest heat possible to produce the required degree of hardness is to be recommended; this in combination with slow carbonizing heats gives excellent results with one quenching heat. In the use of steels suitable for casehardening, experience strongly advises against drawing after hardening. Casehardening steels allow of a much wider range in the heat-treatment than straight hardening steels, and casehardened gears seem decidedly the best for general purposes in machine tools, "clash gears" excepted.

Equipment for Heat-treating Gears. — The Gleason Works are using coal-burning furnaces for carbonizing in preference to oil or gas (see plan of heat-treating department, Fig. 12). For continuous use with slow soaking heats coal is most economical, and with very little attention to the fires, there is no difficulty in maintaining and controlling the heat. The fact that the coal-burning furnaces require practically no attention when run after working hours, and that no power is required for air pressure such as is necessary in the oil or gas burners, is an argument in their favor; the very low cost of fuel is also an important consideration. Furnaces of this kind, however, must be used continuously, day after day, to produce the best results, since it requires about twenty-four hours from the start to bring them up to a carbonizing heat. On the other hand, there is no delay whatever in getting the required heat with oil or gas, and where there is not sufficient work to keep a furnace running continuously, the oil or gas furnace would undoubtedly prove the most satisfactory. An oil or gas furnace is necessary in any case for reheating, so that the same furnace can often be used to advantage on carbonizing as well. Aside from the cost of fuel, the

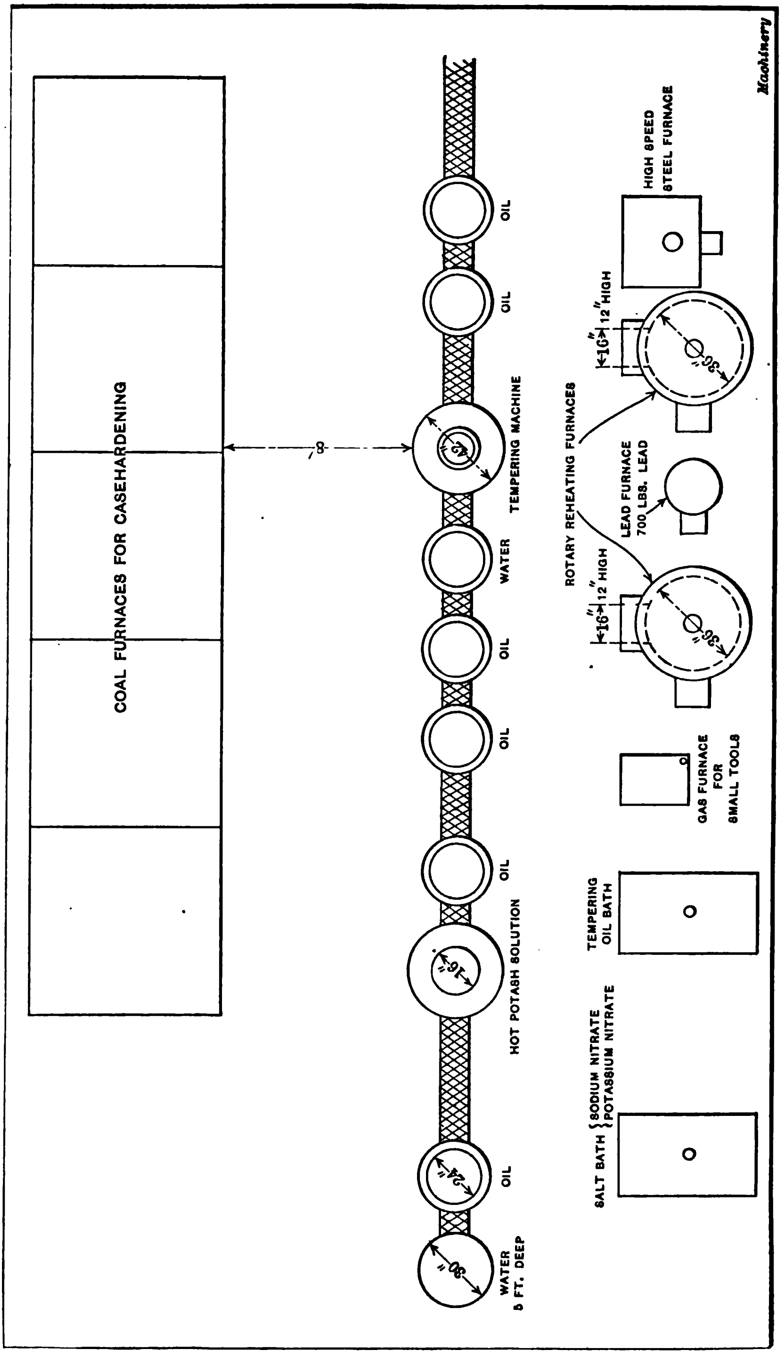


Fig. 12. Plan of Gear-hardening Department—Gleason Works

objection to the use of oil or gas for carbonizing gears is that furnaces of this kind require constant attendance when run after working hours and that extra power is needed for air pressure in the burners. There is always the danger of an interrupted flow of the gas or air to be guarded against.

In regard to the depth of carbon case in gears, $\frac{1}{8}$ the thickness of teeth at the pitch line is recommended, but not more than $\frac{1}{16}$

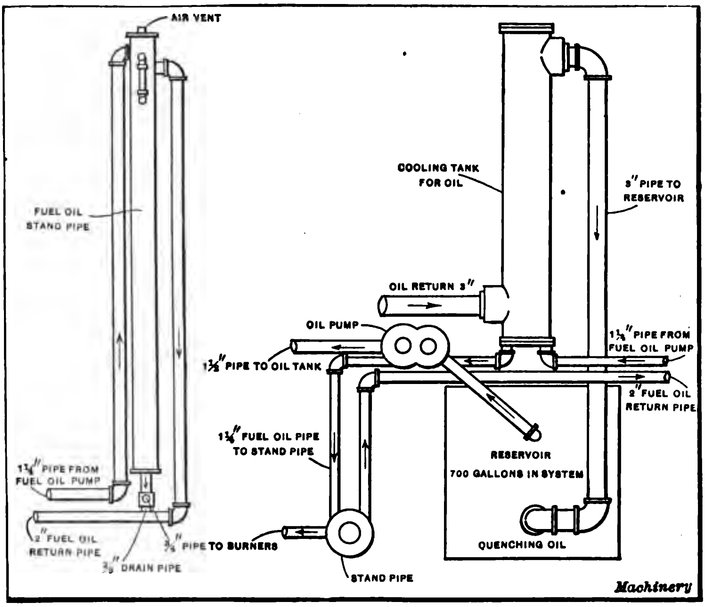


Fig. 13. Diagram showing Quenching Oil Reservoir and Cooling Apparatus

inch deep in the coarsest pitches in machine tools. According to this rule, $\frac{1}{2}$-inch pitch should have $\frac{1}{32}$-inch carbon case.

At the present time, there are several makes of carbonizing compounds extensively used which have proved much more satisfactory than bone or charred leather. Care must be taken to keep these compounds perfectly dry, not only in the packing of the boxes but also after they are taken out to cool; if any water is allowed to leak in when the material is hot, a chemical action sets up which has the effect of blistering the casehardened surfaces of the gears, the same as if they were overheated. Short

pieces of common machinery steel about ½ inch square are generally placed in the top of the boxes for test pieces, and before the work is taken out of the box these pieces should be hardened and broken to make sure that the depth of case is right.

It is a well-known fact that nickel alloy steels require a longer carbonizing heat than straight-carbon steel, and in order to determine the proper depth of case, test pieces of the same material should be used with the plain carbon steel test pieces so

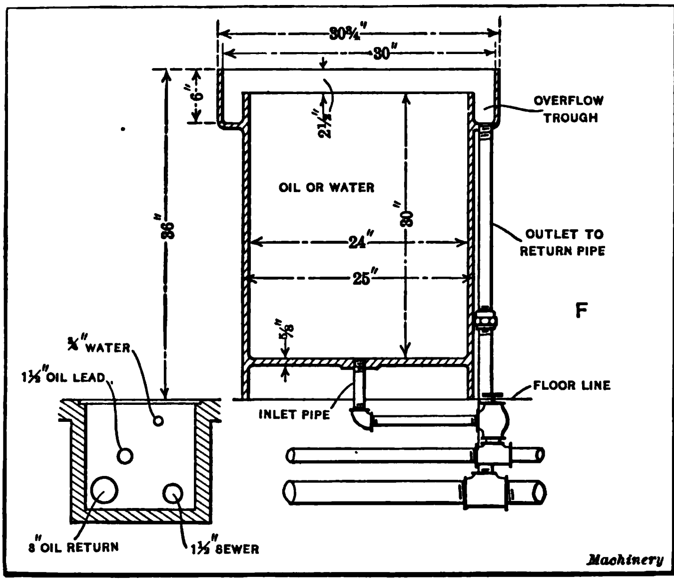


Fig. 14. Oil or Water Quenching Tank

as to make a proper comparison. It does not pay to use plain cast-iron carbonizing boxes; semi-steel is better and cast steel is the best.

Oil used for Quenching. — The Gleason Works use mineral oil for quenching which has a 310-degree flash test and viscosity of 74 inches at 104 degrees. It is a thin petroleum oil which can be bought for 17 cents per gallon by the barrel. With tempering oils like this a safe rule is to quench one pound of steel to a gallon, every four hours, where no special arrangements are made for cooling the oil. With a cooling system, as shown in Fig. 13, it

is possible to quench 3000 pounds of steel in eight hours with 700 gallons of oil. The tempering oil, as shown, is circulated through the inside of a radiator, and the fuel oil is circulated through the outer jacket. The radiator is simply a powerhouse water heater which is adapted for this purpose without change. The fuel oil is used for cooling because of its convenience. Greater efficiency, of course, could be obtained by having a flow of water for the purpose or by increasing the radiating surface. The radiator has thirty feet of cooling surface. There has never been any difficulty with overheating of the oil with this system, no matter how fast the work is put through; occasionally the temperature is as high as 120 degrees but never any higher. Quenching can be done about twice as fast as without the cooling system. Fig. 14 shows the form of oil or water quenching tank used.

Cost of Heat-treated Gears. — In leading up to the final cost of heat-treated gears, the actual cost of labor and materials in the heat-treatment of chrome-nickel straight hardening steels as compared with casehardening in general is here presented.

Cost data for 1000 pounds of gears made of chrome-nickel straight hardening steel:

Labor (one working foreman and two assistants, wages)	$ 9.85
Fuel oil (two hardening furnaces, 60 gallons)	3.00
Quenching oil, 1½ gallon	0.25
Tempering oil, 2 gallons	0.50
Pyrometer ends	0.10
Gas for drawing temper, 1500 cu. ft.	1.45
$15.15 ÷ 1000 = $0.0152 per pound	$15.15

Cost data for 1000 pounds of casehardened gears:

Labor (one working foreman and two assistants, wages)	$ 9.85
Coal (3 furnaces, 200 pounds each)	2.00
Carbonizing compound	1.00
Fuel oil (two hardening furnaces, 60 gallons)	3.00
Quenching oil, 1½ gallon	0.25
Pyrometer ends	0.25
Carbonizing boxes (average)	2.50
Wear and tear on 3 carbonizing furnaces	0.30
$19.15 ÷ 1000 = $0.0192 per pound	$19.15

No account is made of the cost of power or other overhead expenses as they are the same in either case.

Following is a tabulated account of the actual cost of labor and materials in making up a small lot of miter gears complete from bar stock using the various grades of steel referred to:

Taking a miter gear having 18 teeth of 4 pitch, $1\frac{1}{8}$ inch face, $1\frac{1}{2}$ inch bore and an ordinary hub, the weight of the rough bar stock is 11 pounds and the finished gear, $5\frac{1}{2}$ pounds. The labor cost for the machine work would be practically the same for this gear, in any of the standard steels for carbonizing. The only difference in the complete cost would be in the stock. According to this, and taking the labor cost at $1, the cost of the gear complete would be as follows:

Straight carbon steel at 3 cents	$1.44
1 to $1\frac{1}{2}$ per cent natural alloy steel at $4\frac{1}{2}$ cents	1.61
$3\frac{1}{2}$ per cent O. H. nickel steel at 6 cents	1.77
5 per cent O. H. nickel steel at 8 cents	1.99

The cost of machine work is practically the same in either case-hardening straight-carbon steel or any of the nickel alloy case-hardening stock. Heat-treated gears in machine tools are on the side of superior quality and greatest efficiency. It is safe to say that within the next few years, soft steel gears in machine tools will become a thing of the past, just as gears with cast teeth were abandoned twenty years ago.

Heat-treatment Methods used at the Boston Gear Works. — As all the work usually carried on in an up-to-date hardening plant is handled in connection with the manufacture of gears at the Boston Gear Works, a brief review of the methods here in vogue will be of value. In this plant the various heat-treating processes carried out consist of carbonizing and hardening low-carbon and alloy steels, oil hardening high-carbon and alloy steels, heat-treating high-carbon and tungsten tool steels, pack-hardening, annealing and drawing the temper, all the work consisting mainly of gears.

The heat-treatments in use are mainly those recommended by the Iron and Steel Division of the Society of Automobile Engineers, known as the "S. A. E." treatments. These treatments

cover carbon steels with 0.10 to 0.95 per cent carbon; carbon steel screw stock; 3½ per cent nickel steel with 0.15 to 0.50 per cent carbon; low, medium and high nickel-chromium steel with 0.15 to 0.50 per cent carbon; low and medium nickel-chrome-vanadium steel; 1.0 and 1.20 per cent chromium steels with 0.95 and 1.20 per cent carbon; chrome-vanadium steel with 0.15 to 0.95 per cent carbon; and silico-manganese steel with 0.50 per cent carbon.

Kinds of Heat-treatments Used. — For special gear steels, referred to in the following list, special treatments are used which have been largely developed in the plant.

For 0.08 to 0.15 per cent carbon No. 1 machinery steel. Treatment *Z*.

For 0.15 to 0.25 per cent carbon No. 2 machinery steel. Treatment *I*.

For 0.35 to 0.40 per cent carbon No. 3 machinery steel. Treatment *E*.

For 0.40 to 0.50 per cent carbon oil hardening No. 4 machinery steel. Treatment *J*.

For 0.15 to 0.25 per cent carbon 3½ per cent nickel steel. Treatment *G*.

For 0.15 to 0.25 per cent carbon chrome-vanadium steel. Treatment *S*.

Treatment G

1. Carbonize between 1600 and 1750 degrees F. (1650–1700 degrees F. desired).
2. Cool slowly in the carbonizing material.
3. Reheat to from 1450 to 1525 degrees F.
4. Quench.
5. Reheat to from 1300 to 1400 degrees F.
6. Quench.
7. Reheat to from 250 to 500 degrees F. (in accordance with the necessities of the case), and cool slowly.

Treatment S

1. Carbonize at a temperature between 1600 and 1750 degrees F. (1650–1700 degrees F. desired).
2. Cool slowly in the carbonizing mixture.

3. Reheat to from 1600 to 1700 degrees F.
4. Quench.
5. Reheat to from 1475 to 1550 degrees F.
6. Quench.
7. Reheat to from 250 to 550 degrees F., and cool slowly.

Treatment E

1. Heat to from 1500 to 1550 degrees F.
2. Cool slowly.
3. Reheat to from 1400 to 1450 degrees F.
4. Quench.
5. Reheat to from 600 to 1200 degrees F. and cool slowly.

Treatment Z

1. Carbonize at 1600 to 1650 degrees F.
2. Quench direct in oil.
3. Draw in oil at from 400 to 450 degrees F.

Treatment I

1. Carbonize at 1600 to 1650 degrees F.
2. Cool in pot.
3. Reheat from 1550 to 1600 degrees F. (1575 degrees preferred).
4. Quench in oil.
5. Reheat from 1350 to 1400 degrees F. (1375 degrees preferred).
6. Quench in oil.
7. Draw at from 300 to 400 degrees F.

Treatment J

1. Heat to from 1450 to 1500 degrees F. (1480 degrees F. preferred).
2. Quench in oil.
3. Reheat to from 800 to 900 degrees F. (Sometimes drawn as low as 600 degrees F. or as high as 1100 degrees F., according to requirements.)
4. Cool slowly in the air.

There is considerable doubt relative to the proper heat-treatment for 0.35 to 0.40 per cent carbon machinery steel, as it is

neither a casehardening nor oil hardening steel. This steel is largely used in its natural state. Treatment *E* is the one recommended by the S. A. E. for the treatment of 0.40 per cent carbon steel. Treatments *G* and *S* are also standard treatments of the S. A. E.

Tests on Heat-treated Gears. — The method used by the Boston Gear Works for testing the heat-treatment of steel is to cut off $\frac{1}{4}$-inch disks from both ends of a bar after the rough ends have been cut off. The bar is stamped with a number and the disks with the same number. The disks are then heat-treated and broken by a ten-pound drop hammer, the height of drop and number of blows being recorded. Gear teeth are also tested by a Burgess drop hammer testing machine, the standard being a 15-tooth, 6-pitch, $14\frac{1}{2}$-degree involute gear of one-inch face. The average of a number of tests on casehardened standard test gears of No. 2 machinery steel required 31 blows with a ten-pound hammer dropped thirty inches to fracture a tooth.

Carbonizing Methods of the Boston Gear Works. — The depth and quality of the case that is produced depends upon the duration of the heat-treatment, the temperature, the kind of carbonizing material used, the cooling process, and the temperature at which reheating and quenching operations are carried on. For casehardened gears, the best results are obtained with a refined case and fibrous core. Great care is required in packing gears in the pots with the carbonizing material. The pot covers are luted or sealed with fireclay and it is important that no gas escapes. More heats are obtained with malleable iron pots than with those of ordinary cast iron, as the latter burn out more rapidly. Granulated bone, charcoal and commercial carbonizing compounds are used for carbonizing.

The casehardening pots for carbonizing automobile main-drive ring bevel gears have cored centers and lugs on the bottom to allow a more uniform circulation of heat than is secured with the use of ordinary cylindrical pots. The carbonizing material, in this case a compound, is packed around the ring gears in the pots, after which the covers are sealed on with fireclay.

Casehardening. — After carbonizing, the work is carefully reheated and treated according to one of the preceding methods. The quenching equipment includes a cylindrical steel water cooling-tank, three feet in diameter by six feet deep, with a water inlet and outlet. The tank is placed in a vertical position with half of its depth below the floor level. A similar oil cooling-tank, thirty inches in diameter by six feet deep, is placed in an outer water tank, three feet in diameter by six feet deep, which acts as a water jacket. This outer tank has a water inlet and outlet which provides circulation that keeps the oil at a low temperature.

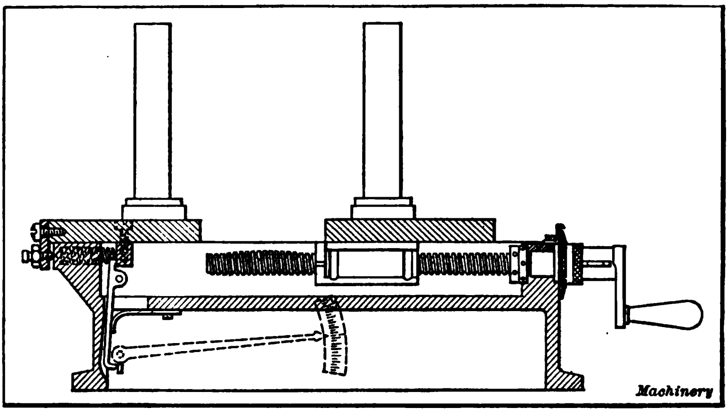


Fig. 15. Device for Testing Gears

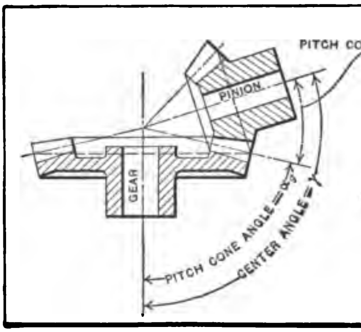
Device for Testing and Measuring Gears. — Various devices have been designed for testing the accuracy of the diameters and tooth outlines of gears. A very simple testing device consists of a cast-iron plate in which are accurately located holes for studs on which the gear to be tested can be placed so that its running with a master gear can be inspected. A more elaborate device for the testing of gears is shown in Fig. 15. This device has been used for several years with very satisfactory results. Differences in diameter and eccentricity of 0.0004 inch can be measured by it and the exact meshing of a couple of spur gears can be accurately tested. The illustration shows a longitudinal section of the device on a cast-iron base with fitted tool slides, each provided with a spindle to hold the gears to be tested. The

slide to the right is moved by means of a long adjusting screw. The one to the left has only limited adjustment but transfers its motion greatly magnified to a pointer placed on the front side of the base. The illustration clearly shows how the motion is transferred to the index which measures the differences in eccentricity and errors in the meshing of the gears. To detect differences in eccentricity, it is most convenient to place the gear to be tested on one spindle and to use a blank with a single tooth on the other. This tooth is meshed in succession with all the teeth of the gear being tested and by observing the different positions of the pointer while each tooth is measured, the eccentricity may be determined with great accuracy.

CHAPTER X

BEVEL GEAR RULES AND FORMULAS

Bevel Gear Definitions.—Bevel gearing is used for transmitting motion between shafts the center lines of which intersect. The teeth of bevel gears are constructed on imaginary pitch cones in the same way that the teeth of spur gears are constructed on imaginary pitch cylinders. In Fig. 1 is shown a drawing of a pair of bevel gears of which the gear has twice as many teeth as the pinion. The latter thus revolves twice for every revolution



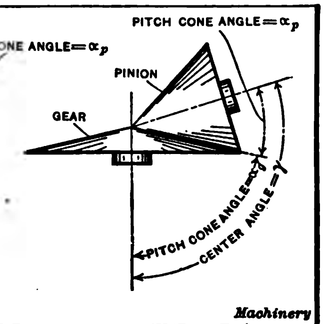


Fig. 1. Bevel Gear and Pinion

Fig. 2. Pitch Cones of Gears in Fig. 1

of the gear. In Fig. 2 is shown (diagrammatically) a pair of conical pitch surfaces driving each other by frictional contact. The shafts are set at the same center angle with each other, as in Fig. 1, and the base diameter of the gear cone is twice that of the pinion cone, so that the latter will revolve twice to each revolution of the former. This being the case, the cones shown in Fig. 2 are the pitch cones of the gears shown in Fig. 1. The term "pitch cone" may be defined as follows: The pitch cones of a pair of bevel gears are those cones which, when mounted on the shafts in place of the bevel gears, will drive each other by

frictional contact in the same velocity ratio as given by the bevel gears themselves.

The pitch cones are determined by their pitch cone angles, as shown in Fig. 2. The sum of the two pitch cone angles equals the center angle, the latter being the angle made by the shafts

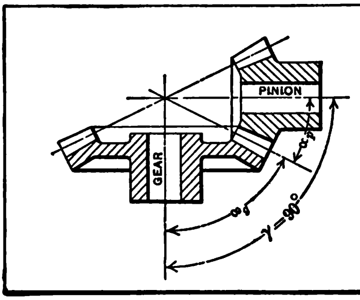


Fig. 3. Right-angle Bevel Gearing

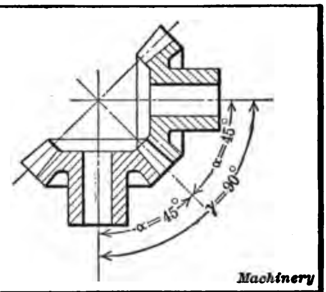


Fig. 4. Miter Gearing

with each other, measured on the side on which the contact between the cones takes place. The center angle and the pitch cone angles of the gear and the pinion are indicated in Fig. 1.

Different Kinds of Bevel Gears. — In Fig. 3 is shown a pair of bevel gears in which the center angle (γ) equals 90 degrees, or in

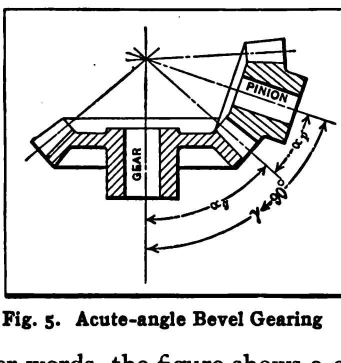


Fig. 5. Acute-angle Bevel Gearing

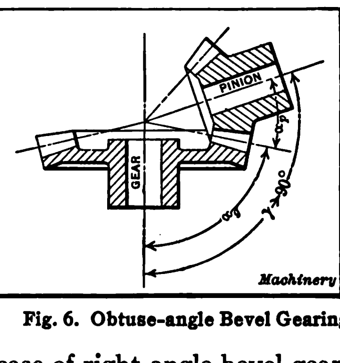


Fig. 6. Obtuse-angle Bevel Gearing

other words, the figure shows a case of right-angle bevel gearing. To the special case shown in Fig. 4 in which the number of teeth in the two gears is the same, the term miter gearing is applied; here the pitch cone angle of each gear will always equal 45 degrees.

When the pitch cone angle is less than 90 degrees we have acute-

angle bevel gearing, as shown in Fig. 5. When the center angle is greater than 90 degrees, we have obtuse-angle bevel gearing, shown in Fig. 6 and also in Fig. 1. Obtuse-angle bevel gearing is met with occasionally in the two special forms shown in Figs. 7 and 8. When the pitch cone angle α_g equals 90 degrees, the gear is called a crown gear. In this case the pitch cone evidently becomes a pitch plane, or disk. When the pitch cone angle of the gear is more than 90 degrees, as in Fig. 8, this member is called an internal bevel gear, and its pitch cone when drawn as in Fig. 2, would mesh with the pitch cone of the pinion on its internal conical surface. These two special forms of gears are of rare occurrence.

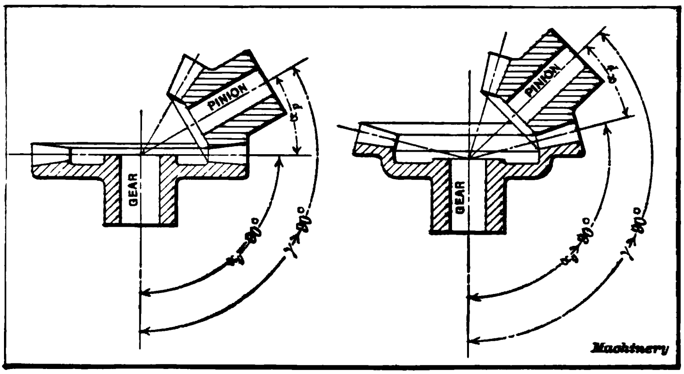


Fig. 7. Crown Gear and Pinion **Fig. 8. Internal Bevel Gear and Pinion**

Bevel Gear Dimensions and Definitions. — In Fig. 9, which shows an axial section of a bevel gear, the pitch lines show the location of the periphery of the imaginary pitch cone. The pitch cone angle is the angle which the pitch line makes with the axis of the gear. The pitch diameter is measured across the gear drawing at the point where the pitch lines intersect the outer edge of the teeth. The teeth of bevel gears grow smaller as they approach the vertex of the pitch cone, where they would disappear if the teeth were cut for the full length of the face. In speaking of the pitch of a bevel gear we always mean the pitch of the larger or outer ends of the teeth. Diametral and circular pitch have the same meaning as in the case of spur gears, the diametral pitch

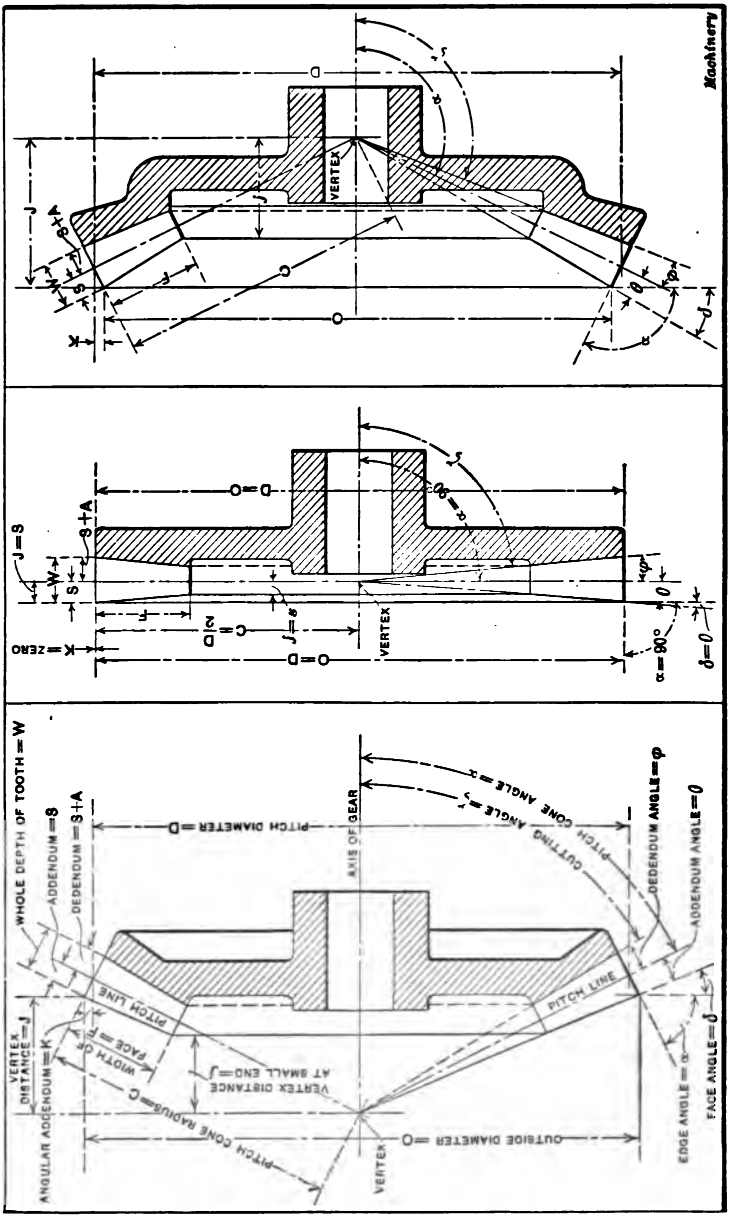


Fig. 9. Dimensions, Definitions and Reference Letters for Ordinary Bevel Gear

Fig. 10. Dimensions and Reference Letters for Crown Gear

Fig. 11. Dimensions and Reference Letters for Internal Bevel Gearing

being the number of teeth per inch of the pitch diameter, while the circular pitch is the distance from the center of one tooth to the center of the next, measured along the pitch diameter at the back faces of the teeth. The addendum is the height of the tooth above the pitch line at the large end. The dedendum (the depth of the tooth space below the pitch line) and the whole depth of the tooth are also measured at the large end.

The pitch cone radius is the distance measured on the pitch line from the vertex of the pitch cone to the outer edge of the teeth. The width of the face of the teeth, as shown in Fig. 9, is measured on a line parallel to the pitch line. The addendum, whole depth and thickness of the teeth at the small or inner end, may be derived from the corresponding dimensions at the outer end, by calculations depending on the ratio of the width of face to the pitch cone radius. (See s, w and t in Fig. 12.)

The addendum angle is the angle between the top of the tooth and the pitch line. The dedendum angle is the angle between the bottom of the tooth space and the pitch line. The face angle is the angle between the top of the tooth and a perpendicular to the axis of the gear. The edge angle (which equals the pitch cone angle) is the angle between the outer edge and the perpendicular to the axis of the gear. The latter two angles are measured from the perpendicular instead of from the axis, for the convenience of the workman in making measurements with the protractor when turning the blanks. The cutting angle is the angle between the bottom of the tooth space and the axis of the gear.

The angular addendum is the height of tooth at the large end above the pitch diameter, measured in a direction perpendicular to the axis of the gear. The outside diameter is measured over the corners of the teeth at the large end. The vertex distance is the distance measured in the direction of the axis of the gear from the corner of the teeth at the large end to the vertex of the pitch cone. The vertex distance at the small end of the tooth is similarly measured.

The shape of the teeth of a bevel gear may be considered as being the same as for teeth in a spur gear of the same pitch and

style of tooth, having a radius equal to the distance from the pitch line at the back edge of the tooth to the axis of the gear, measured in a direction perpendicular to the pitch line. This distance is dimensioned $\frac{D'}{2}$ in Fig. 12. The number of teeth which such a spur gear would have, as determined by diameter D' thus obtained, may be called the "number of teeth in equivalent spur gear," and is used in selecting the cutter for forming the teeth of bevel gears by the formed cutter process.

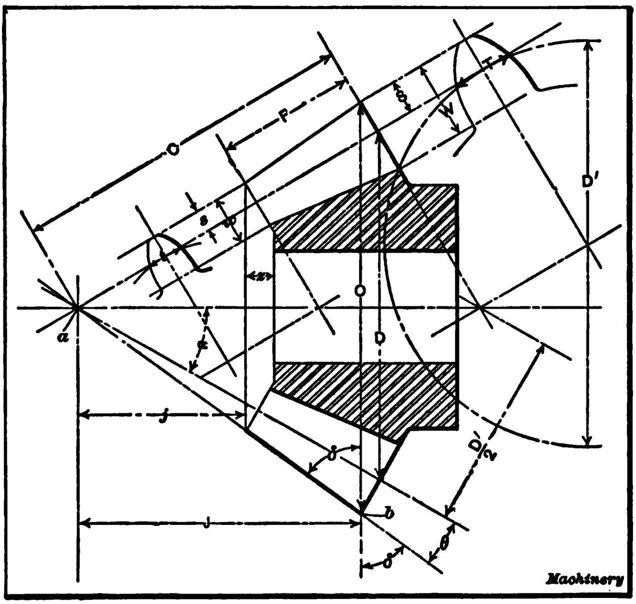


Fig. 12. Diagram used in Connection with Derivation of Formulas for Bevel Gears

In two special forms of gears, the crown gear, Fig. 10, and the internal bevel gear, Fig. 11, the same dimensions and definitions apply as in regular bevel gears, though in a modified form in some cases. In the crown gear, for instance, the pitch diameter and the outside diameter are the same, and the pitch cone radius is equal to ½ the pitch diameter. The addendum angle and the face angle are also the same. The angular addendum becomes zero, and the vertex distance is equal to the adden-

dum. The number of teeth in the equivalent spur gear becomes infinite, or in other words, the teeth are shaped like those of a rack.

When the pitch cone angle is greater than 90 degrees, so that the gear becomes an internal bevel gear, as in Fig. 11, the outside diameter (or edge diameter as it is better called in the case of internal gears) becomes less than the pitch diameter. Otherwise the conditions are the same although many of the dimensions are reversed in direction.

Rules and Formulas. — Rules and formulas for calculating the dimensions of bevel gears are given on the following pages. In these formulas the reference letters below are used:

N = number of teeth;
P = diametral pitch;
P' = circular pitch;
π = 3.1416;
α = pitch cone angle and edge angle;
γ = center angle;
D = pitch diameter;
S = addendum;
$S + A$ = dedendum (A = clearance);
W = whole depth of tooth space;
T = thickness of tooth at pitch line;
C = pitch cone radius;
F = width of face;
s = addendum at small end of tooth;
t = thickness of tooth at pitch line at small end;
θ = addendum angle;
ϕ = dedendum angle;
δ = face angle;
ζ = cutting angle;
K = angular addendum;
O = outside diameter (edge diameter for internal gears);
J = vertex distance;
j = vertex distance at small end;
N' = number of teeth in equivalent spur gear.

Sub$_p$ refers to dimensions applying to pinion (α_p, N_p, etc.).

Sub$_g$ refers to dimensions applying to gear (α_g, N_g, etc.).

It will be noted that directions for the use of these rules are given for each of the six cases of right-angle bevel gearing, miter bevel gearing, acute-angle and obtuse-angle bevel gearing, and crown and internal bevel gears. Examples are given on subsequent pages which show, in detail, the use of the rules and formulas.

Derivation of Formulas for Bevel Gear Calculations. — The derivation of most of these formulas is evident on inspection of Figs. 1 to 12 inclusive, for anyone who has a knowledge of elementary trigonometry. It is not necessary to know how they

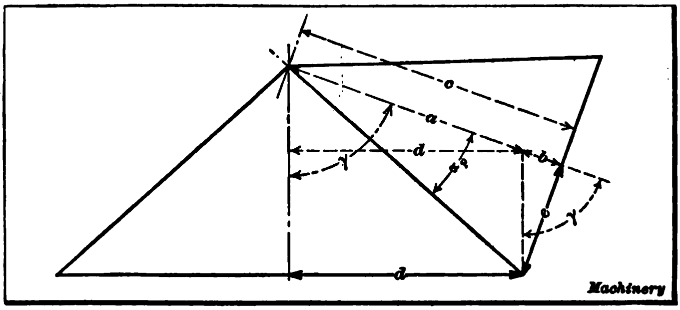


Fig. 13. Diagram for Obtaining Pitch Cone Angle of Acute-angle Bevel Gearing

were derived in order to use them, as all that is needed is the ability to read a table of sines and tangents.

Formulas (5), (6), (7) and (8) are the same as for Brown & Sharpe standard gears. The dimensions at the small end of the tooth given by Formulas (10), (11) and (19) obviously are to the corresponding dimensions at the large end, as the distance from the small end of the tooth to the vertex of the pitch cone is to the pitch cone radius. This relation is expressed by these formulas. The derivation of Formula (20) may be understood by reference to Fig. 12:

$$D' = \frac{D}{\cos \alpha} = \frac{N}{P \times \cos \alpha}, \quad \text{also} \quad D' = \frac{N'}{P}$$

therefore $$\frac{N'}{P} = \frac{N}{P \times \cos \alpha}, \quad \text{or} \quad N' = \frac{N}{\cos \alpha}$$

Formula (21) for checking the calculations will also be understood from Fig. 12, where it will be seen that

$$O = 2\,ab \times \cos\delta, \text{ and that } ab = \frac{C}{\cos\theta}$$

Therefore, $$O = \frac{2\,C \times \cos\delta}{\cos\theta}$$

Formulas (22) to (27) inclusive are simply the corresponding Formulas (1), (9), (14), (15), (16) and (20) when $\alpha = 45$ degrees.

Formula (28) is derived as shown in Fig. 13:

$$c = \frac{e}{\tan\alpha_p}, \quad \text{also,} \quad c = a + b = \frac{d}{\sin\gamma} + \frac{e}{\tan\gamma}$$

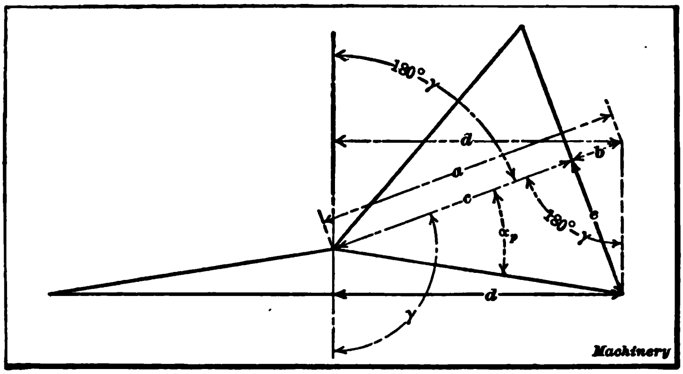


Fig. 14. Diagram for Obtaining Pitch Cone Angle of Obtuse-angle Bevel Gearing

Therefore, $$\frac{e}{\tan\alpha_p} = \frac{d}{\sin\gamma} + \frac{e}{\tan\gamma}$$

Solving for $\tan\alpha_p$, we have: $\tan\alpha_p = \dfrac{e\,(\sin\gamma \times \tan\gamma)}{d\tan\gamma + e\sin\gamma}$

Dividing both numerator and denominator by $e\tan\gamma$, we have:

$$\tan\alpha_p = \frac{\sin\gamma}{\dfrac{d}{e} + \dfrac{\sin\gamma}{\tan\gamma}}$$

Since $d = \dfrac{N_g}{2\,P}$ and $e = \dfrac{N_p}{2\,P}$, and since $\dfrac{\sin}{\tan} = \cos$, we have:

$$\tan\alpha = \frac{\sin\gamma}{\dfrac{N_g}{N_p} + \cos\gamma}$$

Formula (29) is derived by the same process for the other gear. Formulas (31) and (33) are derived from Fig. 14, using the following fundamental equation:

$$\frac{e}{\tan \alpha_p} = \frac{d}{\sin (180^\circ - \gamma)} - \frac{e}{\tan (180^\circ - \gamma)}$$

When solved for $\tan \alpha_p$, this gives Formula (31).

Rule (32), of course, simply expresses the operation of finding whether the pitch cone angle of the gear is less than, equal to, or greater than, 90 degrees. The derivation of Formula (34) is shown in Fig. 15:

$$\sin \alpha_p = \frac{e}{d} = \frac{N_p}{N_g}$$

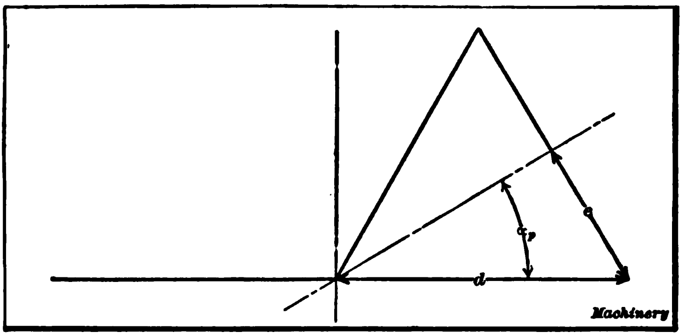
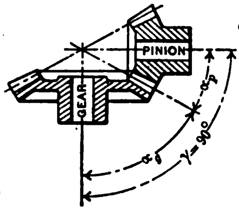


Fig. 15. Diagram for Obtaining Pitch Cone Angle of Pinion to Mesh with Crown Gear

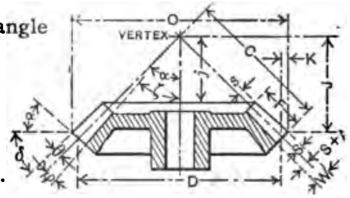
Since in a crown gear the dimension $\frac{D'}{2}$ in Fig. 12 is to be measured parallel to the axis, and will therefore be of infinite length, the form of the teeth will correspond to those of a spur gear having a radius of infinite length, that is to say, to a rack. This accounts for Formula (38).

Formulas (39), (40), (42) and (44) are simply the corresponding Formulas (33), (9), (16) and (20) changed to avoid the use of negative cosines, etc., which occur with angles greater than 90 degrees. These negative functions might possibly confuse readers whose knowledge of trigonometry is elementary. The other formulas for internal gears are readily comprehended from an inspection of Fig. 11.

Rules and Formulas for Calculating Bevel Gears with Shafts at Right Angles



α_p = Pitch cone angle of pinion;
α_g = Pitch cone angle of gear;
N_p = Number of teeth in pinion, etc.



Use Rules and Formulas Nos. 1 to 21 in the order given.

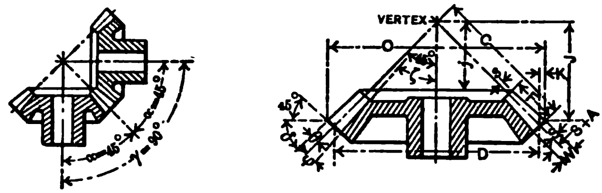
No.	To Find	Rule	Formula
1	Pitch Cone Angle (or Edge Angle) of Pinion.	Divide the number of teeth in the pinion by the number of teeth in the gear to get the tangent.	$\tan \alpha_p = \frac{N_p}{N_g}$
2	Pitch Cone Angle (or Edge Angle) of Gear.	Divide the number of teeth in the gear by the number of teeth in the pinion to get the tangent.	$\tan \alpha_g = \frac{N_g}{N_p}$
3	Proof of Calculations for Pitch Cone Angles.	The sum of the pitch cone angles of the pinion and gear equals 90 degrees.	$\alpha_p + \alpha_g = 90°$
4	Pitch Diameter.	Divide the number of teeth by the diametral pitch; or multiply the number of teeth by the circular pitch and divide by 3.1416.	$D = \frac{N}{P} = \frac{NP'}{\pi}$
5	These dimensions are the same for both gear and pinion. Addendum.	Divide 1.0 by the diametral pitch; or multiply the circular pitch by 0.318.	$S = \frac{1.0}{P} = 0.318\,P'$
6	Dedendum.	Divide 1.157 by the diametral pitch; or multiply the circular pitch by 0.368.	$S + A = \frac{1.157}{P} = 0.368\,P'$
7	Whole Depth of Tooth Space.	Divide 2.157 by the diametral pitch; or multiply the circular pitch by 0.687.	$W = \frac{2.157}{P} = 0.687\,P'$
8	Thickness of Tooth at Pitch Line.	Divide 1.571 by the diametral pitch; or divide the circular pitch by 2.	$T = \frac{1.571}{P} = \frac{P'}{2}$
9	Pitch Cone Radius.	Divide the pitch diameter by twice the sine of the pitch cone angle.	$C = \frac{D}{2 \times \sin \alpha}$
10	Addendum of Small End of Tooth.	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the addendum.	$s = S \times \frac{C - F}{C}$

Rules and Formulas for Calculating Bevel Gears with Shafts at Right Angles

No.	To Find		Rule	Formula
11	These dimensions are the same for both gear and pinion.	Thickness of Tooth at Pitch Line at Small End.	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the thickness of the tooth at the pitch line.	$t = T \times \frac{C-F}{C}$
12		Addendum Angle.	Divide the addendum by the pitch cone radius to get the tangent.	$\tan\theta = \frac{S}{C}$
13		Dedendum Angle.	Divide the dedendum by the pitch cone radius to get the tangent.	$\tan\phi = \frac{S+A}{C}$
14	Face Angle.		Subtract the sum of the pitch cone and addendum angles from 90 degrees.	$\delta = 90° - (\alpha + \theta)$
15	Cutting Angle.*		Subtract the dedendum angle from the pitch cone angle.	$\zeta = \alpha - \phi$
16	Angular Addendum.		Multiply the addendum by the cosine of the pitch cone angle.	$K = S \times \cos\alpha$
17	Outside Diameter.		Add twice the angular addendum to the pitch diameter.	$O = D + 2K$
18	Apex Distance.		Multiply one-half the outside diameter by the tangent of the face angle.	$J = \frac{O}{2} \times \tan\delta$
19	Apex Distance at Small End of Tooth.		Subtract the width of face from the pitch cone radius; divide the remainder by the pitch cone radius and multiply by the apex distance.	$j = J \times \frac{C-F}{C}$
20	Number of Teeth for which to Select Cutter.		Divide the number of teeth by the cosine of the pitch cone angle.	$N' = \frac{N}{\cos\alpha}$
21	Proof of Calculations by Rules Nos. 9, 12, 14, 16 and 17.		The outside diameter equals twice the pitch cone radius multiplied by the cosine of the face angle and divided by the cosine of the addendum angle.	$O = \frac{2C \times \cos\delta}{\cos\theta}$

* See Chapter XIII, paragraph "Cutting Bevel Gears in the Milling Machine."

Rules and Formulas for Calculating Miter Bevel Gearing



Use Rules and Formulas Nos. 22, 4–8, 23, 10–13, 24–26, 17–19, 27 and 21 in the order given. All dimensions thus obtained are the same for both gears of a pair.

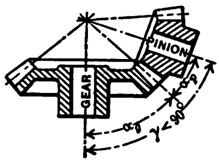
No.	To Find	Rule	Formula
22	Pitch Cone Angle.	Pitch cone angle equals 45 degrees.	$\alpha=45°$
23	Pitch Cone Radius.	Multiply the pitch diameter by 0.707.	$C=0.707\,D$
24	Face Angle.	Subtract the addendum angle from 45°.	$\delta=45°-\theta$
25	Cutting Angle.*	Subtract the dedendum angle from 45°.	$\zeta=45°-\phi$
26	Angular Addendum.	Multiply the addendum by 0.707.	$K=0.707\,S$
27	Number of Teeth for which to Select Cutter.	Multiply the number of teeth by 1.41.	$N'=1.41\,N$

* See Chapter XIII, paragraph "Cutting Bevel Gears in the Milling Machine."

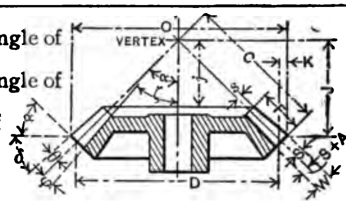
Examples of Bevel Gear Calculations. — A number of examples of calculations are given in the following for practice, covering all the various types shown in Figs. 3 to 8 inclusive. The conditions of the various examples differ from each other only in the center angle. While such great accuracy is not required in the work itself, it will be found convenient in the calculations to use tables of sines and tangents which give readings for minutes to five figures. This permits accurate checking of the various dimensions by Rules and Formulas (3), (21), etc.

Shafts at Right Angles. — Let it be required to make the necessary calculations for a pair of bevel gears in which the shafts are at right angles; diametral pitch = 3, number of teeth in gear = 60, number of teeth in pinion = 15, and width of face = 4 inches.

Rules and Formulas for Calculating Bevel Gears with Shafts at an Acute Angle



α_p = Pitch cone angle of pinion;
α_g = Pitch cone angle of gear;
N_p = Number of teeth in pinion, etc.



Use Rules and Formulas Nos. 28–30, and 4–21 in the order given.

No.	To Find	Rule	Formula
28	Pitch Cone Angle (or Edge Angle) of Pinion.	Divide the sine of the center angle by the sum of the cosine of the center angle and the quotient of number of teeth in the gear divided by the number of teeth in the pinion; this gives the tangent.	$\tan \alpha_p = \dfrac{\sin \gamma}{\dfrac{N_g}{N_p} + \cos \gamma}$
29	Pitch Cone Angle (or Edge Angle) of Gear.	Divide the sine of the center angle by the sum of the cosine of the center angle and the quotient of the number of teeth in the pinion divided by the number of teeth in the gear; this gives the tangent.	$\tan \alpha_g = \dfrac{\sin \gamma}{\dfrac{N_p}{N_g} + \cos \gamma}$
30	Proof of Calculations for Pitch Cone Angles.	The sum of the pitch cone angles of the pinion and gear equals the center angle.	$\alpha_p + \alpha_g = \gamma$

$$\tan \alpha_p = 15 \div 60 = 0.25000 = \tan 14^\circ\, 2' \quad (1)$$

$$\tan \alpha_g = 60 \div 15 = 4.00000 = \tan 75^\circ\, 58' \quad (2)$$

$$\gamma = 14^\circ\, 2' + 75^\circ\, 58' = 90^\circ. \quad (3)$$

$$D_p = 15 \div 3 = 5.000'' \quad (4)$$

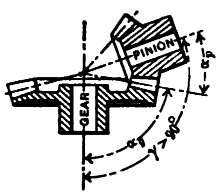
$$S = 1 \div 3 = 0.3333''. \quad (5)$$

$$S + A = \frac{1.157}{3} = 0.3856''. \quad (6)$$

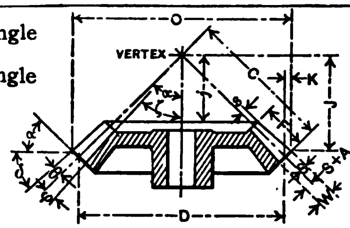
$$W = \frac{2.157}{3} = 0.7190''. \quad (7)$$

$$T = \frac{1.571}{3} = 0.5236''. \quad (8)$$

Rules and Formulas for Calculating Bevel Gears with Shafts at an Obtuse Angle



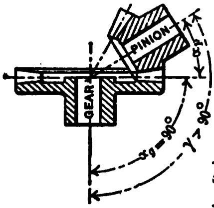
α_p = Pitch cone angle of pinion;
α_g = Pitch cone angle of gear;
N_p = Number of teeth in pinion, etc.



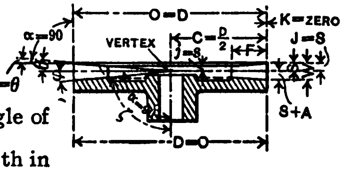
Use Rules and Formulas Nos. 31 and 32 as directed below.

No.	To Find	Rule	Formula
31	Pitch Cone Angle (or Edge Angle) of Pinion.	Divide the sine of 180 degrees minus the center angle by the difference between the quotient of the number of teeth in the gear divided by the number of teeth in the pinion and the cosine of 180 degrees minus the center angle; this gives the tangent.	$\tan \alpha_p = \dfrac{\sin (180° - \gamma)}{\dfrac{N_g}{N_p} - \cos (180° - \gamma)}$
32	Whether Gear is a Regular Bevel Gear, a Crown Gear, or an Internal Bevel Gear.	Add 90 degrees to the pitch cone angle of the pinion. If the sum is greater than the center angle use Rules and Formulas Nos. 33, 30 and 4–21 in the order given. If the sum equals the center angle see rules and formulas for crown gear. If the sum is less than the center angle see rules and formulas for internal bevel gear.	
33	Pitch Cone Angle (or Edge Angle) of Gear.	Divide the sine of 180 degrees minus the center angle by the difference between the quotient of the number of teeth in the pinion divided by the number of teeth in the gear and the cosine of 180 degrees minus the center angle; this gives the tangent.	$\tan \alpha_g = \dfrac{\sin (180° - \gamma)}{\dfrac{N_p}{N_g} - \cos (180° - \gamma)}$

Rules and Formulas for Calculating Crown Gears



α_p = Pitch cone angle of pinion;
N_p = Number of teeth in pinion;
N_g = Number of teeth in gear, etc.



Use Rules Nos. 31 and 4–21 in the order given, for the pinion; use Rules Nos. 30, 4–8, 36, 10–13, 37, 15 and 38 in the order given for the crown gear; if dimensions for crown gear are known, to find center angle and dimensions of pinion, use Rules and Formulas Nos. 34, 35 and 4–21 in the order given.

No.	To Find	Rule	Formula
34	Pitch Cone Angle (or Edge Angle) of Pinion.	Divide the number of teeth in the pinion by the number of teeth in the gear, to get the sine.	$\sin \alpha_p = \frac{N_p}{N_g}$
35	Center Angle.	Add 90 degrees to the pitch cone angle of the pinion.	$\gamma = 90° + \alpha_p$
36	Pitch Cone Radius.	Divide the pitch diameter by 2.	$C = \frac{D}{2}$
37	Face Angle of Gear.	The face cone angle of the gear equals the addendum angle.	$\delta_g = \theta$
38	Number of Teeth for which to Select Cutter.	The teeth are equivalent in form to rack teeth.	$N_g' =$ infinity

$$C = \frac{5}{2 \times 0.24249} = 10.3097'' \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

$$s = 0.3333 \times \frac{6.31}{10.31} = 0.2040'' \quad . \quad . \quad . \quad . \qquad (10)$$

$$t = 0.5236 \times \frac{6.31}{10.31} = 0.3204'' \quad . \quad . \quad . \quad . \qquad (11)$$

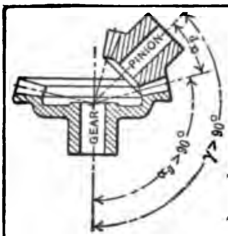
$$\tan \theta = \frac{0.3333}{10.3097} = 0.03233 = \tan 1° \, 51' \quad . \quad . \qquad (12)$$

$$\tan \phi = \frac{0.3856}{10.3097} = 0.03740 = \tan 2° \, 9' \quad . \quad . \quad . \qquad (13)$$

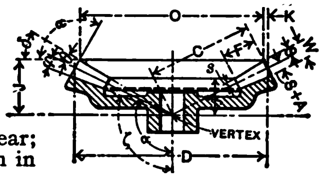
$$\delta = 90° - (14° \, 2' + 1° \, 51') = 74° \, 7' \quad . \quad . \qquad (14)$$

$$\zeta = 14° \, 2' - 2° \, 9' = 11° \, 53' \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

Rules and Formulas for Calculating Internal Bevel Gears



δ_g = Face angle of gear;
N_p = Number of teeth in pinion;
N_g = Number of teeth in gear, etc.



Use Rules and Formulas Nos. 31 and 4–21 inclusive for the pinion; use Rules and Formulas Nos. 39, 30, 40, 41, 15, 42, 43, 18, 19, 44 and 21 in the order given for the gear.

No.	To Find	Rule	Formula
39	Pitch Cone Angle (or Edge Angle) of Gear.	Divide the sine of 180 degrees minus the center angle by the difference between the cosine of 180 degrees minus the center angle and the quotient of the number of teeth in the pinion divided by the number of teeth in the gear; subtract the angle whose tangent is thus found from 180 degrees.	$\tan\alpha_a = \dfrac{\sin(180-\gamma)}{\cos(180-\gamma) - \dfrac{N_p}{N_g}}$ $\alpha_g = 180 - \alpha_a$
40	Pitch Cone Radius.	Divide the pitch diameter by twice the sine of 180 degrees minus the pitch cone angle.	$C = \dfrac{D_g}{2\sin(180-\alpha_g)}$
41	Face Angle of Gear.	Subtract 90 degrees from the sum of the pitch cone angle and the addendum angle.	$\delta_g = \alpha_g + \theta - 90°$
42	Angular Addendum of Gear.	Multiply the addendum by the cosine of 180 degrees minus the pitch cone angle.	$K_g = S \times \cos(180-\alpha_g)$
43	Outside (or Edge) Diameter of Gear.	Subtract twice the angular addendum from the pitch diameter.	$O_g = D_g - 2K_g$
44	Number of Teeth for which to Select Cutter.	Divide the number of teeth by the cosine of 180 degrees minus the pitch cone angle.	$N_g' = \dfrac{N_g}{\cos(180-\alpha_g)}$

$$K = 0.3333 \times 0.97015 = 0.3234'' \quad . \quad . \quad . \quad (16)$$

$$O = 5.000 + 2 \times 0.3234 = 5.6468''. \quad . \quad . \quad (17)$$

$$J = \frac{5.6468}{2} \times 3.51441 = 9.9225'' \quad . \quad . \quad . \quad (18)$$

$$j = 9.9225 \times \frac{6.31}{10.31} = 6.0726'' \quad . \quad . \quad . \quad . \quad (19)$$

$$N' = \frac{15}{0.97015} = 15.4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$5.6468'' \cong \frac{20.6194 \times 0.27368}{0.99948} = 5.6461''. \quad . \quad . \quad (21)$$

This gives all the data required for the pinion. Rules (5) to (13) inclusive apply equally to the gear and the pinion, so we have only calculations by Rules and Formulas (4) and (14) to (21) to make, though it is well to calculate Formula (9) a second time as a check for the same calculation for the pinion.

$$D = \frac{60}{3} = 20.000'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$C = \frac{20}{2 \times 0.97015} = 10.3077''. \quad . \quad . \quad . \quad . \quad (9)$$

$$\delta = 90 - (75^\circ\, 58' + 1^\circ\, 51') = 12^\circ\, 11'. \quad . \quad (14)$$

$$\zeta = 75^\circ\, 58' - 2^\circ\, 9' = 73^\circ\, 49'. \quad . \quad . \quad . \quad . \quad (15)$$

$$K = 0.3333 \times 0.24249 = 0.0808'' \quad . \quad . \quad . \quad (16)$$

$$O = 20 + 2 \times 0.0808 = 20.1616'' \quad . \quad . \quad . \quad (17)$$

$$J = \frac{20.1616}{2} \times 0.2159 = 2.1764'' \quad . \quad . \quad . \quad (18)$$

$$j = 2.1764 \times \frac{6.31}{10.31} = 1.3320'' \quad . \quad . \quad . \quad . \quad (19)$$

$$N' = \frac{60}{0.24249} = 247 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$20.1616'' \cong \frac{20.6154 \times 0.97748}{0.99948} = 20.1615''. \quad . \quad . \quad (21)$$

This gives the calculations necessary for this pair of gears, which are shown dimensioned in Fig. 1, Chapter XI. There are two or three other dimensions, such as the over-all length of the pinion, etc., which depend on arbitrary dimensions given the

gear blank. Directions for calculating these are given in Chapter XI in connection with Fig. 1 of that chapter.

Acute Angle Bevel Gearing. — Let it next be required to calculate the dimensions of a pair of bevel gears whose center angle is 75 degrees, the number of teeth in the pinion 15, the number of teeth in the gear 60, the diametral pitch 3, and the width of face 4 inches. This is the same as the first example, except for the center angle. Following the directions given in the chart we have:

$$\tan \alpha_p = \frac{0.96593}{\frac{60}{15} + 0.25882} = 0.22681 = \tan 12° \, 47'. \quad . \quad (28)$$

$$\tan \alpha_g = \frac{0.96593}{\frac{15}{60} + 0.25882} = 1.89837 = \tan 62° \, 13' \quad . \quad (29)$$

$$\gamma = 12° \, 47' + 62° \, 13' = 75° \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Formulas (4) to (8) as in first example; also, $C = 11.2989''$, $s = 0.2154''$, $t = 0.3382''$, $\theta = 1° \, 41'$, $\phi = 1° \, 57'$, $\delta = 75° \, 32'$, $\zeta = 10° \, 50'$, $K = 0.3251''$, $O = 5.6502''$, $J = 10.9501''$, $j = 7.0748''$, and $N' = 15.3$, also,

$$5.6502'' \cong \frac{22.598 \times 0.24982}{0.99957} = 5.6483'' \quad . \quad . \quad (21)$$

For the gear, the additional calculations give: $C = 11.303''$, $\delta = 26° \, 6'$, $\zeta = 60° \, 16'$, $K = 0.1553''$, $O = 20.3106''$, $J = 4.9748''$, $j = 3.2142''$, $N' = 129$.

$$20.3106'' \cong \frac{22.606 \times 0.89803}{0.99957} = 20.3096'' \quad . \quad . \quad (21)$$

The above calculations are not all given in full, as most of them are merely re-duplications of formulas previously used.

Obtuse Angle Bevel Gearing. — Let it be required to calculate the dimensions of the same set of gears but with the center angle of 100 degrees. This being an example of obtuse angle gearing, we apply Formula (31) as follows:

$$\tan \alpha_p = \frac{0.98481}{\frac{60}{15} - 0.17365} = 0.25738 = \tan 14° \, 26' \quad . \quad (31)$$

and thus discover that it is an example of regular obtuse angle gearing, since

$$14° \, 26' + 90° = 104° \, 26' > 100°. \quad . \quad . \quad . \quad . \quad (32)$$

The remaining calculations for the angles are as follows:

$$\tan \alpha = \frac{0.98481}{\frac{15}{60} - 0.17365} = 12.8986 = \tan 85^\circ\, 34' \quad . \quad . \quad (33)$$

$$\gamma = 14^\circ\, 26' + 85^\circ\, 34' = 100^\circ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

and the calculations for the other dimensions as per the table.

Crown Gear. — Suppose it is required to make a crown gear and a pinion for the same number of teeth, pitch and face as in the previous example. What are the additional calculations necessary? Following the proper formulas in the order given by the chart, we have:

$$\sin \alpha_p = \frac{15}{60} = 0.25000 = \sin 14^\circ\, 29' \quad . \quad . \quad . \quad . \quad (34)$$

$$\gamma = 90^\circ + 14^\circ\, 29' = 104^\circ\, 29' \quad . \quad . \quad . \quad . \quad (35)$$

The other calculations are similar to those already given.

Internal Bevel Gear. — Let it be required to design a pair of bevel gears of the same number of teeth, pitch and face, in which the center angle is 115 degrees. This being an example of obtuse angle gearing, we use Formula (31):

$$\tan \alpha_p = \frac{0.90631}{\frac{60}{15} - 0.42262} = 0.25334 = \tan 14^\circ\, 13' \quad . \quad . \quad (31)$$

Thus, according to Rule (32), we find that

$$14^\circ\, 13' + 90^\circ = 104^\circ\, 13' < 115^\circ \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

showing that the gear is an internal bevel gear. Applying the rules and formulas for internal bevel gearing, we have:

$$\tan \alpha_a = \frac{0.90631}{0.42262 - \frac{15}{60}} = 5.25032 = \tan 79^\circ\, 13'$$

$$180^\circ - 79^\circ\, 13' = 100^\circ\, 47' \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

$$\gamma = 100^\circ\, 47' + 14^\circ\, 13' = 115^\circ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

$$C = \frac{20}{2 \times 0.98234} = 10.1797'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

$$\delta = 100^\circ\, 47' + 1^\circ\, 53' - 90^\circ = 12^\circ\, 40' \quad . \quad . \quad . \quad . \quad . \quad (41)$$

$$\zeta = 98^\circ\, 37', \text{ and } K = 0.0624''$$

$$O = 20 - 2 \times 0.0624 = 19.8752'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

$$N' = \frac{60}{0.1871} = 320 \text{ (internal)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The calculations for the pinion and the other calculations for the gear are similar to those already given.

How to Avoid Internal Bevel Gears. — When Rule (32), in any given case, shows that the large gear will be an internal bevel gear, such as shown in Fig. 16, this construction may be avoided without changing the position of the shafts, the numbers of the teeth in the gear, the pitch of the teeth, or the width of face. This is done simply by subtracting the given center angle from 180 degrees, and using the remainder as a new center angle in calculating a set of acute angle gears by Rules and Formulas (28), (29), (30), etc. A pair of bevel gears calculated on this basis

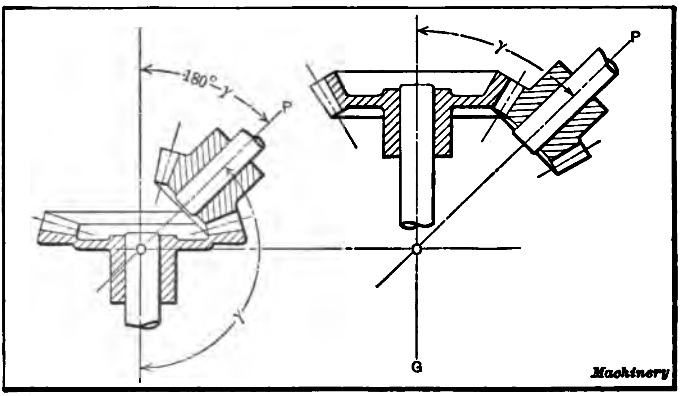


Fig. 16. Internal Bevel Gearing, a Type to be Avoided

Fig. 17. Acute-angle Bevel Gears substituting Internal Gearing

corresponding to those in Fig. 16 are shown in Fig. 17. It will be seen that the contact takes place on the other side of the axis *OP* of the pinion.

It is necessary to avoid internal bevel gears as they can only be used with cast teeth, it being practically impossible to cut them. It may be that some forms of templet planing machines will do this work, if the pitch cone angle is not too great, but no form of generating machine will do it.

Systems of Tooth Outlines used for Bevel Gearing. — Five systems of tooth outlines are commonly used for bevel gearing. They are the cycloid, the standard 14½-degree involute, the 20-degree involute and the 15- and 20-degree octoid.

The cycloidal form of tooth is obsolete for cut bevel gears, and is rarely met with nowadays even for cast gears. It requires very careful workmanship, and is difficult or impossible to generate. It is also a bad shape to form with a relieved cutter, as the cutting edge tends to drag at the pitch line, where for a short distance the sides of the teeth are nearly or quite parallel. For spur gearing it has a few points of advantage over the involute form of tooth, but in the case of bevel gearing these are nullified by the impossibility of generating the teeth in practicable machines. The cycloidal form of tooth need not be seriously considered for bevel gears.

Involute Teeth. — Most bevel gears are made on the involute system, of either the standard $14\frac{1}{2}$-degree pressure angle, or the 20-degree pressure angle. In spur gear teeth the pressure angle may be defined as the angle which the flat surface of the rack tooth makes with the perpendicular to the pitch line. The 20-degree tooth is consequently broader at the base and stronger in form than the $14\frac{1}{2}$-degree tooth. This same difference applies to bevel gears. Most bevel gears that are milled with formed cutters are made to the $14\frac{1}{2}$-degree standard, as cutters for this shape are regularly carried in stock. The planed gears, made by the templet or generating principles, are nowadays often made to the 20-degree pressure angle, both for the sake of obtaining stronger teeth, and for avoiding undercutting of the flanks of the pinions as well. This undercutting is due to the phenomenon of "interference," as it is called, which is minimized by increasing the pressure angle.

Octoid Teeth. — If a manufacturer is asked to plane a pair of involute bevel gears on the Bilgram, Gleason or other similar generating machine, he will not produce involute teeth, but something "just as good." This "just as good" form was invented by Mr. Bilgram, and was named "Octoid" by Mr. Geo. Grant. In generating machines the teeth of the gears are shaped by a tool which represents the side of the tooth of an imaginary crown gear. The cutting edge of the tool is a straight line, since the imaginary crown gear has teeth whose sides are plane surfaces. It can be shown that the teeth of a true involute crown

gear have sides which are very slightly curved. The minute difference between the tooth shapes produced by a plane crown tooth and a slightly curved crown tooth is the minute difference between the octoid and involute forms. Both give theoretically correct action. The customer in ordering gears never uses the word "octoid," as it is not a commercial term; he calls for "involute" gears.

Formed Cutters for Involute Bevel Gear Teeth. — For $14\frac{1}{2}$-degree involute teeth, the shapes of the standard cutter series furnished by the makers of formed gear cutters for bevel gears are commonly used. There are 8 cutters in the series, to cover the full range from the 12-tooth pinion to a crown gear. The various cutters are numbered from 1 to 8, as given in the table below.

Formed Cutters for Involute Bevel Gear Teeth

Number of Cutter	Number of Teeth for which Cutter is Used	Number of Cutter	Number of Teeth for which Cutter is Used
1	From 135 to a Rack	5	From 21 to 25
2	From 55 to 134	6	From 17 to 20
3	From 35 to 54	7	From 14 to 16
4	From 26 to 34	8	From 12 to 13

It should be remembered that the number of teeth in this table refers to the number of teeth in the equivalent spur gear, as given by Rule (20), which should always be used in selecting the cutter used for milling the teeth of bevel gears. The standard bevel gear cutter is made thinner than the standard spur gear cutter, as it must pass through the narrow tooth space at the inner end of the face. As usually kept in stock, these cutters are thin enough for bevel gears in which the width of face is not more than one-third the pitch cone radius. Where the width of face is greater, special cutters have to be made, and the manufacturer should be informed as to the thickness of the tooth space at the small end; this will enable him to make the cutter of the proper width.

Special Forms of Bevel Gear Teeth. — In generating machines (such as the Bilgram and the Gleason) it is often advisable to depart from the standard dimensions of gear teeth as given by Rules and Formulas (1) to (44). For instance, where the pinion is made of bronze and the gear of steel, the teeth of the former can be made wider and those of the latter correspondingly thinner, so as to somewhere nearly equalize the strength of the two. Again, where the pinion has few teeth and the gear many, it may be advisable to make the addendum on the pinion larger and the dedendum correspondingly smaller, reversing this on the gear, making the addendum smaller and the dedendum larger. This is done to avoid interference and consequent undercut on the flanks of pinions having a small number of teeth. Such changes are easily effected on generating machines and instructions for doing this for any case will be furnished by the makers.

CHAPTER XI

STRENGTH AND DESIGN OF BEVEL GEARS

The Materials Used for Making Bevel Gears. — The same materials are used in general for making bevel gears as for spur gears and each has practically the same advantages and disadvantages for both cases. In general, the strength of different materials is roughly proportional to the durability.

Cast iron is used for the largest work, and for smaller work which is not to be subjected to heavy duty. In cases where great working stress or a sudden shock is liable to come on the teeth, steel is ordinarily used. Such gears are made from bar stock for the smallest work, from drop forgings for intermediate sizes made on a manufacturing basis, and from steel castings for heavy work. The softer grades of steel are not fitted for high-speed service, as this material abrades more rapidly than cast iron. This objection does not apply to hardened steels, such as used in automobile transmission gears.

As in the case of spur gearing it is quite common to make the gear and pinion of different materials. This is advantageous from the standpoint of both efficiency and durability, since two dissimilar metals work on each other with less friction than similar metals, as is well known. Cast iron and steel, and steel and bronze are common combinations. In general, the pinion should be made of the stronger material, since it is of weak form; and it should be made of the more durable material, as it revolves more rapidly and each tooth comes into working contact more times per minute than do those of the larger mating gear. In a steel and cast iron combination, then, the pinion should be of steel, while the gear is of cast iron. In a steel and bronze combination, the pinion should be of steel and the gear of bronze, though this is more costly than when the materials are reversed.

A wide range of physical qualities is now available in steel, both for parts small enough to be made from bar stock, and for those made from drop forgings. Recent improvements have also given almost as much flexibility in the choice of steel castings. Gears made from high-grade steels may be subjected to heat-treatments which increase their durability and strength amazingly.

Rawhide and fiber are quite largely used for pinion blanks in cases where it is desired to run gearing at a very high speed and with as little noise as possible. There is a little more difficulty in building up a rawhide blank properly for a bevel gear than for a spur gear. Fiber, which is used in somewhat the same way, has the merit of convenience and comparative inexpensiveness, as it may be purchased in a variety of sizes of bars, rods, tubes, etc., ready to be worked up into pinion blanks at short notice. It is not so strong as rawhide, and is difficult to machine owing to its gritty composition. For light duty at high speed it does very well. For large, high-speed gearing it was formerly a common practice to use inserted wooden teeth on the gear, meshing with a solid cast-iron pinion. This construction is seldom used for cut gearing.

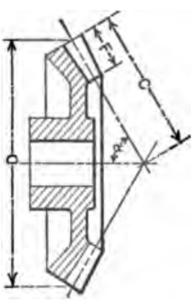
Strength of Bevel Gear Teeth. — The Lewis formula is the one generally used for calculating the strength of gears. Mr. Myers, in an article on the "Strength of Gears" in the December, 1906, number of MACHINERY, gives Mr. Barth's adaptation of this formula for calculating the strength of bevel gears. The rules and formulas in the following are condensed from the method given in the article referred to.

The factors to be taken into account are the pitch diameter of the gear, the number of revolutions per minute, the diametral pitch (or circular pitch as the case may be), the width of face, the pitch cone radius, the number of teeth in the gear and the maximum allowable static fiber stress for the material used. From this may be found the maximum allowable load at the pitch line, and the maximum horsepower the gear should be allowed to transmit.

The reader familiar with the Lewis formula will note that Rule and Formula (3) is the same as for spur gears with the exception

of the additional factor $\frac{C - F}{C}$. This factor is an approximate one which expresses the ratio of the strength of a bevel gear to that of a spur gear of the same pitch and number of teeth, the decrease being due to the fact that the pitch grows finer toward the vertex. This factor is approximate only and should not be used for cases in which F is more than $\frac{1}{3}C$; but since no bevel gears should be made in which F is more than $\frac{1}{3}C$, the rule is of

Factors for Calculating Strength of Bevel Gears



$$N' = \frac{\text{No. of teeth}}{\cos \alpha}$$

Table of Outline Factors (Y) for 14½° and 20° Involute

N'	Outline Factor = Y		N'	Outline Factor = Y	
	14½° Involute (Std.)	20° Involute		14½° Involute (Std.)	20° Involute
12	0.210	0.245	27	0.314	0.349
13	0.220	0.261	30	0.320	0.358
14	0.226	0.276	34	0.327	0.371
15	0.236	0.289	38	0.336	0.383
16	0.242	0.295	43	0.346	0.396
17	0.251	0.302	50	0.352	0.408
18	0.261	0.308	60	0.358	0.421
19	0.273	0.314	75	0.364	0.434
20	0.283	0.320	100	0.371	0.446
21	0.289	0.327	150	0.377	0.459
23	0.295	0.333	300	0.383	0.471
25	0.305	0.339	Rack	0.390	0.484

universal application for good practice. As the width of face is made greater in proportion to the pitch cone radius, the increase of strength obtained thereby grows proportionately smaller and smaller, as may be easily proved by analysis and calculation. Actually the advantage of increasing the width of face is even less than is indicated by calculation, since the unavoidable deflection and disalignment of the shaft is sure at one time or another to throw practically the whole load on the weak inner ends of the teeth, which thus have to carry the load without help from the large pitch at the outer ends.

Rules and Formulas for the Strength of Bevel Gears. — The method for calculating the strength of bevel gearing is practi-

cally the same as that used for spur gears. The accompanying tables of "Rules and Formulas for the Strength of Bevel Gears," and "Factors for Calculating the Strength of Bevel Gears," in combination with the table "Working Stresses used in the Lewis Formula for the Strength of Gear Teeth," in Chapter III,

Rules and Formulas for the Strength of Bevel Gears

D = pitch diameter of gear in inches;
R = revolutions per minute;
V = velocity in ft. per min. at pitch diam.;
S_s = allowable static unit stress for material;
S = allowable unit stress for material at given velocity;
F = width of face;
N' = No. of teeth in equivalent spur gear (see diagram in table on opposite page);
Y = outline factor (see table, opposite page);
P = diametral pitch (if circular pitch is given, divide 3.1416 by circular pitch to obtain diametral pitch);
C = pitch cone radius;
W = maximum safe tangential load in pounds at pitch diam.;
H.P. = maximum safe horsepower.

Use Rules and Formulas Nos. 1 to 4 in the order given.

No.	To Find	Rule	Formula
1	Velocity in Feet per Minute at the Pitch Diameter.	Multiply the product of the diameter in inches and the number of revolutions per minute by 0.262.	$V = 0.262\,DR$
2	Allowable Unit Stress at given Velocity.	Multiply the allowable static stress by 600 and divide the result by the velocity in feet per minute plus 600.	$S = S_s \times \frac{600}{600+V}$
3	Maximum Safe Tangential Load at Pitch Diameter.	Multiply together the allowable stress for the given velocity, the width of face, the tooth outline factor and the difference between the pitch cone radius and the width of face; divide the result by the product of the diametral pitch and the pitch cone radius.	$W = \frac{SFY(C-F)}{PC}$
4	Maximum Safe Horsepower.	Multiply the safe load at the pitch line by the velocity in feet per minute, and divide the result by 33,000.	$\text{H.P.} = \frac{WV}{33{,}000}$

"Strength and Durability of Spur Gearing," give all the necessary information for calculating the strength of bevel gears. The formulas and factors given are based on the use of the diametral pitch of the gears, and constants Y given in the factor table are figured for diametral pitch. If the circular pitch is given, it should be transformed into diametral pitch by dividing 3.1416 by the circular pitch. By means of the Formulas (1) to (4), the horsepower which can be transmitted by a gear of given pitch diameter and diametral pitch, running at a given number of revolutions per minute, can be found. The formulas should be used in the order given. The allowable static unit stress S_s is found from the first line (velocity = 0) in the table of "Working Stresses used in the Lewis Formula for the Strength of Gear Teeth," given in Chapter III. The allowable working stress S at any velocity may also be found directly from the table. An example showing the use of these rules and formulas is given herewith.

Calculate the maximum load at the pitch line which can be safely allowed for the bevel gears in Fig. 1, if the maximum allowable static stress for the pinion is 20,000 pounds, and for the gear, 8,000 pounds per square inch; the pinion runs at 300 revolutions per minute. The calculations for the pinion are as follows:

$$N' = \frac{15}{\cos 14^\circ} = 15.5, \text{ approx.}$$

$$V = 0.262 \times 5 \times 300 = 400 \text{ feet per minute (about)} \quad . \quad . \quad . \quad (1)$$

$$S = 20{,}000 \times \frac{600}{600 + 400} = 12{,}000 \text{ pounds per square inch.} \quad (2)$$

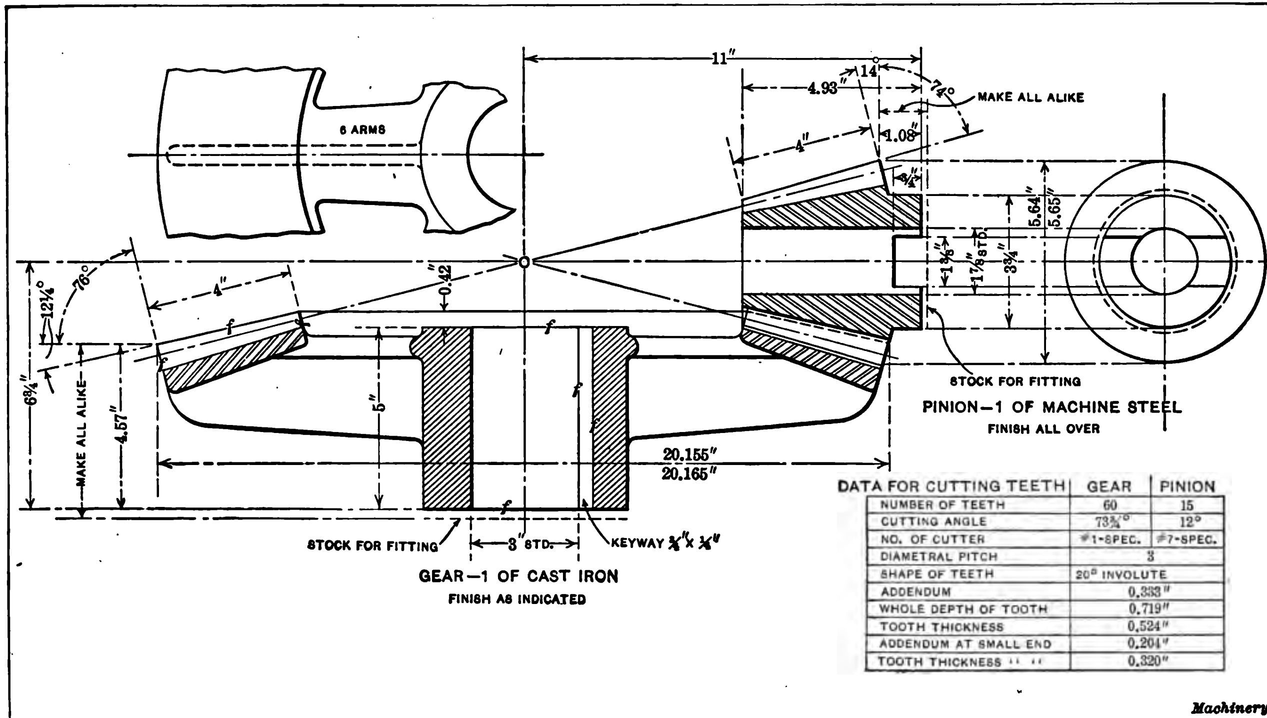
$$W = \frac{12{,}000 \times 4 \times 0.292 \times 6.3}{3 \times 10.3} = 2860 \text{ pounds} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the gear, the velocity is the same as for the pinion. The necessary calculations are as follows:

$$N' = \frac{60}{\cos 76^\circ} = 250, \text{ approx.}$$

$$S = 8000 \times \frac{600}{600 + 400} = 4800 \text{ pounds per square inch} \quad . \quad . \quad (2)$$

$$W = \frac{4800 \times 4 \times 0.467 \times 6.3}{3 \times 10.3} = 1830 \text{ pounds} \quad . \quad . \quad . \quad . \quad . \quad (3)$$



DATA FOR CUTTING TEETH	GEAR	PINION
NUMBER OF TEETH	60	15
CUTTING ANGLE	73¾°	12°
NO. OF CUTTER	#1-SPEC.	#7-SPEC.
DIAMETRAL PITCH	3	
SHAPE OF TEETH	20° INVOLUTE	
ADDENDUM	0.333"	
WHOLE DEPTH OF TOOTH	0.719"	
TOOTH THICKNESS	0.524"	
ADDENDUM AT SMALL END	0.204"	
TOOTH THICKNESS " "	0.320"	

Fig. 1. Model Working Drawing for Bevel Gears to be cut with Formed Milling Cutter

The gear is, therefore, the weaker of the two, and thus limits the allowable tooth pressure. The maximum horsepower this gearing will transmit safely is found as follows:

$$\text{H.P.} = \frac{1830 \times 400}{33{,}000} = 22 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Durability is practically of as much importance as strength in proportioning bevel gears, but unfortunately no information is as yet available for making satisfactory comparisons of durability, so that the usual procedure is to design the gears for strength alone, assuming then that they will not wear out within the lifetime of the machine in which they are used.

Simplified Formulas for the Strength of Bevel Gears. — In Chapter IV, simplified formulas for the strength of gears were given by means of which the circular pitch to transmit a given horsepower at a given speed could be found. The formulas relating to bevel gears, without their derivation, are repeated in the following for the sake of completeness. These formulas are based on the assumption that the pinion (for which the strength usually is calculated) has 15 teeth. If the number of teeth in the pinion is other than 15, multiply the horsepower, as given in the formulas below, by $(0.027\,N + 0.6)$, in which N = number of teeth. In the formulas:

P' = circular pitch;
H.P. = horsepower to be transmitted;
R.P.M. = revolutions per minute of the pinion;
D = pitch diameter of 15-tooth pinion.

	Stress according to the Lewis Tables	Stress ⅔ of that given in the Lewis Tables
Cast-iron Bevel Gear P' =	$\sqrt[5]{\dfrac{5.0\,\text{H.P.}^2}{\text{R.P.M.}}}$	$\sqrt[5]{\dfrac{11.0\,\text{H.P.}^2}{\text{R.P.M.}}}$
Cast-steel Bevel Gear P' =	$\sqrt[5]{\dfrac{0.8\,\text{H.P.}^2}{\text{R.P.M.}}}$	$\sqrt[5]{\dfrac{1.8\,\text{H.P.}^2}{\text{R.P.M.}}}$

$$D = 4.77\,P'.$$

The formula in the column headed "Stress According to the Lewis Tables" is used for ordinary conditions and steady drive.

The column headed "Stress Two-thirds of that given in Lewis Tables" should be used for gears subjected to rapidly varying loads or where the drive is often started and stopped.

Bearing Pressures Due to the Action of Bevel Gears Under Load. — The action of a pair of bevel gears under load produces a radial pressure on the bearings due to the tendency of the driver to climb onto the driven gear. The pressure angle of the involute teeth also produces a force tending to push the gears out of engagement. This angular thrust may be resolved into a radial bearing pressure and a direct end thrust on the shaft.

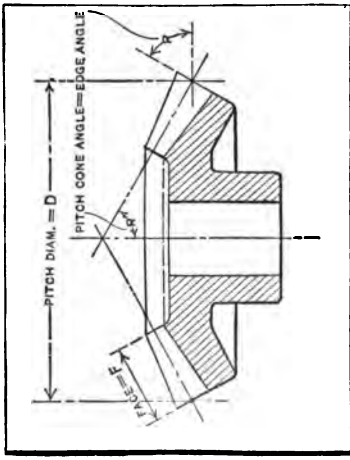


Fig. 2. Gear Dimensions required to determine Radial Pressure and End Thrust

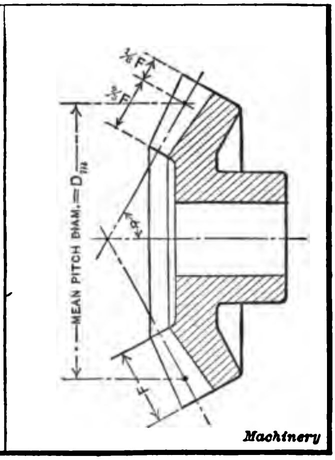


Fig. 3. Mean Pitch Diameter defined for Purposes of Calculation

The amount of radial bearing pressure and end thrust due to a given load on a given gear may be calculated by the rules and formulas presented in the following. We must have the following data:

1. The pitch diameter, width of face, and edge angle of the gear; the edge angle is the same as the pitch cone angle. These dimensions are shown in Fig. 2.

2. The horsepower and revolutions per minute, or the torque in inch-pounds to which the gear will be subjected.

3. The pressure angle of the teeth; that is, whether they are $14\frac{1}{2}$-degree standard teeth, or 20-degree, or other shape. It

makes no difference whether the teeth are of standard length or of the "stub-tooth" variety.

We will evidently get different driving pressures on the teeth, depending on whether we use the pitch diameter at the large end or at the small end of the teeth for our calculations. We should evidently use a diameter somewhere between these two to get correct results. An accurate calculation would require consideration of the elasticity of the tooth, which would be somewhat complicated. It will, however, be accurate enough for practical purposes to locate our mean pitch circle at one-third the width of face from the large end of the tooth. The diameter thus located, as shown in Fig. 3, may be considered the mean pitch diameter for the purposes of this calculation. This mean pitch diameter may be obtained either by measuring an accurate drawing or by the following calculation:

Rule 1. To find the mean pitch diameter, multiply two-thirds the width of face by the sine of the edge angle, and subtract the product from the pitch diameter.

For the tangential tooth pressure we have:

Rule 2. To find the tangential tooth pressure (which equals the direct radial pressure on the bearing), divide the torque in inch-pounds by one-half the mean pitch diameter; or,

Rule 3. Multiply the horsepower transmitted by 126,050, and divide by the product of the revolutions per minute and the mean pitch diameter.

The next step is to find the thrust due to the tooth pressure and the angularity of the gear in the direction $x-x$ in Fig. 4. This may be done by the regular parallelogram of forces, or by calculation as follows:

Rule 4. To find the angular thrust of the gear, multiply the tangential tooth pressure by the tangent of the pressure angle of the tooth. (Note: For a $14\frac{1}{2}$-degree tooth the tangent is 0.2586, for a 15-degree, 0.2680; for 20 degrees, 0.3640; for $22\frac{1}{2}$ degrees, 0.4142).

This angular thrust, as shown in Fig. 4, may be resolved into two components, one of which is the direct thrust on the shaft (which may be taken care of by a thrust bearing), and the other

an additional radial pressure on the bearing. These quantities may be derived by the parallelogram of forces or by the following calculation:

Rule 5. To find the direct thrust on the shaft, multiply the angular thrust by the sine of the edge angle.

Rule 6. To find the additional bearing pressure due to the angular thrust, multiply the angular thrust by the cosine of the edge angle.

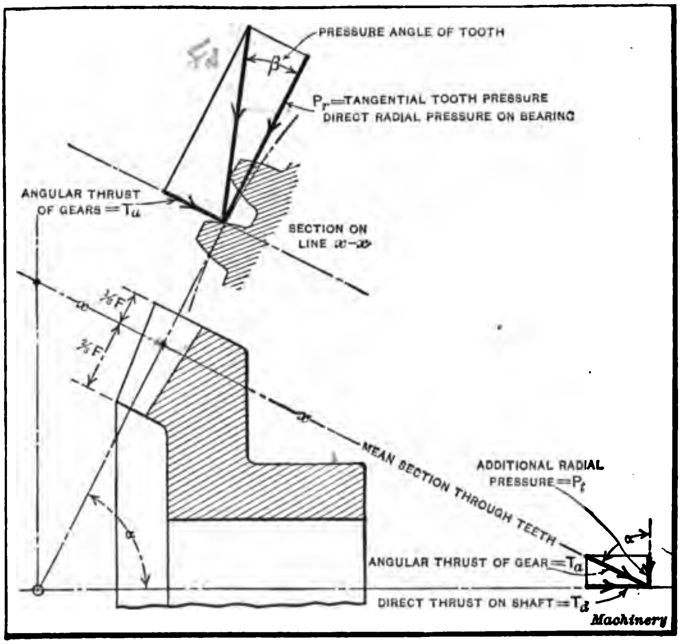


Fig. 4. Finding Direct Thrust and Additional Radial Bearing Pressure

It is evident from Fig. 5 that the bearing pressure due to the angularity of the teeth is at right angles to that due to the tangential tooth pressure carrying the load. The resultant total pressure on the bearings may be obtained by the parallelogram of forces, or by calculation as follows:

Rule 7. To find the total radial pressure on the bearings, add together the squares of the tangential tooth pressure and the additional radial thrust on the bearing due to the angular thrust, and extract the square root of the sum.

Formulas Embodying the Rules Given. — The foregoing rules may be expressed in the following formulas:

$$D_m = D - (\tfrac{2}{3} F \times \sin \alpha) \quad (1)$$

$$P_r = \frac{2M}{D_m} \quad (2)$$

$$P_r = \frac{126{,}050 \text{H.P.}}{RD_m} \quad (3)$$

$$T_a = P_r \times \tan \beta \quad (4)$$

$$T_d = T_a \times \sin \alpha \quad (5)$$

$$P_t = T_a \times \cos \alpha \quad (6)$$

$$P_b = \sqrt{P_r^2 + P_t^2} \quad (7)$$

in which,

D = pitch diameter;
F = width of face;
α = edge angle (same as pitch cone angle);
D_m = mean pitch diameter;
M = torque or turning moment in inch-pounds;
R = revolutions per minute;
P_r = direct radial pressure on bearing = tangential tooth pressure;
β = pressure angle of involute tooth;
T_a = angular thrust of gears;
T_d = direct thrust on shaft due to angular thrust of gears;
P_t = additional radial pressure on bearings due to angular thrust;
P_b = total radial pressure on bearing;
H.P. = horsepower transmitted.

Example of Calculation of Thrust. — As an example, take the case of the gear shown in Figs. 4 and 5, the dimensions of which are as follows: Pitch diameter = 6 inches; face = $1\frac{1}{8}$ inch; edge angle = 63 degrees 26 minutes; horsepower = 19, or torque = 1500 inch-pounds at 800 R.P.M.; pressure angle of tooth = 20 degrees.

The radial pressure and thrust on the bearing are found by calculation from the given rules and formulas, as follows.

Graphical calculations giving the same results by the parallelogram of forces are indicated in Figs. 4 and 5.

$$D_m = 6 - \tfrac{3}{4} \times 0.894 = 5.330 \text{ inches.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$P_r = \frac{3000}{5.33} = 560 \text{ pounds, about.} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$P_r = \frac{126,050 \times 19}{800 \times 5.33} = 560 \text{ pounds, about.} \quad . \quad . \quad (3)$$

$$T_a = 560 \times 0.364 = 204 \text{ pounds.} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$T_d = 204 \times 0.894 = 182 \text{ pounds.} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$P_t = 204 \times 0.447 = 91 \text{ pounds.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$P_b = \sqrt{560^2 + 91^2} = 567 \text{ pounds.} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

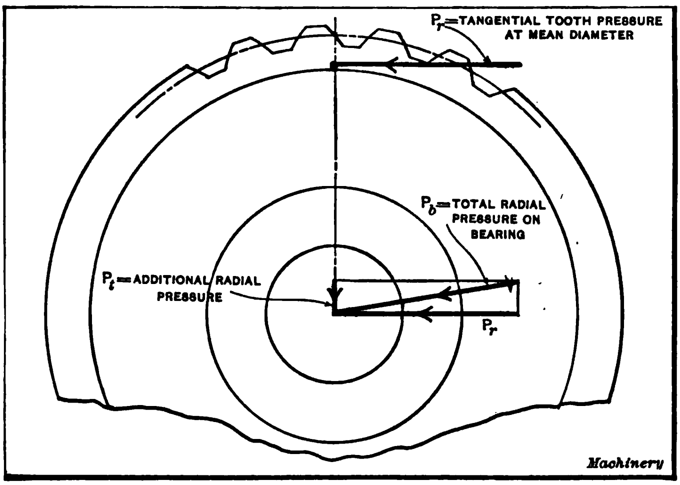


Fig. 5. Finding the Total Radial Pressure on the Bearing

Effect of Pressure Angle. — It will be seen from (7) that the pressure angle of the tooth has a practically negligible effect in increasing the radial bearing pressure. In the case of spur gears, this effect is so small that the angular pressure need not be reckoned with, and in bevel gears the effect is even smaller, since a good share of the angular pressure is transmitted into thrust. It is, therefore, not necessary in practice to carry the calculations beyond the fifth rule or formula.

Total Bearing Pressure. — The total radial bearing pressure thus found is, of course, distributed between the bearings of the

shaft, usually two in number, in accordance with the principle of moments. (See MACHINERY'S Handbook, pages 338 and 339.) The pressure on each bearing resulting from the gear action, as calculated above, must be combined by the parallelogram of forces in the same way as shown in Fig. 5 with any other pressures arising in these bearings from other gears, belt pulleys, brakes or similar loads on the same shaft.

Design of Bevel Gearing. — So far we have dealt with design as relating to calculations. In the following will be discussed

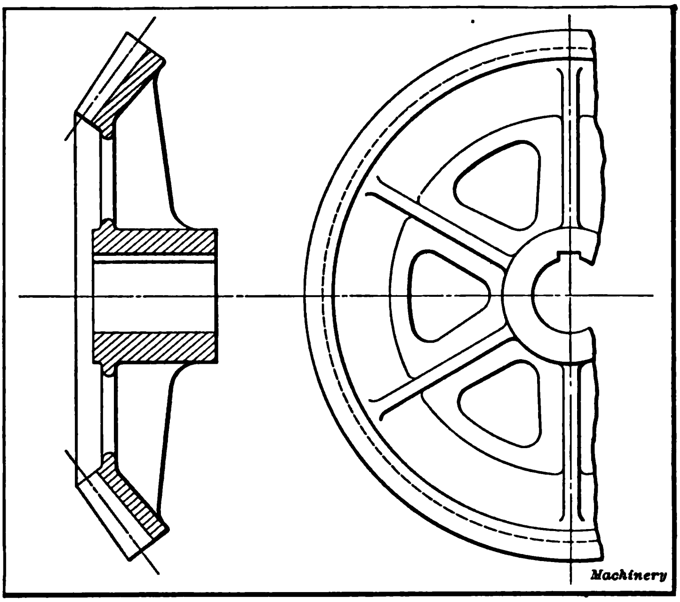


Fig. 6. T-arm Style of Bevel Gear for Heavy Work

the application of the calculated dimensions, the determination of the factors left to the judgment, and the recording of the design in the drawing.

Bevel Gear Blanks. — Various forms may be given to the blanks or wheels on which bevel gear teeth are cut, depending on the size, material, service, etc., to be provided for. The pinion type of blank is shown in Fig. 12, Chapter X, and is used mostly, as indicated by the name, for gears of a small number of teeth and small pitch cone angle. Where the diameter of the bore

comes too near to the bottoms of the teeth at the small end, it is customary to omit the recess indicated by dimension z, and leave the front face of the pinion blank as in the case shown in Fig. 1 of the present chapter.

For gears of a larger number of teeth, the web type shown in Fig. 9, Chapter X, is appropriate. This does not require to be finished all over, as the sides of the web, the outside diameter of the hub, and the under side of the rim may be left rough if desired.

A steel gear suitable for very heavy work is shown in Fig. 6 of the present chapter. Here the web is reinforced by ribs. The web may be cut out so that the rim is supported by T-shaped arms, as shown. This makes a very stiff wheel and at the same time a very light one, when its strength is considered. Where the pitch cone angle is so great that the strengthening rib would be rather narrow at the flange, it may be given the form shown in Fig. 1 in place of that shown in Fig. 6.

General Considerations Relating to Design. — The performance of the most carefully designed and made bevel gears depends to a considerable extent on the design of the machine in which they are used. When the shafts on which a pair of bevel gears are mounted are poorly supported or poorly fitted in their bearings, the pressure of the driving gear on the driven causes it to climb up on the latter, throwing the shafts out of alignment. This in turn causes the teeth to bear with a greater pressure at one end of the face (usually on the outer end) than the other, thus making the tooth more liable to break than is the case where the pressure is more evenly distributed. It is important, therefore, to provide rigid shafts and bearings and careful workmanship for bevel gearing.

The question of alignment of the shafts should be considered in deciding on the width of face of the gear. Making the width of the face more than one-third of the pitch cone radius adds practically nothing to the strength of the gear even theoretically, since the added portion is progressively weaker as the tooth is lengthened, as has been explained. In addition to this, there is the danger that through springing of the shafts or poor work-

manship, the load will be thrown onto the weak end of the tooth, thus fracturing it. For this reason it may be laid down as a definite rule that there is nothing to be gained by making the face of the bevel gear more than one-third of the pitch cone radius.

The Brown & Sharpe gear book gives a rule for the maximum width of face allowable for a given pitch. The width of face should not exceed $2\frac{1}{2}$ times the circular pitch or 8 divided by the diametral pitch. This rule is also rational since the danger to the teeth from the misalignment of the shaft increases both with the width of face and with the decrease of the size of the tooth, so that both of these should be reckoned with. In designing gearing it is well to check the width of face from the rule relating to the pitch cone radius and that relating to the pitch as well, to see that it does not exceed the maximum allowed by either.

Model Bevel Gear Drawing. — It is not enough for the designer to carefully calculate the dimensions of a set of bevel gearing. In addition to this he has the important task of recording these dimensions in such a form that they will be intelligible to an intelligent workman, and will plainly furnish him every point of information needed for the successful completion of the work without further calculation. A drawing which practically fills these requirements is shown in Fig. 1. The arrangement of this drawing and the amount and kind of information shown on it are based on the drafting-room practice of the Brown & Sharpe Mfg. Co., with a few slight changes.

In general, the dimensions necessary for turning the blank have been given on the drawing itself, while those for cutting the teeth are given in tabular form. All the dimensions were calculated from Rules (1) to (21) in Chapter X, and may be checked for practice by the reader. It will be noticed that limits are given for the important dimensions. This should always be done for manufacturing work which is inspected in its course through the shop. It ought to be done even when a single gear is made, as it is exceedingly difficult to properly set a gear if the workman does not work close enough. There is no sense, however, in asking

him to work to thousandths of an inch on blanks like these, so he should be given some notion as to the accuracy required by limits such as shown.

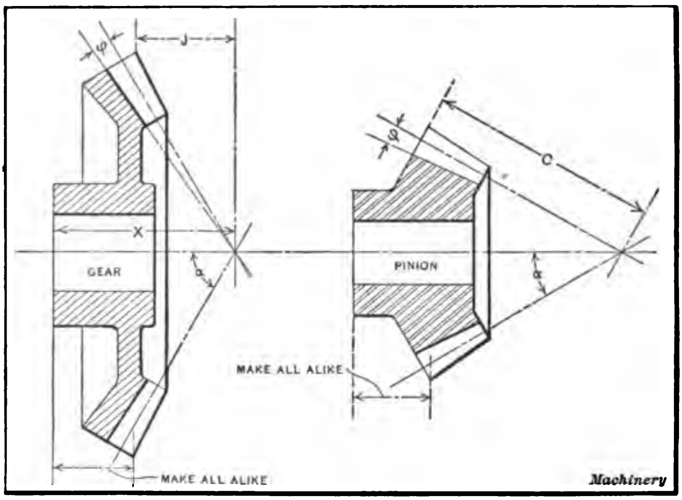
It is assumed that the gears are to be cut with rotary cutters. It is unusual to do this with a pitch as coarse as this, though there are machines on the market capable of handling such work. In gear-cutting machines using form cutters, the blanks are located for axial position by the rear face of the hub. It is necessary also to leave stock at this place for fitting the gears in the machine. It will be seen that the dimension for bevel gears and pinions from the outside edge of the blank to the rear face of the hub is marked "Make all alike." This means that the same amount of stock should be left on all the gears in a given lot so that after the machine is set for one of them, it will not be necessary to alter the adjustment for the remainder.

There are one or two dimensions which are not given directly by Rules (1) to (21). One of these is the distance 4.57 inches from the outside edge of the teeth to the finished rear face of the hub of the gear. This dimension is commonly scaled from an accurate drawing, but it may be calculated by subtracting the vertex distance from the distance between the pitch cone vertex O and the rear face of the hub. This gives $6\frac{3}{4} - 2.1764$ equals 4.57 inches (about) as dimensioned. Another dimension not directly calculated is the over-all length of the pinion. This may be obtained by subtracting the vertex distance at the small end (marked j in illustrations with rules and formulas in Chapter X) from the distance between the vertex and the rear face of the hub, giving 4.93 inches as shown.

In the tabular dimensions for cutting the teeth, most of the figures are self-explanatory. The fact that in this particular case a 20-degree form of tooth has been adopted to avoid the undercut in small pinions (see Chapter X, section on "Systems of Tooth Outlines used for Bevel Gearing") is indicated in the table.

The number of cutter is selected from the table "Formed Cutters for Involute Bevel Gear Teeth," Chapter X, in accordance with the number of teeth (N') in equivalent spur gear, as deter-

mined by Rule (20) in the formulas in the same chapter. This is 15.4 for the pinion, and 247 for the gear, giving a No. 1 and No. 7 cutter respectively. These cutters are marked special, owing to the fact that they are 20 degrees involute instead of $14\frac{1}{2}$ degrees. They would be special under any circumstances, however, since the width of face for these gears (4 inches) is more than $\frac{1}{3}$ the pitch cone radius, which figures out to 10.3097 inches. Standard bevel gear cutters are only made thin enough to pass through the teeth at the small end when the width of face is not



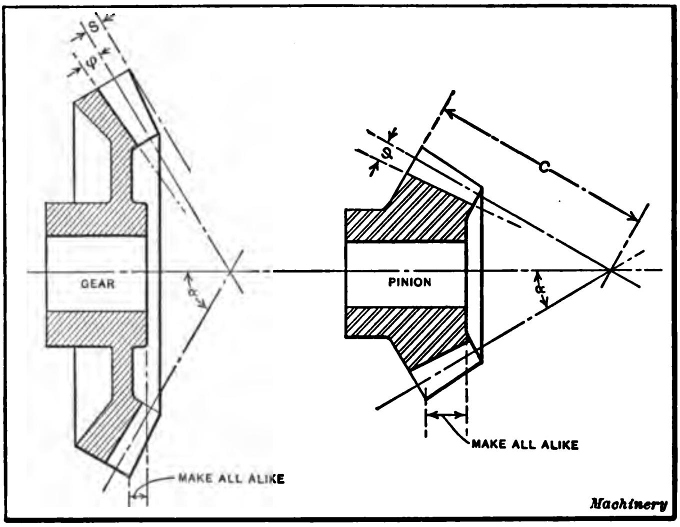
Figs. 7 and 8. Additional Dimensions for Gears to be cut by the Templet Planing Process or the Gleason Generating Machine

more than $\frac{1}{3}$ the pitch cone radius. For this reason cutters thinner than the standard would have to be used.

In bevel pinions of the usual form, such as shown in Fig. 12, Chapter X, dimension *z* there given has to be furnished. This may be scaled from a carefully made drawing, or may be calculated by subtracting the length of the bore of the pinion from the over-all length, the latter being obtained as described for the pinion in Fig. 1. Such dimensions do not need to be given in thousandths of an inch on moderately large work. It is also not necessary to give the angles any closer than the quarter degree, as few machines are furnished with graduations which

can be read finer than this. In order to check the calculations carefully, however, it is wise, as previously described, to make them with considerable accuracy, using tables of sines and tangents which read to five figures. After the dimensions are calculated, they may be put in more approximate form for the drawing.

The gear drawing in Fig. 1 is dimensioned more fully, perhaps, than is customary, especially in shops having a large gear-cutting department, where the foreman and operators are experienced and have access to tables and records of data for bevel gear cut-



Figs. 9 and 10. Additional Dimensions for Gears to be cut on the Bilgram Generating Machine

ting. Every dimension given is useful, however, and it is a good plan to include them all, especially on large work.

Dimensioning Drawings for Gears with Planed Teeth. — The machine on which the teeth of a gear are to be cut determines to some extent the dimensions which the workman needs, so this should be taken into account in making the drawing. For gears which are to be cut on a templet planing machine, the dimensions given in Fig. 1 may be followed in general. Further dimensions are needed, however, to set the blank so that the vertex of the pitch cone corresponds with the central axis of the machine. For

gears with pitch cone angle greater than 45 degrees, this may be obtained from dimension X, as given in Fig. 7, or, better, from dimension J. For gears smaller than 45 degrees, C (Fig. 8) may be given.

There are two commercial forms of gear generating machines in general use in this country for planing the teeth of bevel gears. These are the Gleason and Bilgram machines. Since the methods of supporting the gears are different, the drawings should be dimensioned to suit, if it is known beforehand how they are to be cut. For the Gleason machine the dimensioning shown in Figs. 7 and 8 should be given, in addition to that shown in Fig. 1. The angles α and ϕ, the pitch cone angle and dedendum angle, respectively, may well be put in the table of dimensions instead of on the drawing. The distance from the outside corner of the teeth to the rear face of the hub should be made alike for all similar gears in the lot, the same as for gears which are to be cut by the form cutters or the templet process. The cutting angle may be omitted from the drawing.

The method of dimensioning for the Bilgram gear planer is shown in Figs. 9 and 10. Angles α and ϕ should be given in the table as before. Dimension S is used for setting on gears of large pitch cone angle, and dimension C or the pitch cone radius for those of small pitch cone angle (less than 45 degrees). It is a good idea to give both of these dimensions for both gear and pinion, so that the setting may be checked by two different methods. In this machine the dimension to be marked "Make all alike" should be given as shown.

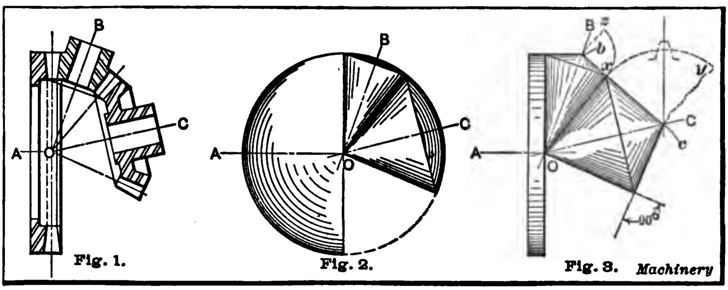
CHAPTER XII

METHODS FOR FORMING THE TEETH OF BEVEL GEARS

It will be impossible in the comparatively short space we are able to devote to the subject of forming the teeth in bevel gears to do more than give the bare outlines of the ingenious mechanisms which have been devised for this work. Almost any one of the methods to be described would require several pages and many illustrations to explain the details of its application, and special books devoted to this subject alone are available. (See "Gear Cutting Machinery," by R. E. Flanders.) We can, however, in the comparatively short descriptions here given, get an understanding of the principles of operation of each method.

Comparison between Spur and Bevel Gear Cutting. — The changes required in spur gear cutting devices to adapt them for cutting bevel gears, made necessary by the difference in the nature of the two forms of gearing, are illustrated in Figs. 1, 2 and 3. The action of a pair of mating spur gears may be seen and studied on the plane perpendicular to their axes. To be understood correctly, the action of bevel gearing must be observed on a spherical surface. In Fig. 1 are shown three bevel gears with axes *OA*, *OB* and *OC*. The bevel gear on axis *OA* is of the form known as the "crown gear." It is practically a rack bent in a circle about center *O*. Pinion *OB* and gear *OC* are familiar types of bevel gears. In Fig. 2 are shown the pitch surfaces of the gears in the preceding figure. It will be seen in that figure that the pitch lines of the gear on the axis *OC*, for instance, converge at the center *O*. These pitch lines represent a conical pitch surface which is shown cut out from a sphere on axis *OC* in Fig. 2. In a similar way the cone about axis *OB* represents the pitch surface of the pinion in Fig. 1, while the plane face of the hemisphere at the left in Fig. 2 is the pitch surface of the

crown gear of the preceding figure. If we wish to draw accurate representations of the teeth of the bevel gears in Fig. 1, in order to study their action in the same way that we can when drawing the teeth of spur gears on the plane surface of the drawing-board, we would have to draw them on surfaces of the sphere from which the pitch cones in Fig. 2 are cut. The pitch circles, etc., of the various gears would be struck from centers located at the points where the various axes OA, OB and OC break through the surface of the sphere. Except for the different surfaces on which the drawing would be done, the procedure would be identical with that for spur gears. It should be noted that straight lines on spherical surfaces are represented by great circles — that is to say, by the intersection with the surface of planes passing through the center of the sphere.



Figs. 1, 2 and 3. Illustrating the Spherical Basis of the Bevel Gear and Tredgold's Approximation for developing the Outlines of the Teeth on a Plane Surface

Tredgold's Approximation. — Owing to the impracticability of the sphere as a drawing-board, a process known as "Tredgold's" is usually followed for approximately laying out the teeth of bevel gears. This is shown in Fig. 3 applied to the same case as in the two preceding figures. The conical pitch surfaces vanishing at the center O are identical with those in Fig. 2, as is also the plane circular face of the crown gear. For the bevel gear and pinion, however, the teeth are supposed to be drawn and the action studied on surfaces of cones complementary to the pitch cones — that is, on the cones with apexes at c and b. The surface of these cones can be developed on a flat piece of paper, as shown for that on axis OC in Fig. 3, in which case the pitch line becomes xy, as

illustrated. Teeth drawn on this pitch line, as for a spur gear, may be laid out on the conical surface and used as the outlines of bevel gear teeth. The difference in the shape of tooth obtained under the same system as the two methods shown in Figs. 2 and 3, is so slight as to be negligible, except perhaps, in gears having very few teeth. Whatever the method pursued for laying out or studying the action, all the elements of which the teeth are formed consist of straight lines which meet at the center *O* of the pitch cones; consequently the teeth grow small toward the inner end, vanishing at the center if they are carried that far.

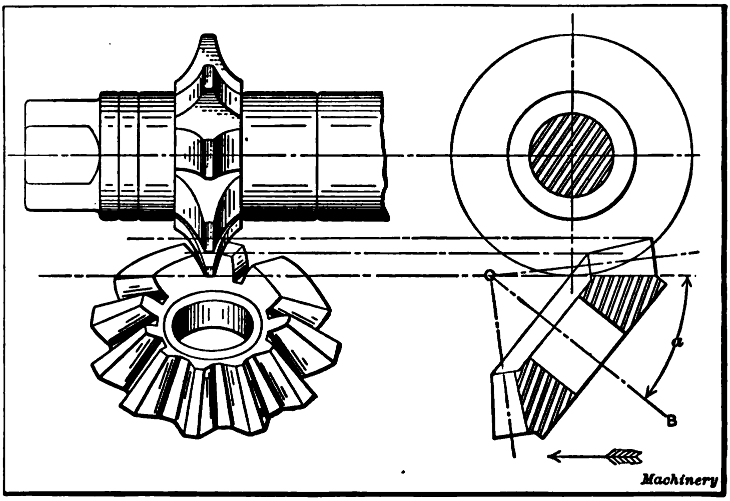


Fig. 4. Shaping the Teeth of a Bevel Gear by the Formed Cutter Process

Five Principles of Action. — All of the five principles of action on which spur gear teeth may be formed (the formed tool, the templet, the odontographic, the describing-generating and the molding-generating principles) may be also applied to the cutting of bevel gears, although the describing-generating principle has never been so used.

The Formed Tool Principle. — The application of this principle is illustrated in Fig. 4, where a form cutter is shown shaping one side of the tooth of a bevel gear. The gear blank is tipped up to cutting angle *a* and fed beneath the cutter in the direction of the arrow. It will be immediately seen from an examination of the

figure that the form tool process is by necessity approximate. It is evident that the right-hand side of the cutter is reproducing its own unchanging outline along the whole length of the face of the tooth at the right. This form should not be unchanging for, as has been explained, the teeth and the space between them grow smaller towards the apex of the pitch cone, where they finally vanish, so it is evident that the outline of a tooth at the small end should be the same as that at the large end, but on a smaller scale, and not a portion of the exact outline at the large end, as produced by the formed tool process and as shown in the figure. To make this error as small as possible, it is customary to use a cutter which gives the proper shape at the large end,

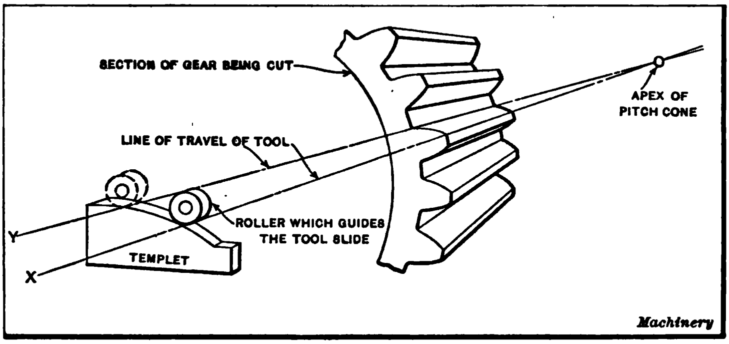


Fig. 5. The Templet Principle for Forming the Teeth of Bevel Gears

and set the blank so that the tooth is cut to the proper thickness at the small end. This leaves the top of the tooth at the small end too thick, an error which is often remedied by filing. Of course, the principle is the same with the formed planer or shaper tool as with the formed milling cutter, and the errors involved in the process are also identical. It is evident that but one side of the tooth space can be cut at a time, so that at least two cuts around will have to be taken. The method, as applied in practice, will be described in detail in the next chapter.

The Templet Principle. — This principle is illustrated in Fig. 5, in skeleton form only. A former is used which has the same outline as would the tooth of the gear being cut, if the latter were extended as far from the apex of the pitch cone as the position

in which the former is placed. The tool is carried by a slide which reciprocates it back and forth along the length of the tooth in a line of direction (OX, OY, etc.), which passes through the apex O of the pitch cone. This slide may be swiveled in any direction and in any plane about this apex, and its outer end is supported by the roller on the former. With this arrangement, in the case shown, as the slide is swiveled inward about the apex, the roll runs up on the former, raising the slide and the tool so as to reproduce on the proper scale the outline of the former on the tooth being cut. Since the movement of the tool is always toward the apex of the pitch cone, the tooth elements vanish

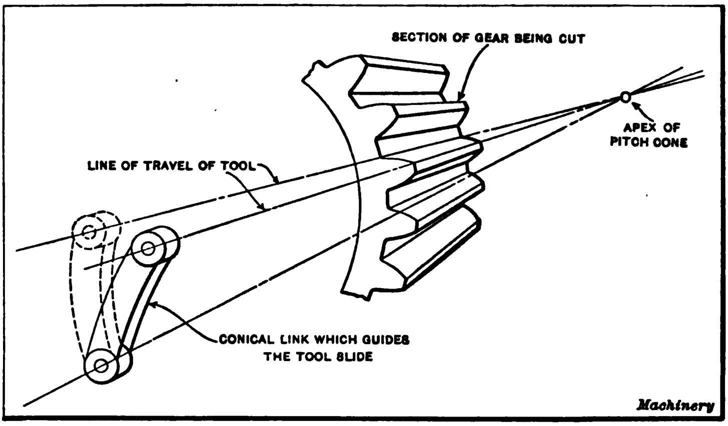


Fig. 6. The Odontographic Principle for Forming the Teeth of Bevel Gears

at this point, and the outlines are similar at all sections of the tooth, although with a gradually decreasing scale as the apex is approached — all as required for correct bevel gearing.

The arrangement thus shown diagrammatically is modified in various ways in different machines, but the movement imparted to the tool in relation to the work is the same in all cases where the templet principle is employed, no matter what the connection between the former and the tool may be.

The Odontographic Principle. — As explained for spur gears in Fig. 3, Chapter VIII, it is often possible to approximate the exact curves required for the teeth of gears by mechanisms which make use of circular arcs or other easily generated curves. In

Fig. 6, of the present chapter, is shown in diagrammatic form an arrangement for obtaining, by means of link work, a close approximation to the exact form of an involute outline, such as might be produced by the templet in Fig. 5, for instance. This true involute outline may be very closely approximated by a circle drawn on the surface of a sphere. To give this required circular movement to the point of the tool, the slide on which the tool reciprocates may be constrained by a link as shown, pivoted at the base to the frame of the machine, and at the upper end to the slide. The axes of these pivots should pass through the apex of the pitch cone, as required by the spherical nature of the bevel gear. This

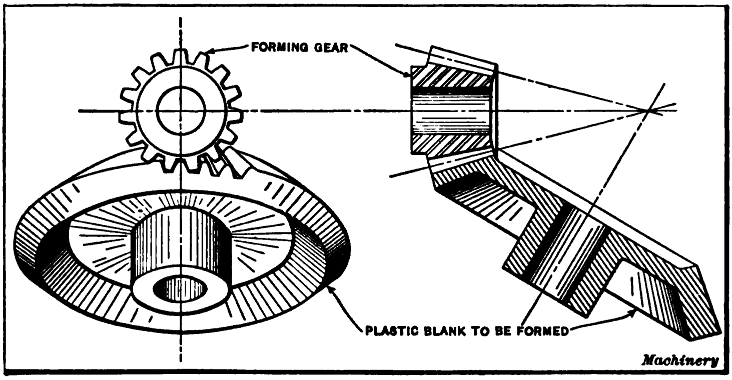


Fig. 7. The Impression Operation applied to Forming the Teeth of a Bevel Gear by the Molding-generating Process

link work (which is thus of the "conical" type) if properly proportioned and located, will guide the tool slide and the tool point in very nearly the same way as a properly constructed templet, used as shown in Fig. 5.

The Molding-generating Principle. — The counterpart of the spur gear process shown in Fig. 5, Chapter VIII, is illustrated for the bevel gears in Fig. 7. Here a correctly formed gear is being rotated in the proper position and in the proper ratio with a plastic blank. This operation, as in the case of the spur gear, forms teeth in the plastic blank which are properly shaped to mesh with the forming gear or with any other gear of the same series. Fig. 6, Chapter VIII, has no possible counterpart in the cutting of bevel gears.

To fully understand the principle outlined in the previous paragraph, suppose we have a bevel gear blank made of some plastic material, such as clay or putty. By transposing Formula (34), Chapter X, $\sin \alpha_p = \frac{N_p}{N_g}$ to read $N_g = \frac{N_p}{\alpha_p}$, it is evidently possible to make a crown gear which will mesh properly with any bevel gear, such as the one we wish to form. If this crown gear and the plastic blank are properly mounted with relation to each other and rolled together, the tooth of the crown gear will form tooth spaces and teeth of the proper shape in the blank. This is the foundation principle of the molding-generating method.

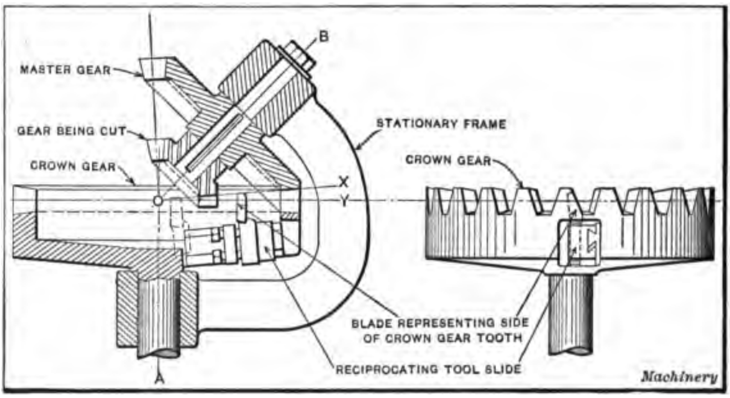


Fig. 8. The Shaping or Planing Operation applied to the Molding-generating Principle of Forming Teeth of Bevel Gears

In practice we have blanks of solid steel or iron to machine instead of those made from putty or clay, so the operation has to be modified accordingly. Fig. 8 shows in diagrammatic form an apparatus for using the shaping or planing operation with the molding-generating principle. Here the crown gear is of larger diameter than is required to mesh with the gear being cut, and it engages a master gear keyed to the same shaft as the gear being cut, and formed on the same pitch cone. If the teeth of the crown gear, instead of being comparatively narrow as shown, were extended clear to the vertex O, they would mesh properly with the gear to be cut. The tooth, as shown, has a line of movement such that the point of the tooth travels along line OX, which is the corner of a tooth of an imaginary extension of the

crown gear. This crown gear has a plane face (see reference to "octoid" form of tooth, Chapter X), and the cutting edge of the tooth is straight and set to mesh the face of the tooth. As it is reciprocated by suitable mechanism (not shown) the cutting edge represents a face of the imaginary crown gear tooth. If now the master gear and crown gear are rolled together and the reciprocating tool starts in at one side of the gear to be cut and passes out at the other, the straight cutting edge of the tooth will generate one side of a tooth in the gear to be cut in the same way as if the extended tooth of the crown gear were rolling its shape on one side of the tooth of a plastic blank. This simple mechanism has, of course, to be complicated by provisions for cutting both sides of the tooth, and for indexing the work from one tooth to the other so as to complete the entire gear. Arrangements have to be made also to make the machine adjustable for bevel gears of all angles, numbers of teeth and diameters within its range.

The use of the three principles illustrated in Figs. 4, 5 and 8 is not limited to the cutting operation shown for each case. In Fig. 4, for instance, a formed planer or shaper tool may be used as well as a formed milling cutter. Templet machines have been made in which a milling cutter is used instead of a shaper tool. This is true also of the molding-generating principle shown in Fig. 8.

Four Methods of Operation — By Impression. — The same four methods of operation as for spur gears may be applied to the molding-generating principle, and quite generally to the other principles as well. Instead of using for illustration a rack as the generating member, we will have to use its bevel gear counterpart, the crown gear shown in Fig. 1. The impression method would simply consist of rolling the crown gear on axis *OA* and the pinion on axis *OB* together, when, if the latter were formed of a plastic material, the teeth of the crown gear would produce in its smaller mate corresponding tooth spaces and teeth of the proper shape.

By Shaping or Planing. — There is but one form of tooth to which the planing operation of molding-generating is adapted.

This is the form in which the crown gear has teeth with plane sides, which may be cut with a straight-sided tool. If the drawing of an involute rack were wrapped around the periphery of the disk in Fig. 3, about axis AO, and the tooth outlines thus determined used in teeth vanishing at O, in the plane of the pitch line, the resulting crown gear would be of this type. In other words, it is Tredgold's approximation of the involute system. In Fig. 8 such a crown gear is shown combined with a simple mechanism for making use of the planing or shaping operation in the molding-generating process. The gear being cut is keyed on a loosely revolving spindle, to which is also keyed a master gear, formed on the same pitch cone and having, in this case, the same number of teeth. This spindle is so set in relation to the axis about which the crown gear revolves, that the master gear and the crown gear mesh together properly, the crown gear being of the required pitch, and having the proper number and shape of teeth for this action. If now the crown gear be rocked about its axis, the master gear will also rock with it, carrying the gear being cut.

The blade is set, as shown in the view at the right, so that its cutting edge coincides with the plane of one of the teeth of the crown gear, and is held in a slide which guides it in such a way that it moves in this plane, and so that its point follows the line OX, radiating from the apex O of the pitch cones. The tool will evidently represent the side of the tooth of an imaginary crown gear, which is adapted to mesh properly with any bevel gear such as that shown being cut, keyed to the master gear and having the same pitch cone shape and number of teeth.

If, with the mechanism so arranged, the crown gear be rotated so as to start the cut at one side of a tooth of the work (which should be first roughly cut to size) the continued rotation of the crown gear will roll the master gear in such a way that the reciprocating blade (representing the side of an imaginary crown tooth meshing with the work) will shape the side of the tooth being cut to the proper form, by the molding-generating process, on the same principle as shown in Fig. 7.

This arrangement, of course, is not a practical working machine as shown, since there is no provision for making it universal

for cutting bevel gears of other pitch cone angles and numbers of teeth, or for indexing the work with relation to the master gear to cut the remaining teeth of the work shown in place. Arranged as shown, however, the machine will cut any gear within its range of the same pitch cone angle and number of teeth as the master gear. To cut a different number of teeth it would only be necessary to alter angle XOY, as required, setting the slide at a greater angle for fewer and larger teeth, or at a less angle for more and smaller teeth.

This principle will be found applied in this and in modified forms in various machines on the market. One of the modifications which will be seen is equivalent to making the crown gear in Fig. 8 stationary, and swinging the frame around it about axis OA, thus rolling the master gear and the work in the same relation to the tool as when the frame is stationary and the crown gear is revolved as just described. Still another possible modification would consist in holding the master gear and work still, while the frame is swung about axis OB. In this case the crown gear would roll on the master gear, rocking the tool slide in such a way as to give the required movement. It is not possible to form a tooth space complete with a single tool, as shown for spur gears, at T_1 in Fig. 8, Chapter VIII, without cutting the tooth space too deep at the outside end. A separate blade has to be used for each side of the space or of the tooth.

By Milling, and by Grinding or Abrasion. — Milling cutters or grinding wheels may be used to represent the space of the tooth, as they represent the rack tooth for spur gears in Figs. 9 and 10, Chapter VIII. In Fig. 9 of the present chapter is shown diagrammatically an arrangement by which two cutters or grinding wheels may be made to represent the two sides of a tooth in such a way that by them a tooth space may be finished complete in the gear to be cut in a mechanism similar to that in Fig. 8, but without requiring the reciprocating movement. The same difficulty arises as in spur gears, of the center of the tooth being cut in deeper than the ends, owing to the circular form of the cutter. This, however, makes no change in the action of the finished gear.

The variety of applications for these various principles and methods of operation is fully as great in bevel gears as in spur gears, and the machines in which they are incorporated apply these principles and methods in an even more ingenious fashion.

Machines for Cutting the Teeth of Bevel Gears by the Formed Tool Process. — A very common method of using the formed tool for cutting bevel gears makes use of the ordinary plain or universal milling machine and adjustable dividing head. The process of cutting bevel gears by this method is described in detail in the next chapter.

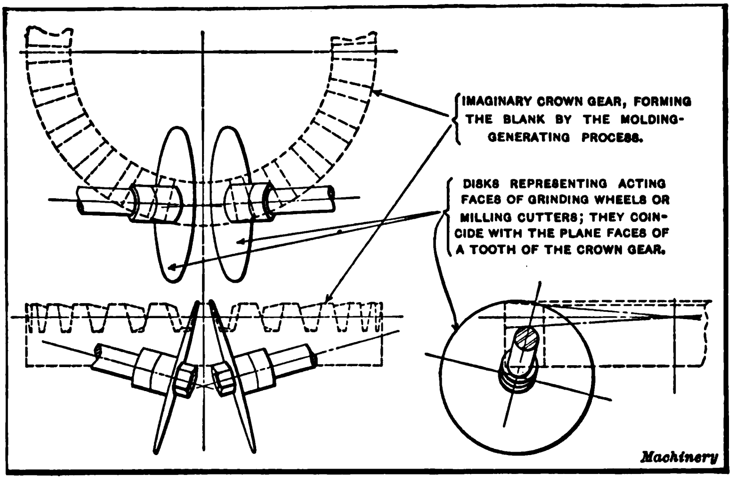


Fig. 9. Diagram suggesting Arrangement of Milling Cutters or Grinding Wheels for Forming the Teeth of Bevel Gears by the Molding-generating Process

Most builders of automatic gear cutting machines furnish them, if desired, in a style which permits the swiveling of the cutter slide or of the work spindle to any angle from 0 to 90 degrees, thus permitting the automatic cutting of bevel gears by the formed cutter process. In some machines of this type the cutter slide is mounted on an adjustable swinging sector. As explained in the next chapter, it is necessary when cutting bevel gears, to cut first one side of the teeth all around and then the other. Between the two cuts the relation of the work and cutter to each other, as measured in a direction parallel to the axis of the cutter

spindle, has to be altered. In the automatic machine this is effected by shifting the cutter spindle axially when the second cut around on the other side of the teeth is taken. Suitable graduations are provided for the angular and longitudinal adjustments.

Bevel Gear Templet Planing Machines. — The templet planing machine most commonly used in this country is the Gleason machine. The tool is carried by a holder reciprocated by an adjustable, quick-return crank motion. The slide which carries this tool-holder may be swung in a vertical plane about the horizontal axis on which it is pivoted to the head, which carries the whole mechanism of tool-holder, slide, crank, driving gearing, etc. This head, in turn, may be swung on a vertical axis about a pivot in the bed. Circular ways guide this movement. The intersection of the vertical and horizontal axes of adjustment (which takes place in mid-air in front of the tool-slide) is the point O in Fig. 5 where the templet principle is shown in diagrammatic form. The blank is mounted on a spindle carried by a head which is adjustable in and on the top of the bed of the machine so that the apex of the cone of the gear may be brought to point O by means of the gages which are a part of the equipment of the machine.

Three templets are used, mounted in a holder attached to the front of the bed on the Gleason bevel-gear planing machine. The first of these templets is for "stocking" or roughing out the tooth spaces. It guides the tool to cut a straight gash in each tooth space, removing most of the stock. After each tooth space has been gashed in this fashion, the templet holder is revolved to bring one of the formed templets into position, and a tool is set in the holder so that its point bears the same relation to the shape of the tooth desired as the cam roll does to the templet. The head is again fed in by swinging it around its vertical axis, during which movement the roll runs up on the stationary templet, swinging the tool about its horizontal axis in such a way as to duplicate the desired form on the tooth of the gear. One side of each tooth being thus shaped entirely around, the holder is again revolved to bring the third templet into position. This has a

reverse form from the preceding one adapted to cutting the other side of the tooth. A tool with a cutting point facing the other way being inserted in the holder, each tooth of the gear has its second side formed automatically, as before, completing the gear. The swinging movement for feeding the tool and the indexing of the work are taken care of by the mechanism of the machine without attention on the part of the operator.

Adjustable Former for Bevel Gear Planing. — A convenient method of cutting ordinary bevel gears by the use of a compara-

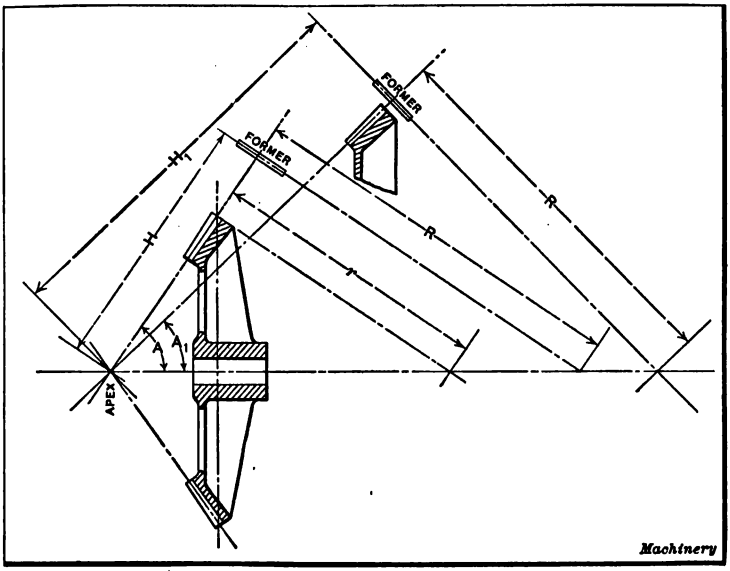


Fig. 10. Planing Gears of Different Angles with the same Former

tively small number of formers is described in the following paragraphs. Bearing in mind the fact that to a given circle there corresponds one and only one shape of involute, one can readily see by referring to Fig. 10 that a pair of formers, one for the upper and one for the lower side of the tooth, would serve for all gears if they could be set at any desired distance, H, from the apex of the pitch cone. If the shape of the former is the same as that of a gear tooth whose pitch radius is R, it will be suitable for cutting the bevel gear indicated by a full section, as the curvature of the gear tooth will be reduced from the curvature of the former in the

same proportion as R is to r; but a bevel gear of any other pitch cone angle and number of teeth, for instance the one shown in part only, having a pitch cone angle A_1, can be cut with the same former, if only this former be set in the new pitch cone at such a distance, H_1, from the apex, that the new pitch radius, R, be the same as it was before. The number of teeth in either of the gears is immaterial so long as the templet is long enough. A long tooth will use the whole of the templet, while a shorter tooth will only use a part of the former.

As stated, it would be possible for one former to cover the whole range of pitch cone angles A, but since on any given machine, distance H has but a limited variation, this necessitates a series of formers in order to include all the gears capable of being cut on the machine. Suppose we have a machine on which a former can be set between 30 and 45 inches from the apex. Let H and H_1 in Fig. 10 represent these two extremes of distance, respectively. It is apparent from this diagram that $\frac{R}{H} = \tan A$. If 2 inches is the smallest value for R to be used on this machine we can, by using it in the above formula with different values of H between 30 and 45, obtain the corresponding values of A which, when laid out on the diagram, Fig. 11, will be represented by the curve *cd*. This diagram has, however, been extended, giving a minimum value to H of 20 inches and a maximum value of 55 inches. In a similar way all the other curves are found, the values of R for each succeeding one being chosen so that each curve intersects the 45-inch line at about the same value for the pitch angle that the preceding curve intersects the 30-inch line, thus always covering the field between 30 inches and 45 inches, the assumed limits of the machine.

Take, for example, a bevel gear with a pitch angle of 30 degrees; according to the diagram the 21-inch former, or a former made for a radius R = 21 inches, is the one to be used, and the reading of the diagram shows that it should be set about $36\frac{1}{4}$ inches from the apex. If the machine allows a shorter or longer adjustment of the former than that assumed above, the $31\frac{1}{2}$-inch former at about 54 inches or the 14-inch former at $24\frac{1}{4}$ inches

from the apex would give the same tooth form. When the pitch radius of the former exceeds 200 inches the involute for any ordinary pitch of tooth is practically a straight line, and a former laid out accordingly may be set at any distance from the apex.

In the above remarks involute formers only have been considered. Owing to the fact that the cycloidal curves vary not

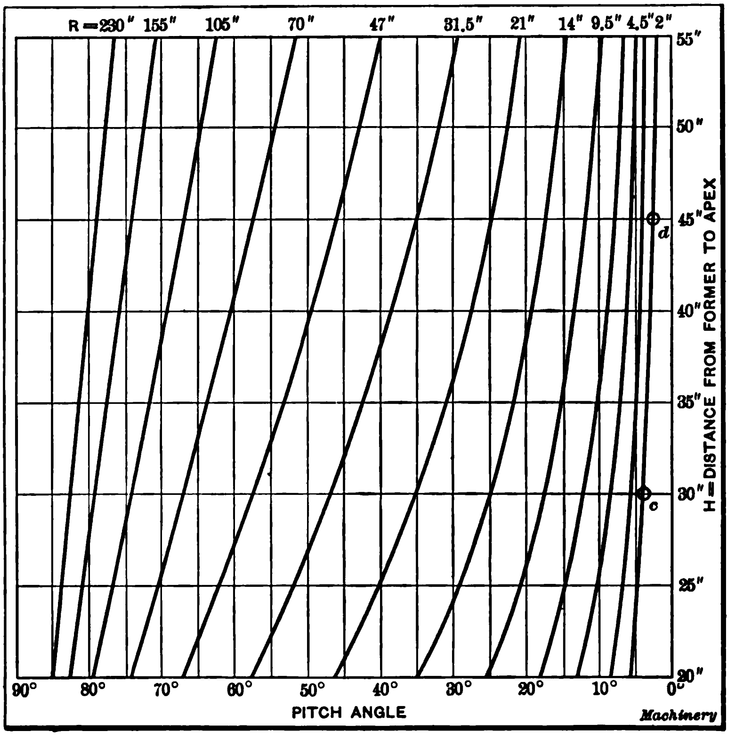


Fig. 11. Diagram for Selecting Formers

only with the pitch radius, but with the pitch as well, and consequently with the number of teeth in the gear, a simple diagram as shown above cannot be obtained for cycloidal formers.

The scheme described allows the use of a smaller number of formers than would otherwise be necessary and practically makes allowance for the errors that would be introduced in cutting a gear, in which the pitch angle is about half-way between those of the two nearest formers in the usual way. So far as

the author knows, planers to which this idea may be applied are not being built at the present time in such a way that the distance from the former to the apex is adjustable; but there are many gear planers in use to which the idea can be applied although they are of an old design. In the later machines, dimension H in Fig. 10 is constant for any given machine, and the formers are made to fit this dimension, being cut in a generating machine by a milling cutter, on a spindle which is pivoted to swing about the apex of the pitch cone in the same way that the tool slide does.

Bevel Gear Generating Machines. — The mechanism illustrated in outline in Fig. 8 is one that has been employed in a number of interesting and ingenious machines. The first application of this principle was made by Mr. Hugo Bilgram of Philadelphia, Pa. This form of machine in the hand-operated style has been used for many years. A more recently developed automatic machine of the same type has also been built. In this the movements operate on the same principle as in Fig. 8, although in a modified form. Instead of rotating the crown gear and master gear together, the imaginary crown gear and, consequently, the tool, remain stationary so far as angular position is concerned, while the frame is rotated about the axis of the crown gear, thus rolling the master gear on the latter and rolling the work in proper relation to the tool. Instead of using crown and master gears, however, a section of the pitch cone of the master gear is used, which rolls on a plane surface, representing the pitch surface of the crown gear. The two surfaces are prevented from slipping on each other by a pair of steel tapes, stretched so as to make the movement positive. A still further change consists in extending the work arbor down beyond center O in Fig. 8, mounting the blank on the lower side of the center so that the tool, being also on the lower side, is turned the reverse from that shown in the diagram. As explained, a tool with a straight edge is used, representing the side of a rack tooth, and this tool is reciprocated by a slotted crank, adjustable to vary the length of the stroke, and driven by a Whitworth quick-return movement. The feed of the machine is effected by swinging the

frame in which the work spindle and its supports are hung about the vertical axis of the imaginary crown gear.

As stated, the machine is automatic. The operator sets the machine and places a previously gashed blank on the work spindle and starts the tool in operation. The mechanism provided will, without further attention, complete one side of all the teeth. The machine may then be readjusted and the tool set for cutting the other side, which will be finished in the same automatic fashion. The mechanism does not operate on the principle of completing one side of one tooth before going to the next. It follows the plan of indexing the work for each stroke of the tool, the rolling action being progressive with the indexing so as to finish all the teeth at once.

The Gleason generating machine differs from the previous machine in employing two tools, one on each side of the tooth. The construction is identical with the mechanism shown in Fig. 8, in having the axes of the tool-slides and of the blank fixed in relation to each other during the operation, the tool-holders and the blank rocking about their axes to give the rolling movement for cutting. The rocking is effected by means of segments of an actual crown gear and master gear. The segment of the crown gear is permanently attached to the face of the rear of the cutter slide frame, while the segment of the master gear (of which there are several furnished with the machine, the one used being chosen to agree with the angle of the gear to be cut) is clamped to the semi-circular arm pivoted at the outer end of the machine at one side, and fastened to the work spindle sleeve on the other. This arm is rocked by a cam mechanism and slotted link.

The cycle of operations is as follows: The machine being adjusted properly in its preliminary position, the tool-slide and the head on which it is mounted are swung back about the vertical axis so that the tools clear the work. The blank being set in the proper position, a cam movement swings the cutter slide head inward until the reciprocating tools reach the proper depth. The cam movement first mentioned now rocks upward the semi-circular arm extending around the front of the machine, rolling the blank and (through the segmental crown and master gears)

the slide, until the tools have been rolled out of contact in one direction, partially forming the teeth as they do so. The arm is then rolled back to the central position and along downward to the lower position, until the tools are rolled out of contact with the tooth in this direction, completing the forming of the proper shape as they do so. The cam then rocks the arm back to the central position, where the cutter-slide head is swung back to clear the tooth, and the work is indexed, after which this cycle of operations is continued for the next tooth. It will be seen that by starting from the central position, going to each extreme and returning, all parts of each tooth are passed over twice, giving a roughing and a finishing chip. The machine is entirely automatic.

CHAPTER XIII

CUTTING THE TEETH OF BEVEL GEARS

SPECIAL directions for operating are furnished by the makers of molding-generating and templet planing machines. As these directions are usually adequate, and apply only to the particular machines for which they are given, this chapter will be confined to giving instructions for cutting teeth by the formed tool method only, as performed on the milling machine.

Cutting Bevel Gears in the Milling Machine. — The first requirement for setting up the milling machine to cut bevel gears is a true-running blank, with accurate angles and diameters. If such a blank cannot be found in the lot of gears to be cut, it will be necessary to turn up a dummy out of wood or other easily worked material. Otherwise the workman is inviting trouble, whatever his method of setting up.

Fig. 1 shows in diagram form the relative positions of the cutter and the work. The spindle of the dividing head is set at the cutting angle, as shown, and the cutter (which has been centered with the axis of the work-spindle) is sunk into the work to the whole depth W, as given by the working drawing.

The Brown & Sharpe Mfg. Co. recommends that for shaping with a formed cutter, the cutting angle be determined by subtracting the *addendum* angle from the pitch cone angle, instead of subtracting the dedendum angle. In other words, the clearance at the bottom of the tooth is made uniform, as shown in Fig. 3, instead of tapering toward the vertex. This gives a somewhat closer approximation to the desired shape.

Setting the Cutter. — The centering may be done by mounting a true hardened center in the taper hole of the spindle, and lining up its point with the mark which will be found inscribed either on the top or on the back face of the tooth of the commercial gear cutter. Setting the cutter to the whole depth W of

tooth to be cut is effected by passing the work back and forth under the revolving cutter and slowly raising it until the teeth of the cutter just bite a piece of tissue paper laid over the edge of the blank. This must be done after centering. The dial on the elevating screw shaft is set at zero in this position, and then the knee is raised an amount equal to the whole depth of the tooth, reading the dial from zero. This is evidently not exactly right, since the measurement should be taken in the direction of the back edge of the tooth, which inclines from the perpendicular an amount equal to the dedendum angle, as shown in Fig. 1. In practice, the slight difference in the value for the whole depth thus obtained is negligible.

Having thus mounted the work at the proper angle and having thus centered the cutter and set it to depth, two tooth spaces should next be cut, with the indexing set by the tables furnished with the dividing head to give the number of teeth required for the gear. Cutting these two spaces leaves a tooth between on which trial cuts are to be made until the desired setting is obtained. The relative positions of the cutter and the work and the shape of the cuts thus produced are shown in the upper part of Fig. 1. It will be seen at once that this does not cut the proper shape of tooth. As explained in Chapter XII, all the elements of the bevel gear tooth vanish at *O*, the vertex of the pitch cone — that is to say, the outer corners of the tooth space should converge at *O* instead of at *A*, and the sides of the tooth spaces at the bottom, instead of having the parallel width given them by the formed cutter, should likewise vanish at *O*. Our next problem is that of so re-setting the machine that we can cut gear teeth as nearly as possible like the true tooth-form in which the elements converge at *O*.

Offsetting and Rolling the Blank to Approximate the Shape of Tooth. — There are a number of ways of approximating the desired shape of bevel gear teeth. Of these we have selected as most practicable the one in which the sides of the tooth at the pitch line converge properly toward the vertex of the pitch cone. Gears cut by this process will show, of course, the proper thickness at the pitch line when measured by the gear tooth caliper at

either the large or the small ends. This method of approximation produces tooth spaces which, at the small end, are somewhat too wide at the bottom and too narrow at the top, or, in other words, the teeth themselves at the small end are too narrow at

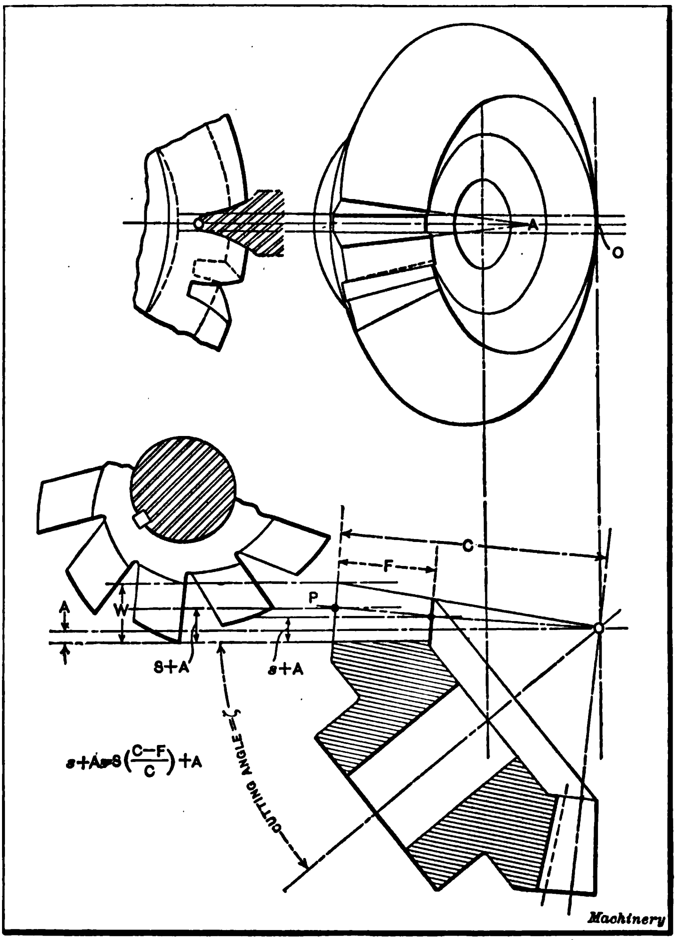


Fig. 1. Relative Positions of the Formed Cutter to the Blank when taking a Central Cut

the bottom and too wide at the top. To make good running gears they must be filed afterward by hand, as described later. When so filed they are better than milled gears cut by other methods of approximation which omit the hand filing.

Set-over. — In the upper part of Fig. 2 is shown a section of the gear in Fig. 1, taken along the pitch cone at *PO*. It will be seen that the teeth at the pitch line converge, but meet at a point considerably beyond the vertex *O*. What we have to do is to move the cutter off the center, so that it will cut a groove, one side of which would pass through *O* if extended that far. The amount by which the cutter is set off the center is known as the "set-

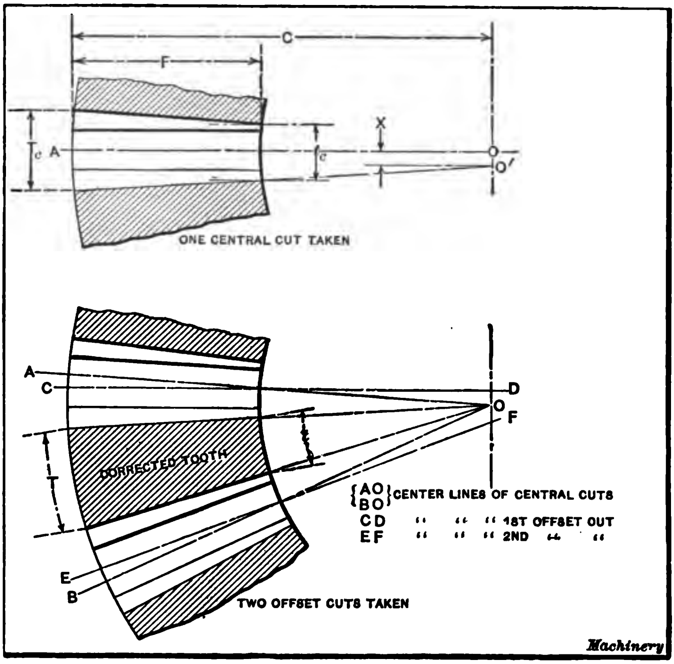


Fig. 2. Section on the Pitch Cone Surface *PO* of Fig. 1 showing Central and Offset Cuts

over." We may take, for instance, for trial a set-over equal to 5 or 6 per cent of the thickness of the tooth at the large end. Move the face of the trial tooth away from the cutter by the amount of this trial set-over, having first, of course, run the cutter back out of the tooth space. Now rotate the dividing head spindle to bring this tooth face back to the cutter again, stopping it where the cutter will about match with the inner end of the space previously cut. Take a cut through in this position.

Trial Cuts. — Having proceeded thus far, index the work to bring the cutter into the second tooth space and move the blank over to a position the other side of the central position by an amount equal to the same set-over, thus moving the opposite face of the trial tooth away from the cutter. Rotate the dividing head spindle again to bring this face toward the cutter until the latter matches the central space already cut at the inner end of the teeth. Take the cut through in this position.

Now with vernier gear tooth calipers or with fixed gages machined to the proper dimensions measure the thickness of the tooth at the pitch line at both large and small ends. (See Fig. 15, Chapter I.) If the thickness is too great at both the large and the small ends, rotate the tooth against the cutter and take another cut until the proper thickness at either the large or small end has been obtained. If the thickness comes right at both ends the amount of set-over is correct. If it is right at the large end and too thick at the small end the set-over is too much. If it is right at the small end and too thick at the large end the set-over is not enough. The recommended trial set-over (5 or 6 per cent of thickness of the tooth at the pitch line at the large end) will probably not be enough, so two or three cuts will have to be taken on each side of the trial tooth, as described, before the proper amount is found.

Having found the proper set-over, the cross-feed screw is set to that amount and the cut is taken clear around the gear. Then the cross-feed screw is set to give the same amount of set-over the other side of the center line and the work is rotated until the cutter matches the tooth spaces already cut at the small end and is run through the work. The tooth will generally be found too thick, so the work spindle is rotated still more until the tooth is of the proper thickness, when the gear is again cut clear around on this second cut.

The number of holes it was necessary to move the index pin on the dividing plate circle between the first and the second cuts to get the proper thickness of tooth, should be recorded. On succeeding gears it will thus only be necessary to take a first cut clear around with the work set over by the required amount on one side

of the center line, and then a second cut around with the work set over on the other side of the center line, rotating the index crank the number of holes necessary to give the proper thickness of tooth between the cuts.

It will be noted that the shifting of the blank by the index crank is only used for bringing the thickness of tooth to the proper dimension. In some cases, particularly in gears of fine pitch and large diameter, this adjustment will not be fine enough — that is to say, one hole in the index circle will give too thick a tooth and the next one too thin a tooth. To subdivide the space between the holes, most dividing heads have a fine adjustment for rotating the worm independently of the crank. Every milling machine should be provided with such an adjustment.

In large gears it is best to take the central cuts shown in Fig. 1 clear around every blank before proceeding with the approximate cuts. This gives the effect of roughing and finishing cuts, and produces more accurate gears. The central cuts may be made in a separate operation with a roughing or stocking cutter if desired. It might also be mentioned that it is common practice to turn up a wooden blank for making the trial cuts shown in Figs. 1 and 2, to avoid the danger of spoiling the work by mistakes in the cut-and-try process.

Positive Determination of the Set-over. — This cut-and-try process, however, may be practically eliminated by calculating the set-over from the following table and formula:

Table for Obtaining Set-over for Cutting Bevel Gears

No. of Cutter	Ratio of Pitch Cone Radius to Width of Face $\left(\frac{C}{F}\right)$												
	$\frac{3}{1}$	$\frac{3\frac{1}{4}}{1}$	$\frac{3\frac{1}{2}}{1}$	$\frac{3\frac{3}{4}}{1}$	$\frac{4}{1}$	$\frac{4\frac{1}{4}}{1}$	$\frac{4\frac{1}{2}}{1}$	$\frac{4\frac{3}{4}}{1}$	$\frac{5}{1}$	$\frac{5\frac{1}{2}}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$
1	0.254	0.254	0.255	0.256	0.257	0.257	0.257	0.258	0.258	0.259	0.260	0.262	0.264
2	0.266	0.268	0.271	0.272	0.273	0.274	0.274	0.275	0.277	0.279	0.280	0.283	0.284
3	0.266	0.268	0.271	0.273	0.275	0.278	0.280	0.282	0.283	0.286	0.287	0.290	0.292
4	0.275	0.280	0.285	0.287	0.291	0.293	0.296	0.298	0.298	0.302	0.305	0.308	0.311
5	0.280	0.285	0.290	0.293	0.295	0.296	0.298	0.300	0.302	0.307	0.309	0.313	0.315
6	0.311	0.318	0.323	0.328	0.330	0.334	0.337	0.340	0.343	0.348	0.352	0.356	0.362
7	0.289	0.298	0.308	0.316	0.324	0.329	0.334	0.338	0.343	0.350	0.360	0.370	0.376
8	0.275	0.286	0.296	0.309	0.319	0.331	0.338	0.344	0.352	0.361	0.368	0.380	0.386

For obtaining the set-over by the above table, use this formula:

$$\text{Set-over} = \frac{T_c}{2} - \frac{\text{factor from table}}{P} \quad . \quad . \quad . \quad (1)$$

P = diametral pitch of gear to be cut;
T_c = thickness of cutter used, measured at pitch line.

Given as a rule this would read: *Find the factor in the table corresponding to the number of the cutter used and to the ratio of pitch cone radius to width of face; divide this factor by the diametral pitch, and subtract the result from half of the thickness of the cutter at the pitch line.*

As an illustration of the use of this table in obtaining the set-over, we will take the following example: A bevel gear of 24 teeth, 6 pitch, 30 degrees pitch cone angle and 1¼ face is to be cut. These dimensions, by the rules given in Chapter X, call for a No. 4 cutter and a pitch cone radius of 4 inches.

In order to get the factor from the table, we must know the ratio of pitch cone radius to width of face. This ratio is $\frac{4}{1.25} = \frac{3.2}{1}$ or about $\frac{3\frac{1}{4}}{1}$. The factor in the table for this ratio with a No. 4 cutter is 0.280. We next measure the cutter at the proper depth of $S + A$ for 6 pitch, which is found in the column marked "depth of space below pitch line" in a regular table of tooth parts, or by dividing 1.157 by the diametral pitch. This gives $S + A$ = 0.1928 inch. We find, for instance, that the thickness of the cutter at this depth is 0.1745 inch. The dimension will vary with different cutters, and will vary in the same cutter as it is ground away, since formed bevel gear cutters are commonly provided with side relief. Substituting these values in the formula, we have:

$$\text{Set-over} = \frac{0.1745}{2} - \frac{0.280}{6} = 0.0406 \text{ inch}$$

which is the required dimension.

The work must now be set off center on one side of the cutter by this amount, taking the usual precautions to avoid errors from back-lash. In this position the cutter is run through the blank, the latter being indexed for each tooth space until it has been cut

around. If a central or roughing cut has been previously taken as suggested in an earlier paragraph, it will be necessary to line up this cut at the small end of the tooth with the cutter. This is done by rotating the tooth space back toward the cutter, either by moving the index crank as many holes in the dial-plate as are necessary, or by means of such other special provisions as may be made for doing this in the index head, independently of the dial-plate.

Having thus cut one side of the tooth to proper dimensions, the work must be set-over by the same amount the other side of the position central with the cutter, taking the same precautions in relation to back-lash as before, and rotating the blank to again line up the cutter with the tooth space at the small end of the tooth. With this setting, take a trial cut as already explained. This will be found to leave the tooth whose side is trimmed in this operation a little too thick, if the cutter is thin enough, as it ought to be, to pass through the small end of the tooth space of the completed gear. This trial tooth should now be brought to the proper thickness by rotating the blank toward the cutter, moving the crank around the dial for the rough adjustment, and bringing it to accurate thickness by such means as may be provided in the head. In the Brown & Sharpe head, this fine adjustment is effected by two thumb-screws near the hub of the index crank, which turn the index worm with relation to the crank.

Testing for Correctness of the Setting. — With reference to the use of the table and formula, the Brown & Sharpe Mfg. Co., after trial in their gear cutting department, say: "We feel fairly confident it is within working limits of being satisfactory." While this sounds encouraging, it will evidently be wise to be sure we are right before going ahead, as the slight approximations involved in the derivation of the formula (to be explained later) may bring the setting not *quite* right, so that the thickness of the tooth at the large and the small ends is not what it ought to be. This point may be tested by measuring the tooth at both the large and the small ends with a vernier caliper, the caliper being set so that the addendum at the small end is in the proper

proportion to the addendum at the large end — that is to say, that it is in the ratio $\frac{C - F}{C}$. In taking these measurements, if the thicknesses at both the large and the small ends, which should be in this same ratio, are too great, rotate the tooth toward the cutter and take another cut until the proper thickness at either the large or small end has been obtained. As already mentioned in connection with the ordinary cut-and-try method, if the thickness is right at the large end and too thick at the small end, the set-over is too much. If it is right at the small end and too thick at the large end, the set-over is not enough, and should be changed accordingly, as is done by the regular "cut-and-try" process. The formula and table herewith given, however, ought to bring it near enough right the first time, and in the general run of work it can be safely relied on.

Use of the Formula for Other Methods of Correction. — It is customary also among workmen expert in cutting bevel gears with formed cutters, to disregard rules and formulas for the selection of the cutters, and depend on their experience to get shapes which require somewhat less filing than would otherwise be necessary. Whenever this dependence on experienced an judgment requires, as it sometimes does, the use of a cutter of finer pitch than that of the teeth of the bevel gear at the large end, the values given in the table are inapplicable. The following formula may then be used:

$$\text{Set-over} = \frac{T_c}{2} - \frac{T_c - t_c}{2} \times \frac{C}{F}$$

in which t_e is the thickness of the cutter measured at a depth $s + A$, obtained as shown in Fig. 1. This has been tried on several widely varying cases with good results. It requires, it will be seen, two measurements of the cutter in place of the single one required when the regular pitch of cutter is used.

Filing the Teeth. — The method of cutting bevel gears just described requires the filing of the points of the teeth at the small end. This can be done "by the eye" very skillfully when the workman is used to it. The operation consists in filing off a triangular area extending from the point of the tooth at the large

end to the point at the small end, thence down to the pitch line at the small end and back diagonally to the point at the large end again. This is shown in Fig. 4, by the shaded outline. Enough is taken off at the small end of the tooth so that the edges of the teeth at the top appear to converge at vertex *O* in Figs. 1 and 2.

Testing Bevel Gear Teeth. — The bevel gears may be tested for the accuracy of the cutting and filing by mounting them in place in the machine and revolving them at high speed, or by mounting them in a testing machine made for the purpose. The marks of wear produced by running them together under pressure, with the back faces flush with each other, should extend the

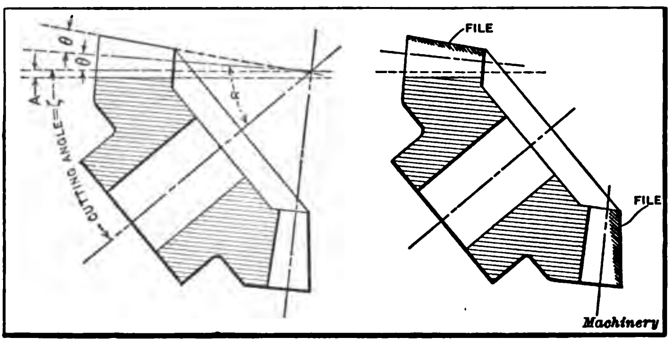


Fig. 3. Parallel Clearance best adapted to Shaping with Formed Cutter

Fig. 4. Shaded Area showing Part Corrected by Filing

whole length of the tooth at the pitch line. If it does not, the amount of set-over allowed in cutting them was at fault, being too little if they bear heavily at the large ends, and too much if they bear heavily at the small ends. The bearing area should also be fairly evenly distributed over the sides of the teeth above the pitch line, from the large to the small end. If it is not, the filing is at fault. The marks of wear will not in any case extend far below the pitch line in a pinion of few teeth.

It is possible to get along without filing by decreasing the amount of set-over so as to make the teeth too thin at the pitch line at the small end, when they are of the right thickness at the large end. This does not give as good running gears, however, as when the method just described is followed.

Cutting Bevel Gears on the Automatic Gear-cutting Machine. — The directions for cutting bevel gears on the milling machine apply in modified form to the automatic gear-cutting machine as well. The set-over is determined in the same way, but instead of moving the work off center, the cutter spindle is adjusted axially by means provided for that purpose. Some machines are provided with dials for reading this movement. The cutter is first centered as in the milling machine, and then shifted — first to the right, and then to the left of this central position.

The rotating of the work to obtain the proper thickness of tooth is effected by unclamping the indexing worm from its shaft (means usually being provided for this purpose) and rotating the worm until the gear is brought to proper position. Otherwise the operations are the same as for the milling machine.

Derivation of Formula for Positive Set-over. — The derivation of the formula and the method of calculating the table need not concern the man who merely desires to use them, as he can employ them with no knowledge of mathematics other than that required for plain subtraction and division. For those, however, who desire to understand the origin of the table, the following explanation will be interesting.

Fig. 2 shows a section such as would be made by turning off the bevel gear blank down to the pitch cone — in other words, it is a section on the conical surface *PO* of Fig. 1. The same references apply to both figures. We find, if we take a cut with the cutter set central, that the side of the tooth space will not pass through the vertex O, but through some point O' at one side. The distance between O' and O is X, which is the amount by which the cutter will have to be offset to bring the side of the tooth space at the pitch line radial with vertex O. A formula can be derived by simple proportion to obtain this offset, in terms of T_c, t_c, F and C. The formula is:

$$X = \frac{T_c}{2} - \left(\frac{T_c - t_c}{2} \times \frac{C}{F}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This determination of the set-over, of course, involves one or

two approximations of minor importance, which will be readily perceived from an examination of the diagrams.

While this formula seems to furnish a means for obtaining by measurement and calculation the amount of set-over, it is rather clumsy. It remains therefore to put it in more usable form. From an examination of the formula, we note that while $\frac{T_c}{2}$ is a variable, depending on the thickness of the cutter, the quantity in parenthesis remains constant as the cutter grows thinner from being ground down. In fact, by taking the measurements T_c and t_c on a one-diametral pitch cutter, and calling them T_c' and t_c' it would be possible to put this in the form:

$$X = \frac{T_c}{2} - \frac{1}{P}\left(\frac{T_c' - t_c'}{2} \times \frac{C}{F}\right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which the quantity $\frac{T_c' - t_c'}{2} \times \frac{C}{F}$ would be constant for all cases of all pitches having the same ratio of $\frac{C}{F}$ and using the same number of cutter.

Now it is possible to put this formula in still simpler form by tabulating the values of $\frac{T_c' - t_c'}{2} \times \frac{C}{F}$, as measured on a one pitch cutter, for different values of $\frac{C}{F}$. This has been done in the table for thirteen values of $\frac{C}{F}$, which cover the major part of the bevel gears cut by the formed tool process. Using the factor as given in the table, the formula reads:

$$X = \text{set-over} = \frac{T_c}{2} - \frac{\text{factor from table}}{P} \quad . \quad . \quad . \quad (1)$$

as we have already given it.

The method of filling in the table will be easily understood. A one-pitch cutter is measured with the Brown & Sharpe gear-tooth caliper at depth $S + A$ for dimension T_c', and at depth $s + A$ for t_c' (see Fig. 1). Of course, $s + A$ and consequently t_c' will vary, as $\frac{C}{F}$ is taken $\frac{3}{1}$, $\frac{3\frac{1}{4}}{1}$, etc., respectively. Having found

these dimensions, the quantities to use in the table are evidently obtained by the formula:

$$\text{Factor from table} = \frac{T_c' - t_c'}{2} \times \frac{C}{F} \quad . \quad . \quad . \quad . \quad (4)$$

The form of table and formula here given is also suitable for recording for future use the amount of set-over obtained by the "cut-and-try" process for other methods of approximation than that here given. Transposing Formula (1) to solve for the factor, we have:

$$\text{Factor from table} = P\left(\frac{T_c}{2} - \text{set-over}\right) \quad . \quad . \quad . \quad (5)$$

in which the measurement as before is taken on the cutter used for the work in hand. By recording the factors for all cases met with, and thus gradually filling in the tables from his own practice, the machinist would be able to put the data in usable form to apply to future jobs.

Practicability of the Milling Process. — The methods obtained with the templet process and, above all, with the generating process, are so much superior to those obtained with the milling cutter that the use of the latter should be avoided wherever possible. It has a legitimate field, however, on gears too small to be cut on any commercial planing machine. In general, it is not considered advisable to plane gears having teeth finer than 10 to 12 diametral pitch. It is allowable, however, to mill gears of coarser pitch which are to run at slow speeds, or which are to be used only occasionally — such, for instance, as the bevel gears used for turning the elevating screws of a planer cross-rail, or those used in connection with other hand-operated mechanisms. Under ordinary conditions it is impracticable to mill bevel gears having teeth coarser than 3 diametral pitch, no matter what the service for which they are to be used.

Approximations Involved in Positive Set-over Calculations. — Two of the approximations involved in this method may be mentioned. In Fig. 1, it will be noticed that the various dimensions W, $S + A$, etc., are taken perpendicular to the bottom of the tooth, instead of perpendicular to the pitch line, as they should be. The setting of the cutter to depth by this usual

method, therefore, involves an error, but it is so slight as to be negligible. The other approximation relates to the use of the uncorrected tooth thickness and addendum for measuring the cutters in preparing the table. The chordal tooth thickness might have been used, but at the cost of a considerable complication in the process. It was found by investigation that this refinement would not affect the final result enough to make it worth while.

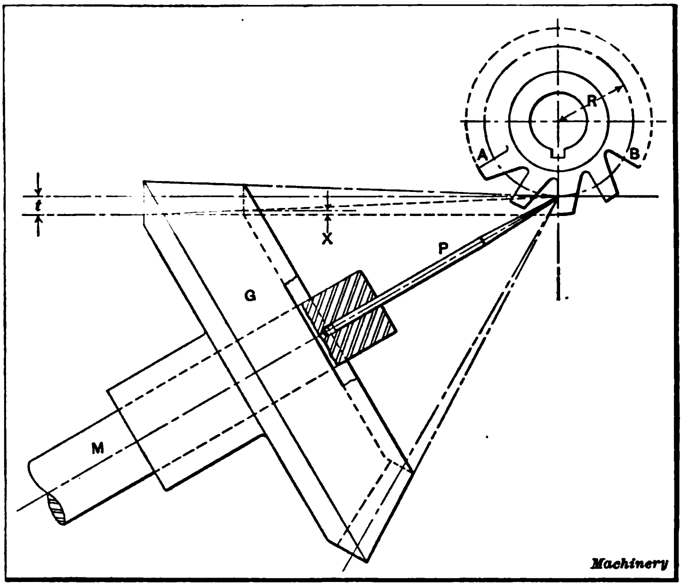


Fig. 5. Method of setting Bevel Gear Cutter

Another Positive Method of Setting Bevel Gear Cutter. — Another method of setting the cutter for milling quiet-running bevel gears is outlined in the following. In using this method, two cuts are required. The set-up for taking the first cut is shown in Figs. 5 and 6, while Figs. 5 and 7 illustrate the set-up for the second cut. As Fig. 5 shows, the *cutting angle equals the pitch cone angle* of the gear. The gear-cutter is set by a pointer *P* which is adjusted so that the end coincides with the apex of the pitch cone. After this pointer is set, the gear-cutter is adjusted until a line on the side of the tooth, representing the pitch circle,

coincides with the end of the pointer. The required number of tooth spaces is then milled, after which the lateral position of the cutter is changed as shown in Fig. 7; that is, the pointer is set to coincide with the pitch circle on the opposite side of the cutter. The teeth are then finished by taking a second series of cuts, as explained later.

In order to locate the pitch circle on the cutter, a little blue vitriol is placed on one of the cutter teeth and a pair of dividers is used to mark the arc of the pitch circle *AB* on this tooth, after

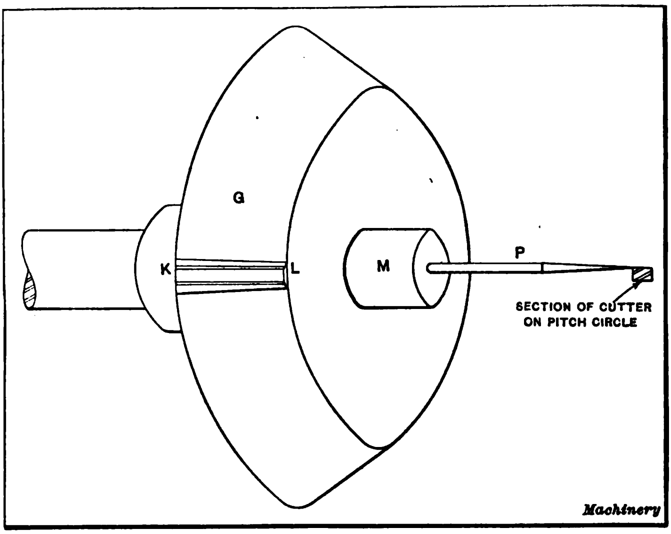


Fig. 6. Method of setting Gear Cutter for taking First Cut

a centering plug has been inserted in the bore of the cutter. The radius *R* of the pitch circle is obtained by subtracting $2t$ from the outside diameter of the cutter and dividing the result by 2. (The distance t or depth of space below the pitch line equals 1.157 divided by the diametral pitch.)

After marking the pitch circle on the cutter, the latter is mounted on the arbor of the milling machine. The gear blank *G* is next mounted on mandrel *M* of the dividing head and is inclined to the pitch cone angle. There is a hole $\frac{1}{8}$ inch in diameter and about 1 inch deep in the center of the mandrel *M* and pointer *P* fits into this hole. The pointer can be moved in and

out by hand, but is tight enough to remain in any position. By placing a straightedge against the face of the gear blank, in two or three different positions, and sliding pointer P in or out, as may be found necessary, its sharp end can be made to coincide with the axis of the cone.

The milling machine table is next adjusted vertically, to the right or left, and laterally until the end of the pointer coincides with the pitch line on one side of the milling cutter, as indicated in Figs. 5 and 6. When this has been done, the table is moved

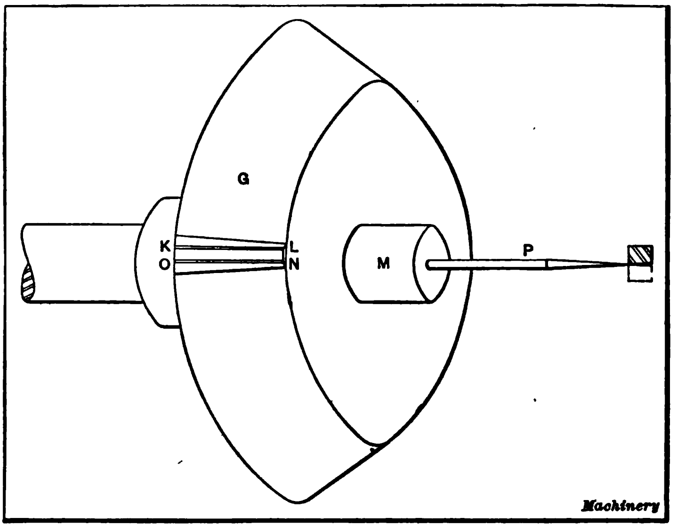


Fig. 7. Method of setting Gear Cutter for taking Second Cut

to the left and the pointer removed from the mandrel, after which the first cut is taken. When the cutter has been located by this method, the pitch line of the tooth face KL coincides with an element of the cone. After the first tooth has been cut, the succeeding teeth are cut by indexing the gear blank in the usual manner.

The pin is now reinserted in the mandrel and its sharp end again located at the apex of the cone. The table is then moved until the relative position of the pin and cutter are as shown in Figs. 5 and 7, the cutter being set on the opposite side of the center line. In Fig. 7, the dotted trapezoid shows the position

occupied by the cutter during the first series of cuts, and the full line its position for the second or finishing cuts.

The table is now moved to the left, the pin removed and the dividing head turned to the left through an angle corresponding to one tooth space or 180 degrees divided by the number of teeth. The machine is now ready for taking the second cut, which is usually very light, seldom exceeding 0.012 to 0.015 inch. With the work located in this way, the pitch line of tooth face *ON* also coincides with the surface of the pitch cone. Fig. 5 shows the amount *X* by which the cut taken by this method exceeds the correct depth at the inner ends of the teeth.

CHAPTER XIV

LONG AND SHORT ADDENDUM GEARS

Object of Gears with Lengthened Addendum. — The bevel driving gears in the rear axle have probably given more trouble to automobile makers and users than any other two gears used on

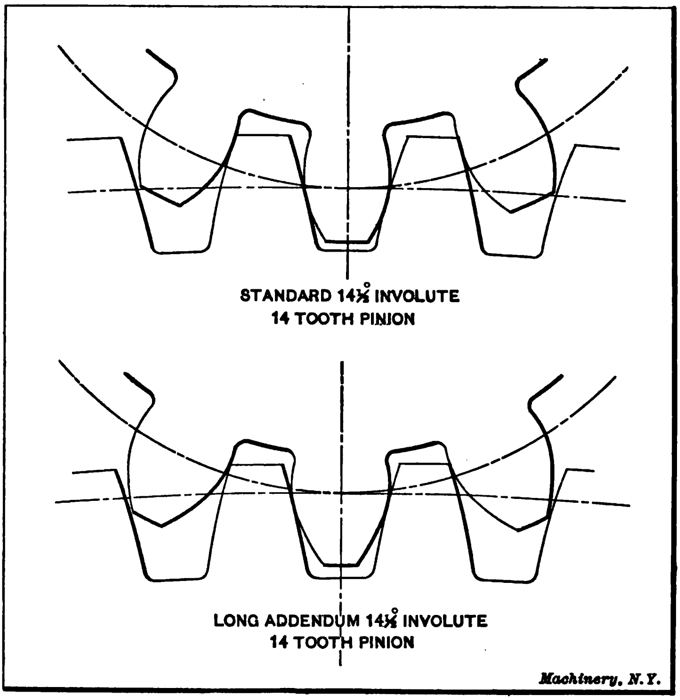


Fig. 1. Comparison between Standard and Long Addendum Gears

the car. For that reason any system of gear tooth design that tends to quieter running, greater strength or durability is deserving of consideration. The system described in the following is not new, although none of the authors of standard gear books in the past have deemed it worthy of more than a passing com-

ment. It is, of course, understood that the tooth is of true involute or of octoid form, depending upon whether it is produced for a spur or bevel gear. The special feature of it is the lengthening of the addendum of the pinion tooth, with a corresponding shortening of the addendum of the gear tooth, the whole depth remaining the same as in the standard tooth. The tables and

Table I. Chordal Thickness of Teeth For Spur Gears, 1 Diametral Pitch, Special Pitch Depth

To obtain chordal thickness of teeth and corrected pitch depth for any diametral pitch other than 1, divide figures in table by diametral pitch required.

20-degree Pressure Angle

For Pinions, Addendum = 0.7 Working Depth		
Number of Teeth	Chordal Thickness	Corrected Pitch Depth
12	1.8545	1.4720
13	1.8554	1.4665
14	1.8567	1.4618
15–16	1.8573	1.4559
17–18	1.8584	1.4495
19–20	1.8592	1.4442
21–22	1.8597	1.4402
23–25	1.8601	1.4361
26–29	1.8606	1.4316
30–34	1.8609.	1.4272
For Gears, Addendum = 0.3 Working Depth		
35– 41	1.2792	0.6107
42– 54	1.2793	0.6085
55– 79	1.2794	0.6060
80–134	1.2795	0.6040
134	1.2795	0.6030

For bevel gears, find chordal thickness of tooth and corrected pitch depth of gear with the same number of teeth as a spur gear having a diameter equal to twice the back cone distance.

formulas given in the following are abstracted from a paper read by Mr. E. W. Weaver before the Society of Automobile Engineers.

With the addendum or face of the driver lengthened, the arc of approach of the gear tooth action is lessened and the arc of recess is increased — becoming all recess and no approach when the driver has only faces and the driven gear only flanks. This

produces particularly smooth-running gears — almost equal, in fact, to spiral gears. As is well known, the friction of the arc of approach is much greater than that of the arc of recess — something on the principle of a man dragging a stick after him or pushing it ahead of him.

Pinions with Small Number of Teeth. — Another advantage of this system is the very great improvement in the shape of the tooth when the pinion has a small number of teeth. It is readily seen in Fig. 1 that the pinion teeth with the long addendum are fully as strong as the gear teeth, while with the standard tooth they are not. This being the case, it is possible, in designing a

Table II. Diametral, Circular and Metric Pitches

Diametral Pitch	Circular Pitch	Nearest Metric Pitch or "Module"	Diametral Equivalent of "Module"	Circular Pitch Corresponding to Module
2¼	1.3962	11.0	2.309	1.3607
2½	1.2566	10.0	2.540	1.2370
2¾	1.1424	9.0	2.822	1.1133
3	1.0472	8.0	3.175	0.9896
3½	0.8976	7.0	3.628	0.8659
4	0.7854	6.0	4.233	0.7422
4½	0.6981	5.5	4.618	0.6803
5	0.6283	5.0	5.080	0.6185
5½	0.5712	4.5	5.644	0.5566
6	0.5236	4.0	6.350	0.4948

rear axle drive, to select a smaller number of teeth for the pinion than could otherwise be used; for instance, if the number of teeth previously used had been 17 and 54, the combination of 15 and 48 would give almost exactly the same ratio and would be fully as strong. With a $5\frac{1}{2}$-inch diameter pitch circle, the outside diameter of the large gear would be decreased somewhat over $\frac{1}{2}$ inch, thereby making the case that much smaller and lighter, and gaining advantages all around.

The disadvantage of using a small number of teeth in a pinion with the standard tooth has been the small amount of stock left between the bore of the shaft and the bottom of the tooth spaces. This is eliminated to a large extent by the decrease of the dedendum of the pinion tooth.

Field of Application. — The field of application is limited to gear sets having a large difference in the number of teeth in the gears and pinions, on account of the gear tooth becoming weaker as the number of teeth decreases. The gear having the lengthened addendum must at all times be the driver, as in reversing the application of power, the arc of recess becomes the arc of approach with its greater friction. This is of no account in the driving gears of a car, as in coasting no power is transmitted, except when there is a propeller-shaft brake.

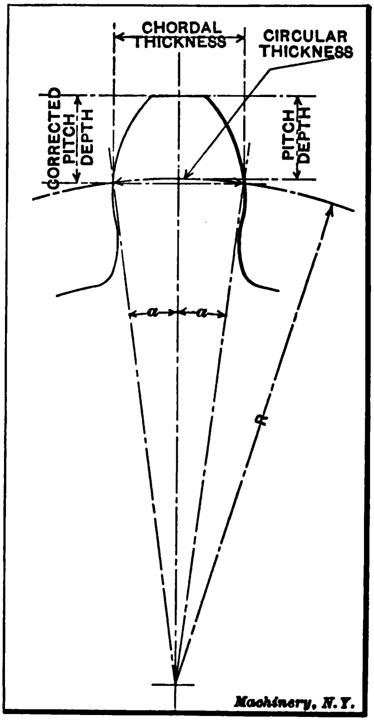


Fig. 2. Nomenclature of Gear Tooth

Efficiency. — One point which has not been touched on is the efficiency, and consequent life, of a gear of this system as compared with one of the ordinary or of the stub form of tooth. No exhaustive scientific tests to determine this have been made, to the author's knowledge, so it is a matter for further demonstration.

Finding Circular Thickness of Tooth. — The addendum of the pinion tooth, of the type described, is arbitrarily taken as 0.7 of its working depth, and 0.3 for the gear tooth, for both $14\frac{1}{2}$- and 20-degree pressure angles. To find the circular thickness of the tooth at the pitch line at these depths, multiply the circular pitch by 0.5659 for the pinion, and by 0.4341 for the gear for $14\frac{1}{2}$-degree pressure angle. For 20-degree pressure angle multiply the circular pitch by 0.5927 and 0.4073, respectively, for the pinion and gear.

The impossibility of getting accurate circular measurements necessitates the calculation of the chordal thickness and cor-

Table III. Formulas for Long Addendum Bevel Gears

Name	Symbol		Formula	
	Pinion	Gear	Pinion	Gear
Number of teeth	N_1	N_2	$N_1 = P_d \times D_1$	$N_2 = P_d \times D_2$
Diametral pitch	P_d	P_d	$P_d = \frac{N_1}{D_1} = \frac{N_2}{D_2}$	$P_d = \frac{N_1}{D_1} = \frac{N_2}{D_2}$
Circular pitch	P_c	P_c	Table II	Table II
Pitch diameter in inches	D_1	D_2	$D_1 = \frac{N_1}{P_d}$	$D_2 = \frac{N_2}{P_d}$
Pitch angle	p_1	p_2	$\tan p_1 = \frac{N_1}{N_2}$	$\tan p_2 = \frac{N_2}{N_1}$
Working depth	W	W	$\frac{2}{P_d}$	$\frac{2}{P_d}$
Addendum	A_1	A_2	$A_1 = 0.7 \times W$	$A_2 = 0.3 \times W$
Dedendum	E_1	E_2	$E_1 = A_2 + G$	$E_2 = A_1 + G$
Clearance	G	G	$G = P_c \times 0.05$	$G = P_c \times 0.05$
Full depth	F	F	$F = W + G$	$F = W + G$
One-half diameter increment	I_1	I_2	$I_1 = A_1 \times \cos p_1$	$I_2 = A_2 \times \cos p_2$
Outside diameter	O_1	O_2	$O_1 = D_1 + 2 I_1$	$O_2 = D_2 + 2 I_2$
Circular thickness	T_1	T_2	$T_1 = P_c \times 0.5927$	$T_2 = P_c - T_1$
Pitch cone distance	C	C	$C = \frac{D_1}{2 \times \sin p_1}$	$C = \frac{D_1}{2 \times \sin p_1}$
Back cone distance	B_1	B_2	$B_1 = C \times \tan p_1$	$B_2 = C \times \tan p_2$
Addendum angle	r_1	r_2	$\tan r_1 = \frac{A_1}{C}$	$\tan r_2 = \frac{A_2}{C}$
Dedendum angle	s_1	s_2	$\tan s_1 = \frac{E_1}{C}$	$\tan s_2 = \frac{E_2}{C}$
Face angle	t_1	t_2	$t_1 = p_1 + r_1$	$t_2 = p_2 + r_2$
Cutting angle	v_1	v_2	$v_1 = p_1 - s_1$	$v_2 = p_2 - s_2$
Distance from crown to pitch line	H_1	H_2	$H_1 = A_1 \times \sin p_1$	$H_2 = A_2 \times \sin p_2$
No. teeth in spur gear having diameter equal to twice the back cone distance	S_1	S_2	$S_1 = 2 \times P_d \times B_1$	$S_2 = 2 \times P_d \times B_2$
Chordal thickness	L_1	L_2	Table I	Table I
Corrected pitch depth	K_1	K_2	Table I	Table I

rected pitch depth. Referring to Fig. 2, let R equal the pitch radius for a spur gear or the back cone distance for a bevel gear.

$$\text{Angle } a = \frac{1}{2} \times \frac{360 \text{ degrees}}{(2\,R \times 3.1416) \div \text{Circular Thickness}}$$

Chordal thickness = $2 \times \sin a \times R$.

Corrected pitch depth = versed $\sin a \times R +$ the given pitch depth.

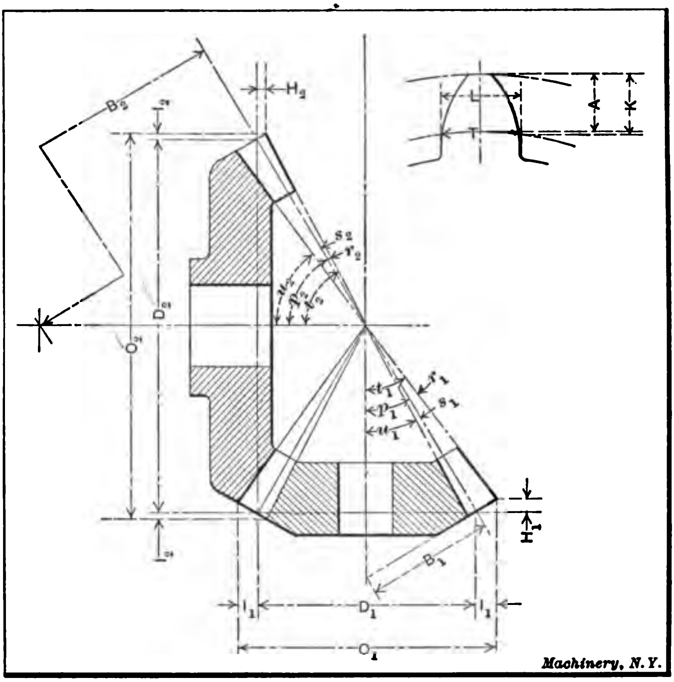


Fig. 3. Symbols used in Formulas in Table III

These values have been tabulated for gears with as large a range of tooth numbers as the system is applicable to with advantage. (See Table I.) The formulas required have also been arranged (in Table III) in the logical routine order for all necessary calculations for a pair of bevel gears of this system; symbols are as indicated in Fig. 3.

As some firms are using the metric pitch or "Module" system, Table II is given for their convenience.

Duplication of Gears having Long and Short Addenda. — The formula according to which the gear addendum is made 0.3 of the working depth, and the pinion addendum 0.7 of the working depth of the tooth, with the working depth equal to twice the standard addendum, was proposed by the Gleason Works and may be considered as standard for this class of gearing in this country. In the examination of foreign-made bevel-gear drives using this style of tooth, it has been found, as explained by Mr. E. W. Baxter, in the June, 1913, number of MACHINERY, that the ratio of the gear addendum to the pinion addendum varies with each set of gears examined, and in order to render the duplication of such gears easier, the following formulas and tables were evolved. In order to explain the method, we will assume that a set of bevel gears has been ordered to duplicate samples which are measured as carefully as possible and the dimensions tabulated as follows:

	Gear	Pinion
Number of teeth	48	13
Measured outside diameter	9.66 inches	3.14 inches
Measured face angle	$76\frac{1}{4}$ degrees	$18\frac{1}{3}$ degrees
Measured cutting angle	$71\frac{1}{4}$ degrees	$13\frac{1}{2}$ degrees
Measured depth of tooth	0.43 inch	0.43 inch

The number of teeth on the pinion being odd, the outside diameter is found by adding to the diameter of the bore, twice the distance from the edge of the bore to the tip of a tooth.

If we figure the outside diameters according to the method used for standard gears and find that the outside diameter of the pinion is larger and that of the gear smaller than the dimensions of the gears to be duplicated, we will know that this pair of gears has been designed with long and short addendum teeth. After a little experience it will not be necessary to figure the standard dimensions, as ordinarily an inspection of the pinion tooth is all that is necessary. By referring to Table IV, we find that the nearest standard pitches having whole depths around 0.43 inch are $\frac{5}{8}$-inch circular pitch (whole depth 0.429 inch) and 5 diametral pitch (whole depth 0.431 inch). We will therefore assume that the gears are 5 diametral pitch, as this will be the

easiest dimension to use in the calculations. The standard outside diameters for 48 and 13 tooth, 5 pitch gears are 9.704 inches and 2.986 inches, while the measured diameters are 9.66 inches and 3.14 inches. As the measured diameter for the gear is smaller and that for the pinion is larger than the standard dimensions, we may safely assume that the gears are cut with long and short addenda. As the center angles remain the same as in the standard gears (on account of the pitch diameters not changing), we have only to look them up in a handbook on gearing or calculate them from the following formula. Calling α the center angle of the gear:

$$\text{Tangent } \alpha = \frac{\text{Number of teeth in gear}}{\text{Number of teeth in pinion}}$$

Center angle, pinion = 90 degrees − α.

Calculating for 48 and 13 teeth, we have:

Tangent $\alpha = \frac{48}{13} = 3.69230$.

Center angle, gear = 74 degrees 51 minutes = α.

Center angle, pinion = 90 degrees − α = 90 degrees − 74 degrees 51 minutes = 15 degrees 9 minutes.

Tabulating the known quantities and representing those that are unknown by suitable letters, we have the following:

Number of teeth on gear	48
Number of teeth on pinion	13
Center angle of gear	74 deg. 51 min.
Cosine of center angle of gear	0.26135
Center angle of pinion	15 deg. 9 min.
Cosine of center angle pinion	0.96524
Measured outside diameter of gear	9.66 inches
Measured outside diameter of pinion	3.14 inches
Pitch diameter of gear	D
Pitch diameter of pinion	d
Addendum of gear (short)	X
Addendum of pinion (long)	Y
Standard addendum of gear and pinion or half the working depth	Z

Z also equals the pitch diameter of either gear divided by the number of teeth.

Pressure angle	$14\frac{1}{2}$ degrees

The pressure angle may be found by taking an impression of the backs of the large gear teeth on paper and measuring with a protractor the angle which the side of the tooth makes with a line drawn through the center of the tooth in the direction of its length.

The formula for finding the outside diameters of a pair of bevel gears when the addenda are unequal in gear and pinion is as follows: Pitch diameter + 2 (special addendum × cosine of center angle) = outside diameter.

Substituting the known and unknown quantities we get:

For the gear, $D + 0.5227\,X = 9.66$ inches = outside diameter . . . . (1)

For the pinion, $d + 1.93048\,Y = 3.14$ inches = outside diameter . . . . (2)

As the clearance and whole depths are the same for both gear and pinion in this style of gearing, the long addendum and the short addendum added together equal the working depth and also equal twice the "diameter addendum" $2\,Z$.

This gives us a third equation.

$$X + Y = 2\,Z \quad \text{or} \quad Z = \frac{X+Y}{2} \qquad (3)$$

As previously stated, Z is the standard addendum for the depth, and Z multiplied by the number of teeth in each gear gives the pitch diameters of the gears. Therefore we derive the following:

$$48\,Z = D \qquad (4)$$

$$13\,Z = d \qquad (5)$$

Substituting these values of D and d in Equations (1) and (2) we get:

$$48\,Z + 0.5227\,X = 9.66$$

Simplifying $$Z + 0.01089\,X = 0.20125 \qquad (6)$$

$$13\,Z + 1.93048\,Y = 3.14$$

Simplifying $$Z + 0.1485\,Y = 0.24154 \qquad (7)$$

Substituting the value of Z from Equation (3) in Equations (6) and (7) and simplifying we get:

$$\frac{X+Y}{2} + 0.01089\,X = 0.20125$$

$$X + Y + 0.02178\,X = 0.4025$$

$$1.02178\,X + Y = 0.4025 \quad \ldots\ldots \quad (8)$$

$$\frac{X+Y}{2} + 0.1485\,Y = 0.24154$$

$$X + Y + 0.297\,Y = 0.48308$$

$$X + 1.297\,Y = 0.48308 \quad \ldots\ldots \quad (9)$$

Multiplying Equation (8) by 1.297 and subtracting Equation (9) we get:

$$\begin{array}{rl} 1.32525\,X + 1.297\,Y & = 0.52204 \\ X + 1.297\,Y & = 0.48308 \\ \hline 0.32525\,X & = 0.03896 \\ X & = 0.11978 \text{ inch} = \text{addendum} \end{array}$$

for gear (short).

Substituting this value for X in Equation (9) we have:

$$\begin{array}{rl} 0.11978 + 1.297\,Y & = 0.48308 \\ 1.297\,Y & = 0.3633 \\ Y & = 0.28010 \text{ inch} = \text{addendum of pinion} \end{array}$$

(long).

We can now get the value of Z by substituting both X and Y in Equation (3).

$$\begin{array}{rl} 0.11978 + 0.28010 & = 2\,Z \\ 0.39988 & = 2\,Z \\ Z & = 0.19994 \text{ inch} = \text{standard addendum.} \end{array}$$

Z is probably 0.200 inch and the pitch for this depth is 5 diametral pitch.

$X = 0.120$ inch which is $\frac{3}{10}$ of 0.400 inch (whole depth)
$Y = 0.280$ inch which is $\frac{7}{10}$ of 0.400 inch

as per the Gleason formula.

Correction for Shrinkage in Hardening. — The large or ring gear is apt to shrink in hardening, the amount of this shrinkage depending upon the size of the gear, the kind of steel and the heat-treatment used. The shrinkage is not often more than 0.02 inch, but even this amount will affect the teeth of the gear by reducing the circular pitch, and if this shrinkage is not compensated for when turning and cutting the gear, the pinion will not mesh with the finished ring gear at the same place that it did

Table IV. Addenda for Diametral and Circular Pitches

(Whole depth = 2.157 × addendum)

Addendum in Inches	Whole Depth in Inches	Diametral Pitch	Circular Pitch		Metric Pitch Module	Addendum in Inches	Whole Depth in Inches	Diametral Pitch	Circular Pitch		Metric Pitch Module
			Fraction, Inches	Decimal, Inches					Fraction, Inches	Decimal, Inches	
0.125	0.270	8	...	....	...	0.174	0.375	5¾	6/11	....	...
0.126	0.272	..	...	....	3.2	0.175	0.378	..	...	....	...
0.127	0.274	..	...	0.4000	...	0.176	0.380	..	...	....	...
0.128	0.276	..	...	....	...	0.177	0.382	..	5/9	....	4.5
0.129	0.278	7¾	...	....	...	0.178	0.384	..	...	....	...
0.130	0.280	..	...	....	3.3	0.179	0.386	..	9/16	0.5625	...
0.131	0.282	..	...	....	...	0.180	0.388	..	...	....	...
0.132	0.285	..	...	....	...	0.181	0.390	..	...	....	4.6
0.133	0.287	7½	5/12	....	...	0.182	0.393	5½	4/7	....	...
0.134	0.289	..	...	....	3.4	0.183	0.395	..	...	....	...
0.135	0.291	..	...	....	...	0.184	0.397	..	...	....	...
0.136	0.293	..	3/7	....	...	0.185	0.399	..	...	....	4.7
0.137	0.295	..	...	....	...	0.186	0.401	..	7/12	....	...
0.138	0.298	7¼	...	....	3.5	0.187	0.403	..	...	....	...
0.139	0.300	..	7/16	0.4375	...	0.188	0.406	..	...	....	...
0.140	0.302	..	...	....	...	0.189	0.408	..	19/32	....	4.8
0.141	0.304	..	4/9	....	...	0.190	0.410	5¼	...	....	...
0.142	0.306	..	...	....	3.6	0.191	0.412	..	3/5	0.6000	...
0.143	0.308	7	...	....	...	0.192	0.414	..	...	....	...
0.144	0.311	..	...	....	...	0.193	0.416	..	...	....	4.9
0.145	0.313	..	5/11	....	...	0.194	0.419	..	...	....	...
0.146	0.315	..	...	....	3.7	0.195	0.421	..	...	....	...
0.147	0.317	..	...	....	...	0.196	0.423	..	...	....	...
0.148	0.319	6¾	...	....	...	0.197	0.425	..	...	....	5.0
0.149	0.321	..	7/15	....	...	0.198	0.427	..	...	....	...
0.150	0.324	..	...	....	3.8	0.199	0.429	..	5/8	0.6250	...
0.151	0.326	..	...	....	...	0.200	0.431	5	...	....	...
0.152	0.328	..	...	....	...	0.201	0.434	..	...	....	5.1
0.153	0.330	..	...	....	3.9	0.202	0.436	..	7/11	....	...
0.154	0.332	6½	...	....	...	0.203	0.438	..	...	....	...
0.155	0.334	..	...	....	...	0.204	0.440	..	...	....	...
0.156	0.337	..	...	....	...	0.205	0.442	4⅞	...	....	5.2
0.157	0.339	..	...	....	4.0	0.206	0.444	..	...	....	...
0.158	0.341	..	...	....	...	0.207	0.447	..	...	....	...
0.159	0.343	..	1/2	0.5000	...	0.208	0.449	..	...	....	...
0.160	0.345	6¼	...	....	...	0.209	0.451	..	21/32	....	5.3
0.161	0.347	..	...	....	4.1	0.210	0.453	4¾	...	....	...
0.162	0.349	..	...	....	...	0.211	0.455	..	...	....	...
0.163	0.352	..	...	....	...	0.212	0.457	..	2/3	0.6666	5.4
0.164	0.354	..	...	....	...	0.213	0.459	..	...	....	...
0.165	0.356	..	...	....	4.2	0.214	0.462	..	...	....	...
0.166	0.358	..	...	....	...	0.215	0.464	..	...	....	...
0.167	0.360	6	...	....	...	0.216	0.466	4⅝	...	....	5.5
0.168	0.362	..	...	....	...	0.217	0.468	..	...	....	...
0.169	0.365	..	17/32	....	4.3	0.218	0.470	..	...	....	...
0.170	0.367	..	...	....	...	0.219	0.472	..	11/16	0.6875	...
0.171	0.369	..	...	....	...	0.220	0.475	..	...	....	5.6
0.172	0.371	..	...	....	...	0.221	0.477	..	...	....	...
0.173	0.373	..	...	....	4.4	0.222	0.479	4½	...	....	...

Table IV. Addenda for Diametral and Circular Pitches. — (*Continued*)

(Whole depth = 2.157 × addendum)

Addendum in Inches	Whole Depth in Inches	Diametral Pitch	Circular Pitch: Fraction, Inches	Circular Pitch: Decimal, Inches	Metric Pitch Module	Addendum in Inches	Whole Depth in Inches	Diametral Pitch	Circular Pitch: Fraction, Inches	Circular Pitch: Decimal, Inches	Metric Pitch Module
0.223	0.481	..	7/10	0.7000	...	0.272	0.587	..	...	....	6.9
0.224	0.483	..	...	....	5.7	0.273	0.589	..	6/7	....	...
0.225	0.485	..	...	....	...	0.274	0.591	..	...	....	...
0.226	0.488	..	...	....	...	0.275	0.593	..	...	....	7.0
0.227	0.490	..	5/7	....	...	0.276	0.595	3 5/8	13/15	....	...
0.228	0.492	4 3/8	...	....	5.8	0.277	0.598	..	...	....	...
0.229	0.494	..	23/32	....	...	0.278	0.600	..	7/8	0.8750	...
0.230	0.496	..	...	....	...	0.279	0.602	..	...	....	7.1
0.231	0.498	..	8/11	....	...	0.280	0.604	..	...	....	...
0.232	0.500	..	...	....	5.9	0.281	0.606	..	...	....	...
0.233	0.503	..	...	....	...	0.282	0.608	..	...	....	...
0.234	0.505	..	...	....	...	0.283	0.611	..	8/9	....	7.2
0.235	0.507	4 1/4	...	....	...	0.284	0.613	..	...	....	...
0.236	0.509	..	...	....	6.0	0.285	0.615	..	...	....	...
0.237	0.511	..	...	....	...	0.286	0.617	3 1/2	9/10	0.9000	...
0.238	0.513	..	...	....	...	0.287	0.619	..	...	....	7.3
0.239	0.516	..	3/4	0.7500	...	0.288	0.621	..	29/32	....	...
0.240	0.518	..	...	....	6.1	0.289	0.623	..	10/11	....	...
0.241	0.520	..	...	....	...	0.290	0.626	..	...	....	...
0.242	0.522	4 1/8	...	....	...	0.291	0.628	..	...	....	7.4
0.243	0.524	..	...	....	...	0.292	0.630	..	11/12	....	...
0.244	0.526	..	...	....	6.2	0.293	0.632	..	...	....	...
0.245	0.529	..	...	....	...	0.294	0.634	..	...	....	...
0.246	0.531	..	...	....	...	0.295	0.636	..	...	....	7.5
0.247	0.533	..	...	....	...	0.296	0.639	3 3/8	...	....	...
0.248	0.535	..	7/9	....	6.3	0.297	0.641	..	14/15	....	...
0.249	0.537	..	25/32	....	...	0.298	0.643	..	15/16	0.9375	...
0.250	0.539	4	...	....	...	0.299	0.645	..	...	....	7.6
0.251	0.541	..	...	....	...	0.300	0.647	..	...	....	...
0.252	0.544	..	...	....	6.4	0.301	0.649	..	...	....	...
0.253	0.546	..	...	....	...	0.302	0.651	..	...	....	...
0.254	0.548	..	...	....	...	0.303	0.654	..	...	....	7.7
0.255	0.550	..	4/5	0.8000	...	0.304	0.656	..	...	....	...
0.256	0.552	..	...	....	6.5	0.305	0.658	..	...	....	...
0.257	0.554	..	...	....	...	0.306	0.660	..	...	....	...
0.258	0.557	3 7/8	...	....	...	0.307	0.662	..	...	....	7.8
0.259	0.559	..	13/16	0.8125	...	0.308	0.664	3 1/4	31/32	....	...
0.260	0.561	..	9/11	....	6.6	0.309	0.667	..	...	....	...
0.261	0.563	..	...	....	...	0.310	0.669	..	...	....	...
0.262	0.565	..	...	....	...	0.311	0.671	..	...	....	7.9
0.263	0.567	..	...	....	...	0.312	0.673	..	...	....	...
0.264	0.569	..	...	....	6.7	0.313	0.675	..	...	....	...
0.265	0.572	..	5/6	0.8333	...	0.314	0.677	..	...	....	...
0.266	0.574	..	...	....	...	0.315	0.680	..	...	....	8.0
0.267	0.576	3 3/4	...	....	...	0.316	0.682	..	...	....	...
0.268	0.578	..	...	....	6.8	0.317	0.684	..	...	....	...
0.269	0.580	..	27/32	....	...	0.318	0.686	..	1	1.0000	...
0.270	0.582	..	...	....	...	0.319	0.688	..	...	....	8.1
0.271	0.585	..	...	....	...	0.320	0.690	3 1/8	...	....	...

before the ring gear was hardened. If we suppose that this shrinkage was not taken into account and amounted to 0.02 inch, the measured outside diameter of the large gear would be 9.64 inches and that of the small gear 3.14 inches, no appreciable shrinkage being evident in the pinion on account of its small size. By substituting these values in the preceding formulas, we can find whether or not the correct dimensions can be found under these conditions. If this calculation is made, it will be found that the small amount that the ring gear will shrink will not alter the results enough to cause any serious error.

Results of Calculation. — From the above data, we can figure the correct face and cutting angles and outside diameters, using the measured values to check the calculations. The full data is tabulated below:

Long and Short Addendum Gears with $14\frac{1}{2}$-Degree Pressure Angle

Ratio, $3\frac{9}{13}$ to 1
Diametral pitch, 5
Working depth, 0.4000 inch
Whole depth, 0.4314 inch

	Gear	Pinion
Addendum	0.1200 inch	0.2800 inch
Dedendum	0.3114 inch	0.1514 inch
Face angle	76 deg. 14 min.	18 deg. 22 min.
Center angle	74 deg. 51 min.	15 deg. 9 min.
Cutting angle	71 deg. 16 min.	13 deg. 24 min.
Pitch diameter	9.600 inches	2.600 inches
Outside diameter	9.662 inches	3.140 inches
Difference between special and standard addendum angles	55 minutes	55 minutes
Difference between Bilgram system and standard addendum angles for the above ratio	59 minutes	59 minutes

CHAPTER XV

SKEW BEVEL GEARS

Characteristics of Skew Bevel Gears. — The skew bevel gear is a special form sometimes used to connect a pair of shafts which are not parallel and which do not intersect. Skew bevel gears have straight teeth which bear on each other along a straight line; a plane through the center of the tooth, however, intersects the axis of the gear instead of passing through the axis as in ordinary bevel gearing. The difficulty of producing correctly shaped teeth in skew bevel gears has been the most important reason why these gears are so little used, and, generally speaking, the spiral gear and worm gear appear to be the most practical solution of the problem of connecting by a single pair of gears two shafts which are not in the same plane. In the following, however, one method for producing skew bevel gear teeth will be described.

Skew Bevel Gear Tooth which can be Produced by the Molding-generating Process. — In the January, 1913, number of MACHINERY, Mr. J. M. Bartlett proposes a method for generating the teeth of skew bevel gears by the molding-generating process. This method is based on the assumption that Sang's theory holds true for skew gears as well as for spur and bevel gears. In a system of interchangeable involute spur gears, each gear will mesh perfectly with a rack the pitch surface of which is a plane, and the teeth of which can each be swept out by a single stroke of a straight-edged planing tool. In a system of interchangeable bevel gears (octoidal system), each gear will mesh perfectly with a crown gear the pitch surface of which is a plane disk, and the teeth of which can each be swept out by a single stroke of a straight-edged planing tool. In the proposed system of interchangeable skew gears, each gear will mesh perfectly with a "rack" the pitch surface of which is a "right helicoid," and the

tooth surfaces of which are hyperbolic paraboloids capable of being swept out by a single stroke of a straight-edged planing tool. The teeth in these gears do not vanish at the gorge, do not have to be undercut to avoid interference, are reversible, and give the same obliquity of action throughout their length. By using sections of the hyperboloids some distance from the gorge, drives of this type can be made more efficient than is possible with either worm or spiral gears.

Methods of Transmitting Power between Non-parallel, Non-intersecting Shafts. — The problem of transmitting rotary motion to non-parallel, non-intersecting shafts by means of toothed wheels has been solved in several ways, *viz.*, by spiral gears, by several types of worm-gears, and by skew bevel gears. While spiral gears possess the advantage of working perfectly when the center distance is slightly altered, or when either gear is shifted along the shaft, they have the disadvantage of rapid wear on account of point contact. A worm mating with a helical gear (straight faced) possesses the same advantages, but is also subject to the same disadvantages.

A straight worm mating with a hobbed wheel (the most common type) has line contact, and consequently better wearing qualities when properly mounted; but a slight displacement of the worm-wheel from its correct position along the shaft will seriously affect both the efficiency and durability of the drive. The Hindley worm and wheel, while theoretically giving line contact only, are so constructed that to the eye there appears to be surface contact. When properly mounted these, too, possess very good wearing qualities, but the slightest displacement of either worm or wheel, or of the shafts, will affect the correct working of the gears.

Types of Skew Bevel Gears. — To students of gearing problems, the solution by means of skew bevel, or hyperboloidal gears, has always seemed the logical one, although up to the present time no form of tooth has been found that possesses the qualities of strength, reversibility, and low pressure angle, and that can be accurately cut or planed. Three notable attempts to produce such gears are worthy of mention:

1. The epicycloidal system of Willis, in which the tooth surfaces are swept out by an element of a hyperboloid simultaneously rolling on the interior of one of the two pitch hyperboloids, and on the exterior of the other. This method of generating the teeth is exactly analogous to that used in connection with the epicycloidal system in spur gearing, but Prof. MacCord has shown that skew teeth produced in this way are not theoretically correct, since the tooth surfaces cannot be tangent to one another for all positions.

2. The Olivier involute system, in which the tooth surfaces are single curved surfaces known as Olivier spiraloids, or helical convolutes. In this system, the teeth vanish at the gorge, and hence it is necessary to form the gears from sections some distance from the gorge, but as we depart from it, the obliquity of action increases so rapidly that it is difficult to find a position where the teeth are satisfactory. Moreover, this method provides for the generation of only one side of the teeth. The backs must be formed by another method.

3. The Beale skew gear is a modified form of the Olivier gear, but is more practical, since the teeth do not vanish at the gorge. They are somewhat undercut, however, and like all the preceding forms are difficult to make.

It does not seem likely that any type of skew gear will ever be of more than theoretical interest, unless the teeth are of such a form that they can be accurately generated by the planing process. To discover this form of tooth, the natural method of procedure is to seek something that will bear the same relation to the hyperboloidal gear that the rack bears to the involute spur gear, or that the crown gear bears to the octoidal bevel gear. Then, assuming that Sang's theory is applicable to skew gears as well as to spur and bevel gears, a tooth section with straight sides can be chosen for this skew rack similar to those of the spur and bevel gear systems, and the teeth can be swept out by a straight-edged planing tool that is made to move in accordance with the proper geometric laws. It will then be true that all hyperboloidal gears that will mesh with this skew rack will mesh with each other.

The process of generating the teeth of a pair of hyperboloidal gears will then at once suggest itself, for we have only to cause our planing tool to pass along the tooth surface of an imaginary skew rack with a reciprocating motion, at the same time moving laterally at the end of each stroke to correspond with the motion

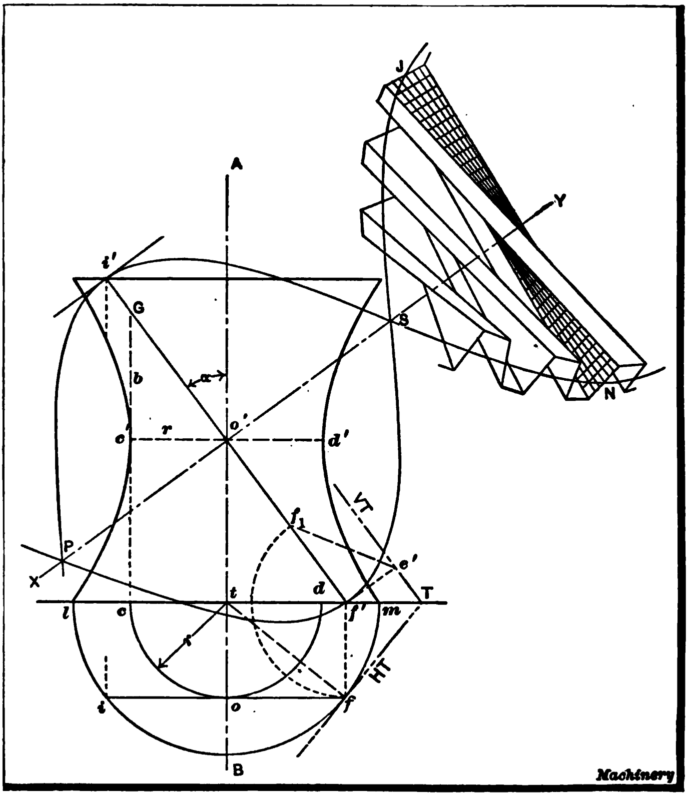


Fig. 1. Theory of Skew Bevel Gears and Requirements for Planing Conjugate Teeth

of the pitch surface of the rack as it rolls upon the pitch hyperboloid. The blank must also turn on its axis a corresponding amount at the end of each stroke, these motions continuing until one side of a tooth has been completed.

Solution of the Problem. — The nature of the skew rack is determined as follows: Let AB, Fig. 1, be the axis of a hyper-

boloid of revolution of one nappe, assumed as the pitch surface of a skew bevel gear. The horizontal projection of the circle of the gorge is *cod*, and the vertical projection, $c'o'd'$. Line $i'f'$ is one of the rectilinear elements of the surface, and is parallel to the plane on which the figure is projected. In this position it is the asymptote of the hyperbola which constitutes the contour lines of the surface. The angle α is the asymptote angle of the hyperbola, $o'c'$ is the radius of the gorge circle, and will be referred to as r; $c'G$ parallel to AB, will be referred to as b. The equation of the hyperbola referred to the axes AB and $c'd'$, origin at o', is:

$$\frac{x^2}{r^2} - \frac{y^2}{b^2} = 1$$

It can easily be proved that any two hyperboloids, so related that they can form the pitch surfaces of skew gears, will have the same value of b. Now $b = r \cot \alpha$. Hence in a set of interchangeable hyperboloidal gears, the product of the gorge radius and the cotangent of the asymptote angle is constant and equal to b, the modulus of the series. Through o, the intersection of $i'f'$ with the gorge circle, pass a line XY, perpendicular to $i'f'$, and parallel to the vertical plane of projection. At any two points on the element $i'f'$ equally distant from o, such as i' and f', draw lines perpendicular to $i'f'$ and tangent to the hyberboloid. The slope of these tangents with respect to a plane perpendicular to XY will be $\frac{b}{o'i'}$. Through i' and f' pass two helices the common axis of which is XY, and whose slope is the same as that of the tangents through i' and f'. These helices will then be tangent to the hyperboloid. Their pitch will be $2\pi b$.

Now let these two helices, and the line XY, be taken as the three directrices of a warped surface. The result will be a right helicoid, all elements of which will be perpendicular to XY. One of these elements is evidently $i'f'$. This helicoid is tangent to the hyperboloid all along the element $i'f'$ (since three common tangent planes can be passed at points on this element). If the helicoid is moved in the direction of XY, it will roll upon the hyperboloid while turning upon its own axis, and successive

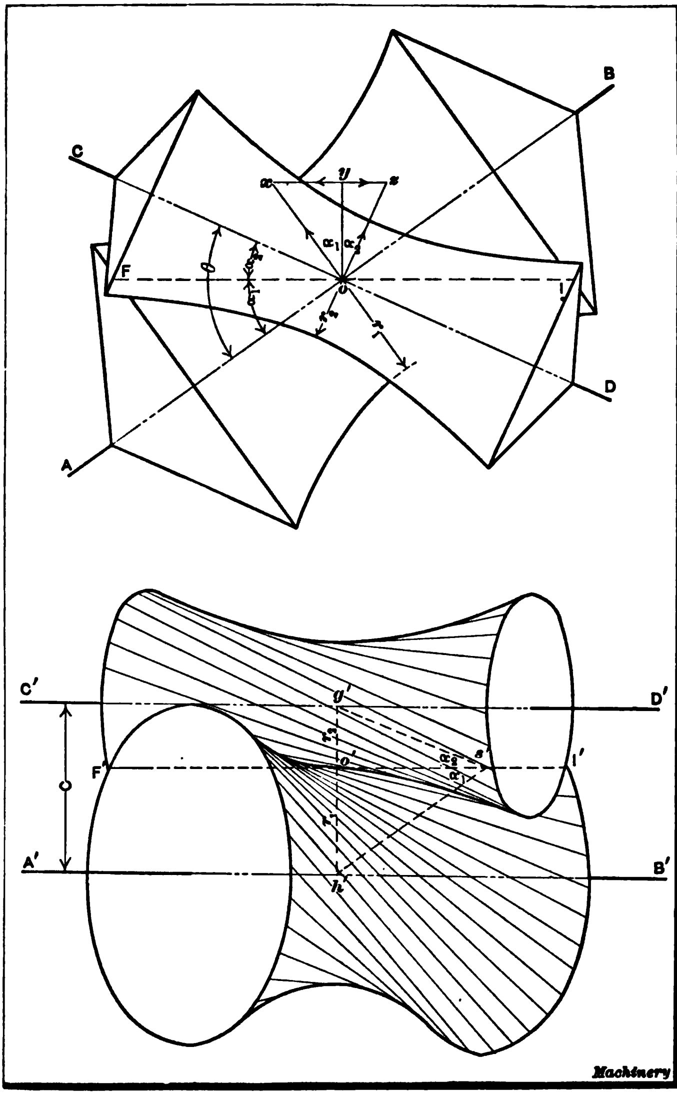


Fig. 2. Pitch Surfaces of Skew Bevel Gears showing Rectilinear Elements of Engagement

elements of the two surfaces will come in contact. The sliding motion that accompanies the rolling will be entirely in the direction of the element of contact.

If we conceive the mate to the hyperboloid of Fig. 1 to be

placed in contact with it along the element *IF* in Fig. 2, and a right helicoid to be constructed tangent to it along *IF*, this helicoid will be identical with the first one. It is then evident that the analogue of the pitch surface of the rack, for hyperboloidal gears, is a right helicoid the pitch of which is $2\pi b$. Now conceive a number of normals to the hyperboloid of Fig. 1 erected at points of the common element $i'f'$ or *IF* in Fig. 2. Each will cut *AB*, and also the axis of the mating hyperboloid. Each will be parallel to any plane perpendicular to *IF*. Hence these normals are elements of a hyperbolic paraboloid which is normal, not only to the two hyperboloids, but also to the right helicoid. By laying off equal distances from *I* and *F* along the normals through those points, and joining them by straight lines, the second generation of the hyperbolic paraboloid will be formed, the plane director of which is parallel to *IF* and *XY*.

Divide the axis *XY* of the helicoid into parts, each equal to the thickness of the proposed rack tooth, and through the several points of division pass rectilinear elements of the helicoid. Then through each of these elements pass a hyperbolic paraboloid normal to the helicoid (in the same way that the first one was passed through *IF*). Now, beginning at one end of the series, turn the first hyperbolic paraboloid, about the helicoidal element as an axis, through, say 15 degrees. Turn the next one through the same angle in the opposite direction, and thus alternate throughout the series. A limited portion of these surfaces, extending equal distances on both sides of the helicoid, will form the surfaces of rack teeth which are analogous in every respect to the rack teeth in the involute spur gear system. They fulfill the condition required by Sang's theory for a conjugating rack, *viz.*, that the faces and flanks be formed of four equal curves arranged in alternate reversion. Hence any two hyperboloidal gears that will mesh properly with this rack will mesh with each other. Three of these rack teeth are shown in Fig. 1.

If we imagine a planing tool the point of which is made to travel along the straight line at the root of the rack tooth, and whose cutting edge, at the same time, is made to turn in such a way that it constantly coincides with some rectilinear element

of the tooth surface (see the tooth at JN, Fig. 1), it only remains for us to supply the mechanism necessary to give the proper motions to the hyperboloidal blank and to the imaginary helicoidal rack, and we shall have a means of accurately generating conjugate teeth for skew gears. The teeth in these gears do not vanish at the gorge, as do those of Olivier and Herrmann. The obliquity of action is the same at points some distance from the gorge as it is at the gorge. The teeth do not have to be undercut to make them work at the gorge; hence they are not weakened near this point. The gears are reversible, and by using sections of the hyperboloids some distance from the gorge, drives of this type can be made more efficient than either worm or spiral gears.

Practical Application of Theory. — Gears produced according to the theoretical principle just outlined have been cut by Mr. Max Uhlmann, of Philadelphia, who states that the theoretical considerations of Bartlett are proved to be correct by the carrying out of the work in practice, except in regard to that part of the theory which relates to under-cutting. Owing to the lengthwise sliding or slip — if it may be so called — it might be expected that the tooth outline of the skew gear would differ from that of the corresponding bevel gear, and to bring out its peculiar features as clearly as possible, Mr. Uhlmann made a pair of gears which extended as near to the gorge circle of the hyperboloid as could be conveniently cut with his present facilities. The dimensions of these gears were as follows:

Largest pitch diameter = 2.649 inches.
Angle of asymptote = 30 degrees.
Number of teeth = 12.
Width of gear face = 1.25 inch.
Diameter of gorge circle = 0.875 inch.

From this data the pitch of the imaginary helicoidal rack figures out 4.761 inches.

The hyperboloidal form of the blanks was turned with the tool set half an inch above the center, and with the compound rest set to the same angle at which it would have been set for turning the corresponding bevel gear blank. The method of turning the blanks will be readily understood by reference to Fig. 3.

Although a pressure angle of 20 degrees was used, there is not only a considerable under-cutting — which is all on one side of the tooth — but the tool also cuts away material on the same side of the tooth at the top; the effect on the opposite side of the tooth is just the reverse so that it will be apparent that the tooth outline is decidedly unsymmetrical. From the study made of this subject, it is believed that this deviation from a symmetrical tooth outline will be most pronounced at some point near the

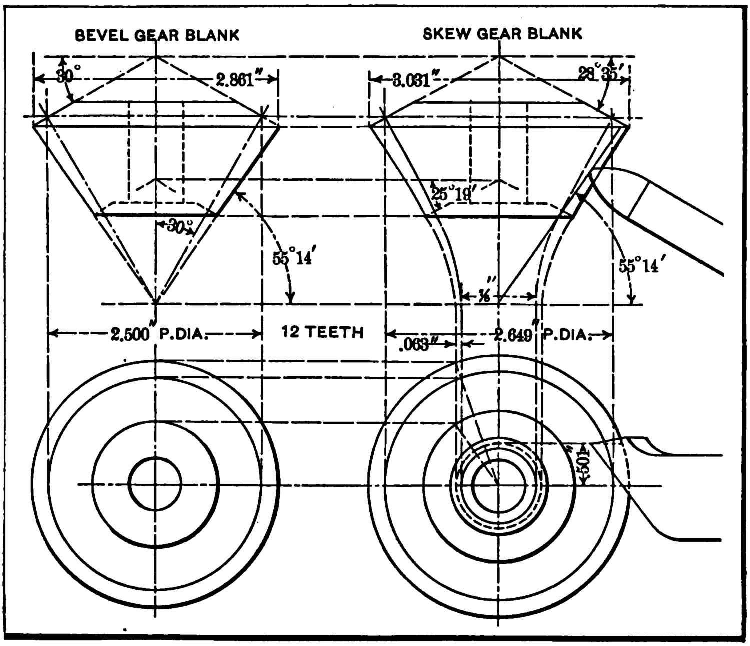


Fig. 3. Turning the Blanks for Skew Bevel Gears and Regular Bevel Gears

gorge circle, and that beyond the gorge circle these conditions reverse so that excessive under-cutting will then appear on the opposite side of the tooth.

In one case of skew bevel gears this excessive under-cutting has been avoided by changing the pressure angle on one side to 25 degrees, and simultaneously changing the pressure angle on the other side to 15 degrees. This produces a somewhat "leaning tooth," but what under-cutting there is on each side is only trifling in its amount.

Skew bevel gears find application in various kinds of machines, but nearly all of these gears are now cast from patterns, while even the best of cut gears of this type, which the trade can so far supply, are far from being good approximations. The method of generating teeth here outlined, should afford a means of producing skew bevel gears which, in point of accuracy, and consequently smooth action, would be equal to generated bevel gears. Where the distance between the shafts is great enough to make the gorge circle very large, it will usually be possible to use other forms of gears; but when this distance is only great enough to allow the shafts to pass each other, thus requiring the gorge circle to be small, as is frequently the case, skew bevel gears are the only form of transmission which will answer the purpose.

INDEX

Acute-angle bevel gearing, 213, 218
Addendum, 10, 16
  angular, 204
  corrected, 33
  for diametral, circular and metric pitches, 288
  gears with long and short, 122, 278, 282
  table, 22, 23
Addendum angle, 204
Angle, addendum, 204
  center, 200
  cutting, 204
  dedendum, 204
  edge, 204
  face, 204
  of pressure or obliquity, 9, 85
  pitch cone, 201
Angular addendum, 204
Annealing steel for gears, 171
Automobile transmission gears, production of, 167

Beale skew bevel gear teeth, 293
Bearing pressures of bevel gears, 231
Bevel gears, 200
  acute-angle, 213, 218
  bearing pressures, 231
  blanks, 236
  calculations, derivation of formulas, 207
  calculations, examples, 212
  crown, 203, 215, 219
  cutting the teeth in, 243, 261, 274
  definitions, 200, 202
  design, 224, 236
  drawings for, 229, 238
  generating machines for, 258
  horsepower transmitted, 227, 230
Bevel gears, internal, 203, 216, 219
  internal, how to avoid, 220
  materials for, 224
  milling the teeth, 261, 274
  milling the teeth, table of "set-over," 266
  miter, 212
  obtuse-angle, 214, 218
  planing, adjustable former for, 255
  right-angle, 210, 212
  rules and formulas, 200, 210
  simplified formulas for strength, 230
  skew, 291
  strength, 224
  teeth, cutters for, 222
  teeth, different systems for, 220
  teeth, filing, 269
  teeth, machines for cutting, 253
  teeth, special forms, 223
  teeth, table of "set-over" when milling, 266
  with long and short addendum, 127, 278, 282
  with long and short addendum, duplication of, 284
  with planed teeth, dimensioning drawings for, 241
Blank diameters, limits for, 36
Blanks for bevel gears, 236
Brass gears, 43
Broaching automobile gears, 172
Bronze gears, 43, 224

Casehardening gears, 49, 60, 175, 177, 184, 195
Cast-iron gears, 38, 40, 47, 224
  working stresses, 47, 52
Cast-steel gears, 38, 40, 225
Center angle in bevel gearing, 200

Center distance, 10, 15
manufacturing limits for, 36
Chart for power transmission by gearing, 82
Chordal pitch, 10
Chordal thickness, 10, 33
gears with long addendum, 279
Circular and diametral pitch, table, 22, 23
Circular pitch, 10, 12
Circular pitch tooth parts, 23
Clearance, 10, 17
of gears cut on gear shaper, 17
Cloth gears, 44
Corrected addendum, 33
Cost of heat-treated gears, 193
Crown gears, 203, 215, 219
Curve, cycloidal, 5
involute, 4
Cutters, for bevel gear teeth, 222
for spur gear teeth, 31
grinding, 31
Cutting angle in bevel gearing, 204
Cutting teeth, in automobile gears, 174
in bevel gears, 243, 261, 274
in spur gears, 136
Cycloidal system of gear teeth, 5, 85
comparison with involute systems, 6
for bevel gearing, 221
odontograph for, 27
used for racks, 6, 7

Dedendum, 10
angle, 204
Definitions, bevel gearing, 200, 202
Definitions of tooth parts, 9
Depth of tooth, 17, 22, 23
diametral and circular pitches, 22, 23
metric pitch, 288
working, 10
Describing-generating principle of gear cutting, 140
Design, bevel gears, 224, 236
spur gears, 129
Device for testing and measuring gears, 198
Diameter, outside, 10, 18
outside of blank, limits for, 36
Diameter, pitch, 10, 14
pitch, limits for, 36
root, 10
Diametral and circular pitch, table, 22, 23
Diametral pitch, 10, 12
relation to width of face, 56
tooth parts, 22
Diametral pitches in full size, 11
Dimensions, of spur gears, 21, 131
of bevel gears, 210
on bevel gear drawings, 229, 238, 241
on rack drawings, 135
on spur gear drawings, 133
Drawings, for bevel gears, 229, 238
for bevel gears with planed teeth, dimensioning, 241
for spur gears, 133, 134
properly dimensioned rack, 135
Drop-forgings for gears, 42, 170
Duplication of gears with long and short addendum, 284
Durability of gears, 65, 230

Edge angle in bevel gearing, 204
Efficiency of spur gears, 66
Epicycloidal curve, 5
Equivalent spur gear, 205

Face angle, 204
Face of gear tooth, 10
Face width, relation to diametral pitch, 56
Fellows gear shaper, clearance of gears cut on, 17
Fellows stub gear-tooth, 84, 94
Fiber gears, 44, 225
Filing bevel gear teeth, 269
Flank of gear tooth, 10
Flutes in hobs, number of, 152
straight or spiral, 153
Forgings for gears, drop, 42, 170
Formed cutters, for bevel gear teeth, 222
for spur gear teeth, 31
grinding, 31
Formed tool principle, for bevel gear cutting, 245
for spur-gear cutting, 138

Former for bevel gear planing, adjustable, 255
Friction wheels, 1

Gear-cutters, for bevel gears, 222
for spur gears, 31
grinding, 31
Gear-cutting machinery, classification, 136
for bevel gears, 253, 258
for spur gears, 145
Gear drawing, model bevel, 229, 238
model rack, 135
model spur, 133, 134
Gear shaper, clearance of gears cut on, 17
Gear teeth, comparison of involute and cycloidal, 6
cutters for milling, 31, 222
cutting bevel, 243, 261
cutting spur, 136
cycloidal, 5, 85
definitions of parts of, 9
dimensions, full size, 11
easing off the points, 74
filing bevel, 269
for bevel gearing, 220
hobbing, 147
involute, 4, 85
involute, variations in tooth shape, 156
machines for cutting bevel, 253, 258
machines for cutting spur, 145
metric or module system, 36, 280, 288
relation of strength to velocity, 67
rolling mill, 95
skew bevel, 293
special forms for bevel, 223
strength of bevel, 225
strength of spur, 51
strength of stub, 96, 99
stub, 84
stub and standard compared, 92
tables of parts of, 22, 23
tests of strength, 59
Gears, bevel, 200
bevel, cutting teeth in, 243, 261, 274
bevel, horsepower transmitted, 227, 230
Gears, bevel, materials for, 224
bevel, strength and design, 224, 236
brass, 43
bronze, 43, 224
cast iron, 38, 40, 47, 52, 224
cast steel, 38, 40, 225
cloth, 44
comparison of heat-treatments, 184
crown, 203, 215, 219
durability, 65, 230
efficiency, 66
equipment for heat-treating, 189
fiber, 44, 225
for machine tools, 46, 182
for rolling mills, 95
hardness testing, 50, 180
heat-treated, cost, 193
heat-treatment, 49, 60, 167, 175, 195
interchangeability of hobbed and milled, 155
internal bevel, 203, 216, 219
internal bevel, how to avoid, 220
internal spur, 24
limits for, 36
make-shift methods for avoiding noisy, 127
materials used for, 38
methods of production, 167
noisy, 108
price, 83
rawhide, 43, 62, 225
rawhide, horsepower transmitted, 63
running-in, 181
simplified formulas for strength, 75
skew bevel, 291
spur, design of, 129
spur, device for testing and measuring, 198
spur, dimensions, 21, 131
spur, horsepower transmitted, 53
spur, principles and dimensions, 1
spur, rules and formulas for, 21
steel, 42, 47, 224
steel used for, 49, 60
strength of, 51, 224
stub-tooth, 84
weight, 83

Gears, with long and short addendum, 122, 278, 282
with long and short addendum, duplication of, 284
Generating hob-tooth shapes, 160
Generating machines for bevel gears, 258
Generating principle, for cutting bevel gears, 248
for cutting skew bevel gears, 291
for cutting spur gears, 140, 142
Grant's odontograph, 27, 28, 30
Grinding gear cutters, 31

Hardening gears, 49, 60, 175, 184, 195
Hardening hobs, 154
Hardening methods, 195
Hardness testing of gears, 50, 180
Heat-treated gears, cost, 193
Heat-treatment of gears, 49, 60, 167
equipment for, 189
for machine tools, 182
methods, 175, 194
Heat-treatment of hobs, 154
Hob teeth, form and dimensions, 150
generating, 160
relief, 151, 154
Hobs for spur gears, designing, 150
heat-treatment, 154
master planing tool for, 164
number of flutes, 152
special, 158
straight or spiral flutes, 153
threading, 153
Hobbed and milled gears, interchangeability of, 155
Hobbing gear teeth, 147
Horsepower transmitted, by bevel gearing, 227, 230
by spur gearing, 53, 63, 82
Hypocycloidal curve, 5

Impact, variation in strength due to, 68, 70
Interchangeability of hobbed and milled gears, 155
Interference, a cause of noisy gearing, 110, 112
Interference, in internal gears, 26
Internal bevel gears, 203, 216, 219
how to avoid, 220
Internal spur gears, 24
chart for number of teeth, 121
interference, 26
rules for designing, 25
with shortened addendum, 125
Involute system of gear teeth, 4, 85
comparison with cycloidal system, 6
for bevel gearing, 221
for racks, 6, 7, 30
odontograph for, 30
variations in tooth shape, 156

Lewis formula, 51, 227
derivation, 57
factors for bevel gears, 226
factors for spur gears, 54
factors for stub-tooth gears, 106
Limits for gearing, 36
Long and short addendum gears, 122, 278, 282

Machine tool gears, 46, 182
Machines, classification of gear cutting, 136
for cutting bevel gear teeth, 253, 258
for cutting spur gear teeth, 145
Materials, for bevel gears, 224
for racks, 45
for spur gears, 38
Measuring gear teeth, 35
Measuring gears, device for, 198
Metric system of gear teeth, 36, 280, 288
Milling bevel gears, 261, 274
method of setting cutter, 274
table of "set-over," 266
Milling cutters, for bevel gear teeth, 222
for gear teeth, grinding, 31
for spur gear teeth, 31
Miter bevel gearing, 212
Module system of gear teeth, 36, 280, 288
Molding-generating principle, for cutting bevel gears, 248
for cutting spur gears, 142
for producing skew bevel gear teeth, 291

Noisy gearing, 108
make-shift methods for avoiding, 127
Number of teeth in gears, 19, 21
Nuttall system of stub gear-teeth, 94

Obliquity angle, 9, 85
Obtuse-angle bevel gearing, 214, 218
Octoid teeth for bevel gearing, 221
Odontograph, for cycloidal teeth, 27
for involute teeth, 30
Odontographic principle, for cutting bevel gears, 247
for cutting spur gears, 139
Oil for quenching gears, 192
Oil-hardened gears, 184
Olivier system for skew bevel gear teeth, 293

Phosphor-bronze gears, working stresses, 52
Pitch, chordal, 10
circular and diametral, 10, 12, 22, 23
diametral, in full size, 11
diametral, relation to width of face, 56
Pitch circle, 10
Pitch cone, 200
Pitch cone angle, 201
Pitch cone radius, 204
Pitch cylinders, 3
Pitch diameter, 4, 10, 14
in bevel gearing, 202
manufacturing limits for, 36
Pitch line, 3
Planing bevel gears, 250
adjustable former for, 255
Planing tools for hobs, 164
Power transmitted, by bevel gearing, 227, 230
by spur gearing, 51, 53, 82
Pressure angle, 9, 85
Pressure on bearings, bevel gears, 231
Price of gears, 83
Production of gears, 167

Quenching oil for gears, 192

Rack and pinion with shortened addendum, 125
Racks, materials for, 45
properly dimensioned drawing for, 135
straightening, 46
teeth, cycloidal, 6, 7
teeth, involute, 7, 30
Rawhide gears, 43, 225
allowable load, 62, 65
power transmitted, 63
strength, 62
with flanges, 64
Relief of hob teeth, 151, 154
Right-angle bevel gearing, 210, 212
Rolling mill gears, 95
Root diameter, 10
Running-in of gears, 181

Sang's theory, 291
Scleroscope test for gears, 50
Shaping bevel gear teeth, 250
Shocks, variation in strength due to, 68, 70
Short addendum gears, 122, 278, 282
Skew bevel gears, 291
Spur gear, equivalent, 205
Spur gears, 1
cutting teeth in, 136
device for testing and measuring, 198
design, 129
dimensions, 12, 21, 131
drawings for, 133, 134
durability, 65
efficiency, 66
for machine tools, 46, 182
hardness testing, 50, 180
heat-treated, cost, 193
heat-treatment, 49, 60, 167, 194
hobbed and milled, 147, 155
horsepower transmitted, 53
internal, 24
machines for cutting, 145
make-shift methods for avoiding noisy, 127
materials used for, 38
noisy, 108
price, 83
principles, 1
production, 167
rules and formulas, 21

Spur gears, running-in of, 181
  strength, 51, 75
  stub-tooth, 84
  weight, 83
  with long and short addendum, 122, 279
Steel castings for gears, 38, 40, 225
Steel, for automobile gears, 168
  for gears in machine tools, 183
  for spur gears, 49, 60
Steel gears, working stresses, 52
Straightening racks, 46
Strength of bevel gears, 224
Strength of gear teeth, 51, 225
  relation to velocity, 67
  simplified formulas, 75
  stub-tooth, 96, 99
  tests, 59
Stub-tooth gears, 84
  advantages, 101
  compared with standard, 92
  dimensions, 94
  factors for Lewis formula, 106
  strength, 96, 99

Teeth, chordal thickness, 10, 33
  comparison of involute and cycloidal, 6
  cutters for bevel gear, 222
  cutters for spur gear, 31
  cutting bevel gear, 243, 261
  cutting, in automobile gears, 174
  cutting spur gear, 136
  cycloidal, 5, 85
  cycloidal, for bevel gearing, 221
  depth, 17, 22, 23
  easing off the points, 74
  filing bevel gear, 269
  for bevel gearing, systems of, 220
  hob, form and dimensions, 150
  hobbing, 147
  involute, 4, 85
  involute, for bevel gearing, 221
  involute rack, 30
  involute, variations in tooth shape, 156
  machines for cutting bevel gear, 253, 258
Teeth, machines for cutting spur gear, 145
  measuring gear, 35
  metric or module system, 36, 280, 288
  milling the teeth of bevel, 261, 274
  number of, 19, 21
  octoid, for bevel gearing, 221
  odontograph for laying out, 27, 30
  rack, 6, 7
  relation of strength to velocity, 67
  shown in full size, 11
  skew bevel gear, 293
  special forms for bevel gear, 223
  strength, 51, 225
  strength of stub gear, 96, 99
  stub gear, 84
  stub gear, and standard compared, 92
  tests of strength, 59
  thickness of, 10, 18, 22, 23
  with shortened addendum, 122, 278
Templet principle, for cutting bevel gears, 246
  for cutting spur gears, 138
Templets for gear hobs, 161
Testing gears, device for, 198
Testing hardness of gears, 50, 180
Thickness of tooth, 10, 18, 22, 23
  chordal, 10, 33
  chordal, for gears with long addendum, 279
Threading hobs, 153
Thrust on bearings, bevel gears, 231
Tooth depth, for diametral, circular and metric pitches, 288
Tooth parts, definitions, 9
  tables, 22, 23
Tooth-shapes for hobs, generating, 160
Tooth thickness, 10, 18, 22, 23
Transmission gears, production of automobile, 167
Tredgold's approximation, 244

Velocity, relation to strength of gear teeth, 67
Vertex distance, 204

Weight of gears, 83
  Willis skew bevel gear teeth, 293
Working depth of teeth, 10, 22, 23